Identification of Volterra Kernels of Nonlinear Systems up to 3rd order

Hiroshi Kashiwagi and Sun Yeping

Faculty of Engineering, Kumamoto University, Kumamoto,860, Japan

Abstract A method for obtaining Volterra kernels of a nonlinear system by use of pseudorandom M-sequences and correlation technique, proposed by the authors in 1993, is further analysed and some applications for identifying nonlinear system having feedback loop are shown.

1 Introduction

Although the identification methods for linear system have been developed by many researchers, the methods for nonlinear system identification are quite few. The reason is that nonlinear systems are complex and difficult to be treated in general.
The authors(Kashiwagi *et al*, 1993a,1993b, 1994a) proposed a new method for obtaining not only the linear impulse response, but also Volterra kernels of nonlinear system simultaneously. A pseudorandom M-sequence, specially chosen beforehand, is applied to the nonlinear system, and the crosscorrelation function between the input and the output is calculated. Then the linear impulse response together with several crosssections of the Volterra kernels are obtained. This method for obtaining Volterra kernels of nonlinear system is further analysed and some applications to actual nonlinear system having feedback loop are shown.

2 Principle of the method

A nonlinear dynamical system is, in general, described as follows.

$$y(t)=\sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1,\tau_2,\cdots\tau_i) \times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2\cdots d\tau_i \qquad (1)$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1,\tau_2,...)$ is called Volterra kernel of i-th order.
When we take the crosscorrelation function between the input $u(t)$ and the output $y(t)$, we have,

$$\phi_{uy}(\tau)=\overline{u(t-\tau)y(t)} = \sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1,\tau_2,\cdots\tau_i) \times\overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)}d\tau_1 d\tau_2\cdots d\tau_i \qquad (2)$$

where $\phi_{uy}(\tau)$ is the crosscorrelation function of $u(t)$ and $y(t)$ and ——— denotes time average.
The difficulty of obtaining $g_i(\tau_1,\tau_2,\cdots\tau_i)$ from $\phi_{uy}(\tau)$ is , in general, due to the difficulty of getting $(i+1)$th moment of the input $u(t)$, because the n-th moment of the signal is very difficult to obtain for actual signals.
When we use an M-sequence as an input to the system, the n-th moment of $u(t)$ can be easily obtained by use of so-called "shift and add property" of the M-sequence. So we can obtain the Volterra kernels $g_i(\tau_1,\tau_2,\cdots\tau_i)$ from simply measuring the crosscorrelation function between the input and output of the nonlinear system.
The $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \qquad (3)$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 10, $1/N$ is smaller than 10^{-3}. So Eqn.(3) can be approximated as a set of impulses which appear at certain τ's.
Eqn.(3) is due to the so-called shift and add property of the M-sequence; that is, for any integer $k_{i1}^{(j)},k_{i2}^{(j)},\cdots k_{i,i-1}^{(j)}$ (suppose $k_{i1}^{(j)}<k_{i2}^{(j)}<\cdots,k_{ii}(j)$), there exists a unique $k_{ii}^{(j)}(modN)$ such that

$$u(t)u(t+k_{i1}^{(j)})u(t+k_{i2}^{(j)})\cdots u(t+k_{i,i-1}^{(j)})=u(t+k_{ii}^{(j)}) \quad (4)$$

where j is the number of a group $(k_{i1},k_{i2},\cdots,k_{i,i-1})$ for which Eqn.(4) holds. We assume that total number of those groups is m_i(that is, $j=1,2,\cdots,m_i$). Note that when $k_{ir}^{(j)}(r=1,2,...i)$ satisfy Eqn.(4), then $2^p k_{ir}^{(j)}$ also satisfy Eqn.(4) for any integer p. Therefore Eqn.(3) becomes unity when

$$\tau_1=\tau-k_{i1}^{(j)},\tau_2=\tau-k_{i2}^{(j)},\cdots\tau_i=\tau-k_{ii}^{(j)} \qquad (5)$$

Therefore Eqn.(2) becomes

$$\phi_{uy}(\tau)\simeq\sum_{i=1}^{\infty}\sum_{j=1}^{m_i} g_i(\tau-k_{i1}^{(j)},\tau-k_{i2}^{(j)},\cdots\tau-k_{ii}^{(j)}) \qquad (6)$$

Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)})$ in Eqn.(6) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$. If the $k_{ii}^{(j)}$ of i-th Volterra kernel g_i are sufficiently apart from each other(say, more than $50\Delta t$, where Δt is the time increment of the measurement time), we can obtain each Volterra kernel $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)})$ from Eqn.(6). Volterra kernels $g_i(\tau_1, \tau_2, \cdots \tau_i)$ are obtained as a set of crosssections along 45 degree lines in $(\tau_1, \tau_2, \cdots, \tau_i)$ space. In order for this to be realized, we have to select suitable M-sequence(As for the selection of M-sequence suitable for obtaining Volterra kernels, see Table 1 of the reference(Kashiwagi,1994a).

