Variable Structure Control of Chaotic Systems

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Abstract

To prevent the stable states from the complex dynamics, the global behavior of the overall system must be considered. Thus, indirect adaptive scheme might result in needless responses. Discrete-time variable structure controllers for a well-known logistic map are designed for two different sliding hyperplanes. Impulse disturbances are fully rejected by the virtue of discrete-time variable sturcture control (DVSC). A numerical example is given to illustrate the effectness of the DVSC.

1. Introduction

Chaos is an ubiquitous and robust nonlinear phenomenon which permeates all fields of science. Roughly speaking, chaos is a more exotic form of steady-state response. It can be seen only in nonlinear systems. The existence of chaotic dynamics in systems which by their very nature are nonlinear poses serious problems for their control. In recent years, much effort has been devoted to the problem of obtaining effective controllers for chaotic systems. As mentioned in [5], various techniques for supressing chaotic oscillations have been suggested. From the most primitive concepts like parameter variation through classical controller applications, to quite sophisticated one like stabilization of unstable periodic orbits embedded within the strange attractor, methods in various aspects are dealt with.

In control engineering approaches, there are feedback control, feedforward control, adaptive control and so on. Chen *et. al.*[2] propose a class of continuous-time linear feedback controllers for a well-known duffing equation. An elaborated proofs for the stability and the convergence property are also sited therein. Indirect adaptive schemes for controlling chaotic systems are popular in these days but these schemes do not give a feasible solution to control, for the complexities are arising in their inherent algorithm like in [6]. Indirect adaptive schemes are compose of three parts, plant to be controlled, adaptation algorithm to estimate the parameters and control law to drive the system to a desired performance. Each of their algorithms is designed to be stable, however, since the estimated paremeters may cause addisional complex dynamics to the overall system, it cannot serve as a perfect solution for the control of chaotic systems. While indirect methods are easy to fail, Lyapunov direct methods or Lyapunov min-max (generally min-sup) controllers [3] can give a feasible solution because it is based on the stability of the overall system, i.e. a Lyapunov function from which all of the adaptation and/or control laws are derived.

Variable Structure Control (VSC) is a typical example of robust control. Changing its structure and thus the contolled process, external disturbances or uncertainties are fully rejected. For continous-time VSC, so many studies are accomplished in the literature. In spite that the continous-time VSC marches on without the end, discrete-time VSC (DVSC) has just started. In this paper, a DVSC of a chaotic system is studied. Though we can say the robustness of DVSC only for impulse disturbances, this paper might provide a start point to a DVSC of chaotic systems as well as any other nonliner systems.

The rest of this paper consists of the following. In section 2, the main shortcoming of indirect adaptive scheme and the previous studies on Lyapunov min-max controllers are addressed. The backgroud of DVSC and the design of DVSC System for logistic map are outlined, and the whole system is simulated in section 3. Finally, we draw some conclusions and discuss about the proposed method.

2. Previous Researches

In general indirect adaptive control system, the parameters of the model are recursively estimated based on the inputs and outputs of the process. Thus, by predicting the expected outcome from the process model, control signal is calculated in such a way that the error between the desired output and the actual output is minimized. Qammar *et. al.* [6] proposed an indirect adaptive controller equipped with quantized least mean square (QLMS) algorithm to control a modified logistic map. Two interesting features are shown in that. One is a projection in the 2-dimensional parameter space of the basin of attraction for the steady state, to show the sensitivity to initial condition. The basin has fractal boundaries with a dimension of approximately 2.5. The other is so called 'average preserving property'. For each stable sequence the mean of the output is equal to the reference point. Thus, the output of the system tracks the reference point from a statistical point of view. Huberman *et. al.* [4] analyzed the dynamics of indirect adaptive systems. On analyzing a certain class of adaptive systems, they showed a relaxation chaos, i.e. a chaotic burst. These complexities are the main drawbacks of the indirect adaptive control of chaotic systems.

The problem of obtaining stabilizing memoryless state feedback controllers for a class of uncertain systems described by difference equations are proposed in [3]. Consider an uncertain discete-time systems of the form

$$x(k+1) = F(k, x(k), u(k)), \qquad (1)$$

subject to

$$u(k) = p(k, x(k)) \qquad (2)$$

namely

$$x(k+1) = F(k, x(k), p(k, x(k))) \qquad (3)$$

Suppose that a Lyapunov candidate given by

$$V(x) = x^T P x \qquad (4)$$

where P in symmetric and positive-definite, is considered for the stability of (3) about zero.