3 Measurement of Volterra kernels up to 3rd order

An example of obtaining Volterra kernels by this method is shown here.

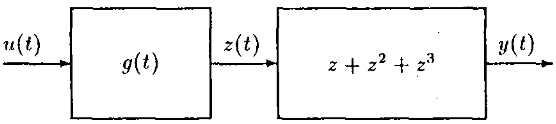

Figure 1: A nonlinear system having up to 3rd Volterra kernels.

The system to be identified is assumed to have up to third Volterra kernel which is actually realized as shown in Fig.1, where $g(t)$ is the impulse response of the linear part of the system.
Then the output $y(t)$ can be written as

$$\begin{aligned} y(t) &= z(t) + z^2(t) + z^3(t) \\ &= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 + \left\{\int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1\right\}^2 \\ &\quad + \left\{\int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1\right\}^3 \\ &= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 \\ &\quad + \int_0^\infty\int_0^\infty g(\tau_1)g(\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\ &\quad + \int_0^\infty\int_0^\infty\int_0^\infty g(\tau_1)g(\tau_2)g(\tau_3) \\ &\quad \times u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)\,d\tau_1\,d\tau_2 d\tau_3 \end{aligned} \tag{7}$$

Therefore Volterra kernels are as follows in this case.

$$\begin{aligned} g_1(\tau_1) &= g(\tau_1) \\ g_2(\tau_1,\tau_2) &= g(\tau_1)g(\tau_2) \\ g_3(\tau_1,\tau_2,\tau_3) &= g(\tau_1)g(\tau_2)g(\tau_3) \end{aligned} \tag{8}$$

When we take the crosscorrelation function between $u(t)$ and $y(t)$, we have

$$\begin{aligned} \phi_{uy}(\tau) &= \int_0^\infty g_1(\tau_1)\overline{u(t-\tau)u(t-\tau_1)}d\tau_1 \\ &+ \int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)}d\tau_1 d\tau_2 \\ &+ \int_0^\infty\int_0^\infty\int_0^\infty g_3(\tau_1,\tau_2,\tau_3) \\ &\times \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)}d\tau_1 d\tau_2 d\tau_3 \end{aligned} \tag{9}$$

The first term in Eqn.(9) becomes,

$$\begin{aligned} &\int_0^\infty g_1(\tau_1)\overline{u(t-\tau)u(t-\tau_1)}d\tau_1 \\ &= -\frac{1}{N}\int_0^\infty g_1(\tau_1)d\tau_1 \\ &+ \int_{\tau-\Delta t}^{\tau+\Delta t} g_1(\tau_1)\left\{\phi_{uu}(\tau-\tau_1) + \frac{1}{N}\right\}d\tau_1 \\ &\simeq \Delta t g_1(\tau) \end{aligned} \tag{10}$$

The second term in Eqn.(9) is,

$$\begin{aligned} &\int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)}d\tau_1 d\tau_2 \\ &= -\frac{1}{N}\int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)d\tau_1 d\tau_2 \\ &+2\sum_{j=1}^{m_2}\int_{\tau-k_{21}^{(j)}-\Delta t}^{\tau-k_{21}^{(j)}+\Delta t}\int_{\tau-k_{22}^{(j)}-\Delta t}^{\tau-k_{22}^{(j)}+\Delta t} g_2(\tau_1,\tau_2) \\ &\times\left\{\phi_{uuu}(\tau-\tau_1,\tau-\tau_2) + \frac{1}{N}\right\}d\tau_1 d\tau_2 \\ &\simeq 2(\Delta t)^2\sum_{j=1}^{m_2} g_2(\tau-k_{21}^{(j)}, \tau-k_{22}^{(j)}) \end{aligned} \tag{11}$$

where $\phi_{uuu}()$ is the third order correlation function of $u(\tau)$. In the same way, the third term in Eqn.(9) becomes,

$$\begin{aligned} &\int_0^\infty\int_0^\infty\int_0^\infty g_3(\tau_1,\tau_2,\tau_3) \\ &\times\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)}d\tau_1 d\tau_2 d\tau_3 \\ &= -2(\Delta t)^3 g_3(\tau,\tau,\tau) + 3(\Delta t)^3\sum_{q=1}^{N} g_3(\tau,q,q) \\ &+6(\Delta t)^3\sum_{j=1}^{m_3} g_3(\tau-k_{31}^{(j)}, \tau-k_{32}^{(j)}, \tau-k_{33}^{(j)}) \end{aligned} \tag{12}$$

Therefore Eqn.(9) becomes

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + (\Delta t)^3 g_3(\tau,\tau,\tau) \\ &+3(\Delta t)^3\sum_{q=1}^{m_1} g_3(\tau,q,q) \\ &+2(\Delta t)^2\sum_{j=1}^{m_2} g_2(\tau-k_{21}^{(j)}, \tau-k_{22}^{(j)}) \\ &+6(\Delta t)^3\sum_{j=1}^{m_3} g_3(\tau-k_{31}^{(j)}, \tau-k_{32}^{(j)}, \tau-k_{33}^{(j)}) \end{aligned} \tag{13}$$

In general case, we have,

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i \sum_{j=1}^{m_i} g_i(\tau-k_{i1}^{(j)}, \tau-k_{i2}^{(j)}, \cdots, \tau-k_{ii}^{(j)}) \end{aligned} \tag{14}$$

Here the function $F(\tau)$ is the sum of the odd order Volterra kernels when some of its argument are equal.

When we use the M-sequence having the characteristic polynomial of $f(x) = 207247$(in octal notation, 16 degree), k_{ij}'s in Eqn.(4) are

$$k_{21} = 655, k_{22} = 658, k_{31} = 735, k_{32} = 738, k_{33} = 741$$

Therefore

$$\begin{aligned}\phi_{uy}(\tau) &= \Delta t g(\tau) + F(\tau) + 2(\Delta t)^2 g_2(\tau - 655, \tau - 658) \\ &+ 6(\Delta t)^3 g_3(\tau - 735, \tau - 738, \tau - 741) + \cdots (15)\end{aligned}$$

Fig. 4 shows an example of the simulation results, when the linear part of the system is of second order with $\zeta = 0.3, \omega_n = 1.0$, where ζ is the damping ratio and ω_n is the natural angular frequency. The linear impulse response is clearly seen for $\tau < 100$, and the 2nd Volterra kernel $g_2(\tau - 655, \tau - 658)$ is obtained from $658 < \tau < 710$ and also $g_3(\tau - 735, \tau - 758, \tau - 741)$ is obtained from $735 < \tau < 770$. In Fig. 4 , o indicate the simulation result and solid line shows the theoretical result, showing a good agreement with each other.

4 Applications

This method of Volterra kernel identification is applied to several actual nonlinear systems. The first one of the application is to a nonlinear system having relay type nonlinearity with dead band as shown in Fig.2. Fig.5 shows the comparison of the actual output(solid line) and the estimated output(denoted as o) which was calculated by use of the measured Volterra kernel.

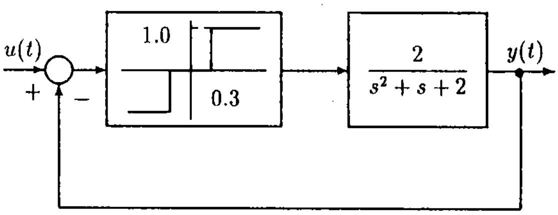

Figure 2: A nonlinear system having relay type with dead band

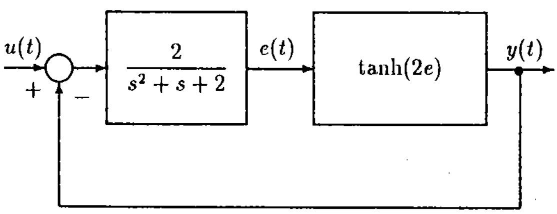

Figure 3: A nonlinear system having tanh-type nonlinearity

Second application of this method is to a nonlinear system having tanh-type nonlinearity with feedback loop as shown in Fig.3.

Fig.6 shows the comparison of the actual output and the estimated output. We see a good agreement between them, showing the validity of the method for nonlinear identification.

5 Conclusion

A method for obtaining Volterra kernel of nonlinear system by use of pseudorandom M-sequence(Kashiwagi 1993a, 1993b, 1994a) is further analysed. A specially chosen M-sequence is applied to the nonlinear system to be identified, and the crosscorrelation function between the input and the output gives us not only the linear impulse response of the linear portion of the system, but also some crosssections of the Volterra kernel $g_i(\tau_1, \tau_2, \cdots \tau_i)$ along some 45 degree lines in $(\tau_1, \tau_2, \cdots \tau_i)$ space.

This method for obtaining Volterra kernel is simulated on the computer for nonlinear systems having up to third order Volterra kernels. And the method is applied to some actual nonlinear system having dead band type nonlinear element with feedback loop.The results show that this method is effective for nonlinear system identification.