Along any solution of (3),

$$V(x(k+1)) = W(F, k, x, p(k, x(k))) \qquad (5)$$

where

$$W(F, k, x, u) := V(F(k, x, u)) \qquad (6)$$

Letting

$$W^*(k, x, u) := \sup\{W(F, k, x, u)\} \qquad (7)$$

a Lyapunov min-sup controller, p is defined if and only if,

$$W^*(k, x, p(k, x)) = \min\{W^*(k, x, u)\} \qquad (8)$$

for all k and x.

In this approach, because the stability is gauranteed under certain conditions, the complexities as in indirect adaptive systems do not appear.

3. Design of Discrete-Time Variable Structure Control Systems

The problem of inducing convergent quasi-sliding regimes on smooth state-space surfaces of nonlinear single-input single-output (SISO) discrete-time controlled system is addressed in [7]. An extention of the notion of relative degree is used. The relative degree determines the time delay undergone by the input signals before they influence the output of the system. The main result of this work is that if the considered system have relative degree 1, there exists a control law which induces a quasi-sliding regime. However, if the definition of convergent sliding regime is defined to be

$$s(k)\,(\,s(k+1) - s(k)\,) < 0 \qquad (9)$$

, which is naively derived from the continuous-time case, the controlled motions are unstable about $s(k) = 0$. A convergent sliding regime is said to exist, if for all k

$$|\,s(k+1)\,| < |\,s(k)\,| \qquad (10)$$

Utilizing the above definition of the sliding mode, Aly *et. al.* [1] suggest a design method of discrete-time variable structure systems (DVSS). Their main study issues are the reachability conditions developed for SISO DVSS, stability conditions of a sliding mode and a modified algorithm to simplify the design procedure for discrete-time linear systems. A method to reduce the chattering along the sliding mode and an investigation of the robust property for the case of system matrix uncertainties are described in [8]. So far, the robustness of uncertainties in discrete-time nonlinear systems are not fully developed. It's because of not only the nonlinearity but also the discrete nature of the system.

As an illustrative example, we consider a well-known logistic map which is described by the following difference equation,

$$x(k+1) = \lambda\, x(k)\,(\,1 - x(k)\,) \qquad (11)$$

for $\lambda \in [0, 4]$ and $x(k) \in [0, 1]$.

As the control parameter, λ is increased, the system undergoes a period-doubling mechanism and finally reaches a chaotic regime. Based on the convergent sliding mode in eq. (10), a DVSC is designed for the system,

$$x(k+1) = \lambda\, x(k)\,(\,1 - x(k)\,) + u(k) \qquad (12)$$

For a point sliding hyperplane, $s(k)$ is defined as

$$s(k) = x(k) - x_d \qquad (13)$$

where x_d is a desired goal dynamic, which can be a time-varying.

From the condition, (10),

$$|\,s(k+1)\,| = |\,x(k+1) - x_d\,| < |\,s(k)\,| \qquad (14)$$

Since the control input takes the following form in general,

$$u(k) = u^{EQ} + u_s(k) \qquad (15)$$

where u^{EQ} is an equivalent control input and $u_s(k)$ is an auxiliary term depending on $s(k)$, $u_s(k)$ is expressed as the following,

$$u_s(k) = \varepsilon\,|s(k)|,\ \ |\varepsilon| < 1 \qquad (16)$$

while u^{EQ} is calculated from

$$|s(k+1)| = |s(k)| \qquad (17)$$

For a line sliding hyperplane, $s(k)$ is defined as

$$s(k) = x(k) - x_d + a\,(x(k-1) - x_d),\ \ |a| < 1 \qquad (18)$$

From exactly the same procedure for a point sliding hyperplane, one can obtain the expression of the control input signal. Simulation results of the overall system for a line sliding hyperplane is depicted in fig. 1. The controller is activated at time index $k = 200$ after which the orbit of the process settles to the desired point after a brief transient. Impulse disturbances, $d = 0.5$ and $d = -1$ are applied at time index $k = 300$ in fig. 2 and fig. 3, respectively. In both cases, after a short trasient, the equilibrium states are recovered successfully. As far as the magnitude of the control signal is admitted, a larger disturbances can also be rejected. Step disturbances, however, are hard to be rejected for the same design of the sliding hyperplane. Some modification algorithm must be needed to overcome the step disturbances or uncertainties.

4. Conclusions

To prevent the stable states from the complex dynamics resulting from the addition of the other nonlinear states, the global behavior of the overall system which includes the controller must be considered. Therefore, if one want to design a controller for suppressing chaotic phenomena, a Lyapunov direct method or a Lyapunov min-sup controller is preferable. Indirect adaptive scheme might result in needless responses.

Discrete-time variable structure controllers for a well-known logistic map are designed for two different sliding hyperplanes. Impulse disturbances are fully rejected by the virtue of DVSC, but for any other disturbance rejection a discrete sliding surface design technique of integral augmented type must be needed. The problem of obtaining robust DVSC, possibly of integral augmented type will be an interesting research topic.