6 References

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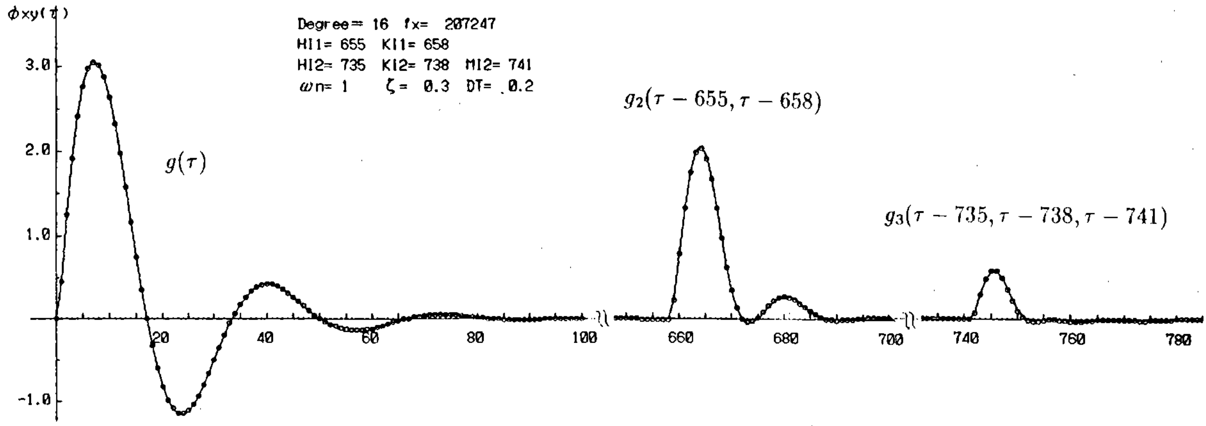

Figure 4: An example of the simulation results showing the crosscorrelation function

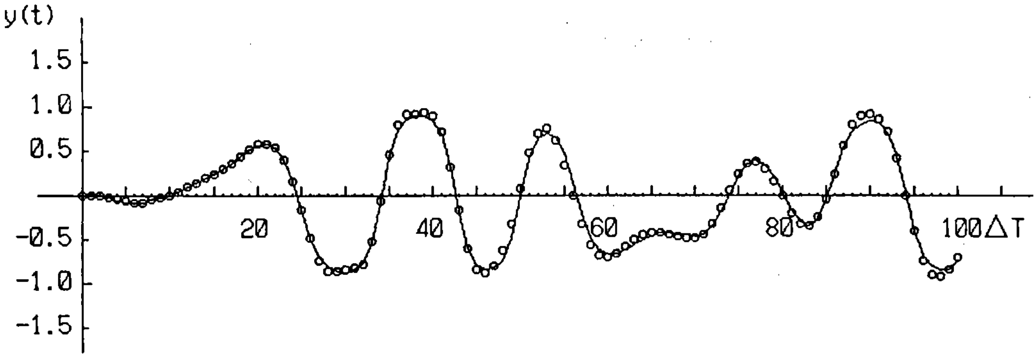

Figure 5: Comparison of the actual output with the estimated one for Figure 2.

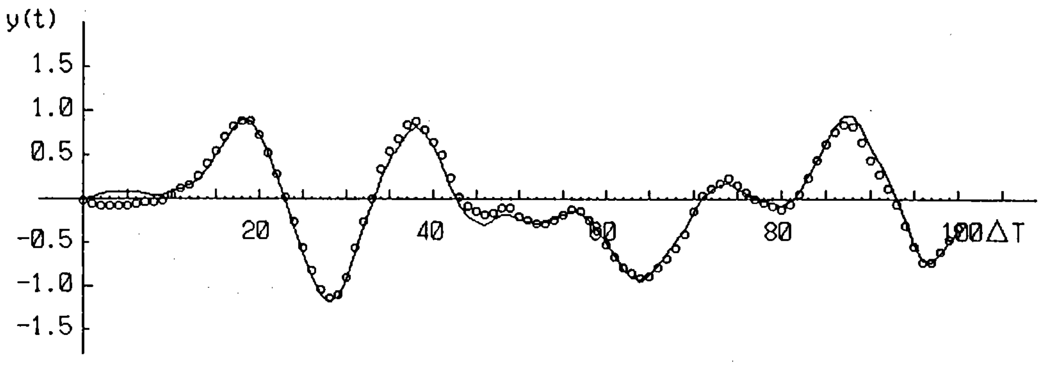

Figure 6: Comparison of the actual output with the estimated one for Figure 3.