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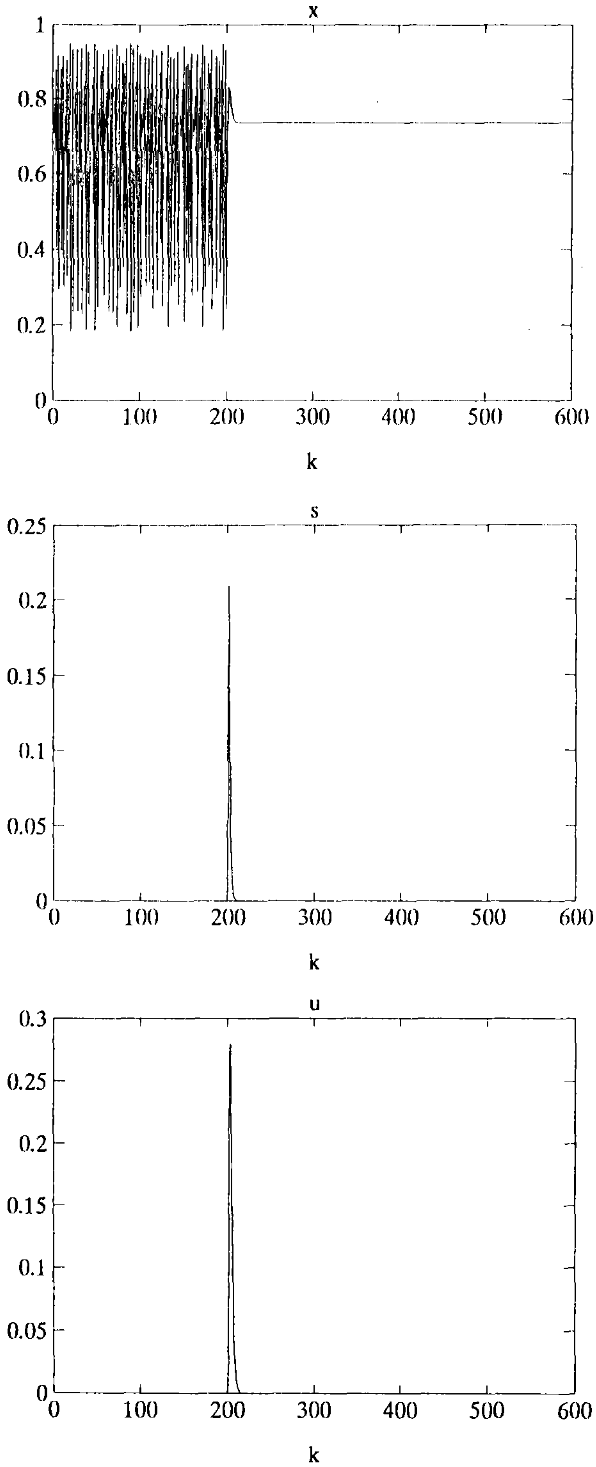

Fig. 1. The output x, sliding function s and control input u with $\lambda=3.8$, $a=-0.5$, $\varepsilon=0.5$, $x_d=0.737$ and $d=0$.

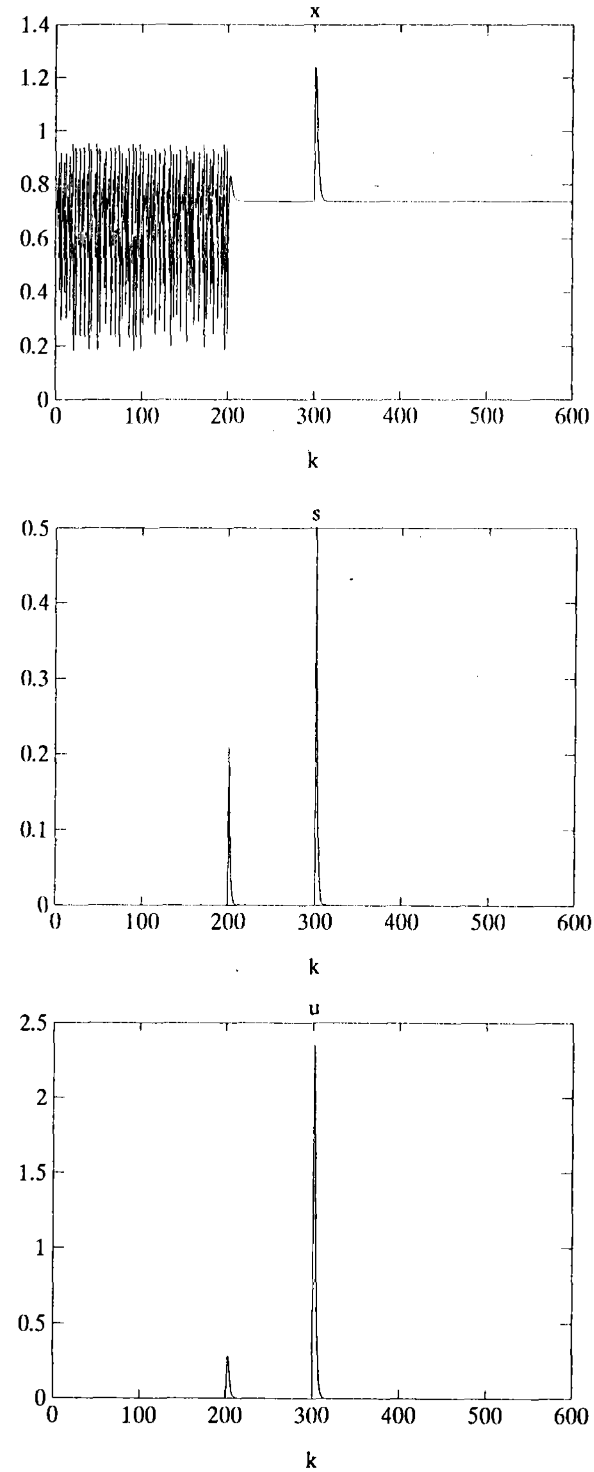

Fig. 2. The output x, sliding function s and control input u with $\lambda=3.8$, $a=-0.5$, $\varepsilon=0.5$, $x_d=0.737$ and $d=0.5$.

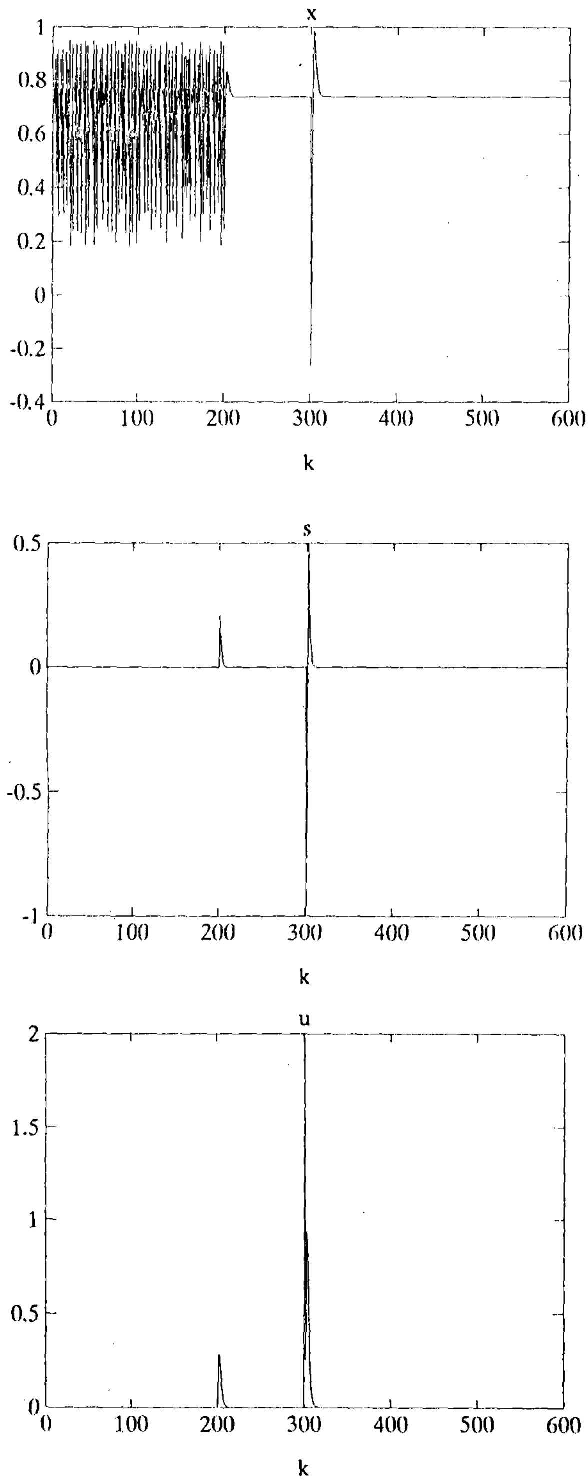

Fig. 3. The output x, sliding function s and control input u with $\lambda=3.8$, $a=-0.5$, $\varepsilon=0.5$, $x_d=0.737$ and $d=-1$.