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CONSTRUCTION OF A ROBUST COMPENSATION CONTROLLER

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Abstract

In this paper a new controller is proposed which gives the resultant system the appointed input-output properties, low sensitivity and robust stability. The proposed controller consists of a reference model and a robust compensator. The reference model determines the input-output properties of the total system and is constructed by using the nominal model of the plant. We can design the reference model by applying design techniques which pay attention to steady robustness and no attention to sensitivity and robust stability, and need all state variables of the plant. The robust compensator is obtained as a solution of the mixed sensitivity problem in H infinity control theory. Therefore, low sensitivity and robust stability are guaranteed in the resultant system. The simulation experiments show that the proposed controller is effective and useful.

1 Introduction

It is required in practical control system design methods that the appointed input-output transmission properties, low sensitivity and robust stability can be realized in the resultant control system.

In this paper we propose a new controller called a robust compensation controller [**RCC**] which can realize these. The **RCC**, which is based on the principle of making a plant follow a reference model, consists of the reference model and a robust compensator. The reference model determines the input-output properties of the resultant system, which is called the robust model following system [**RMFS**], and is constructed by using the nominal model of the plant. The robust compensator is obtained as the solution of the mixed sensitivity problem in H infinity control theory and realizes low sensitivity and robust stability in the **RMFS**. Therefore when we design the reference model, we can utilize system design methods which pay attention to steady robustness and no attention to sensitivity and robust stability, and need not only available state variables but also all state variables of the plant.

2 Basic robust model following system

It is assumed that the plant has multiplicative uncertainty and its input-output pulse transfer matrix is given by $G_0(z)\{I + \Delta(z)\}$, where $\Delta(z)$ expresses uncertainty in the plant. We consider the system shown in Fig.1. Eq.(1) can be derived easily from an inspection of the system in Fig.1.

$$F(z) = -G_0(z)\{I + \Delta(z)\}J(z) - G_0(z)\Delta(z)R(z) \tag{1}$$

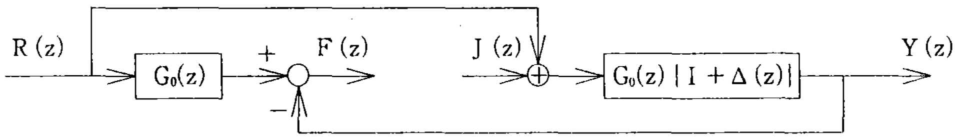

Fig.1 A basic system configuration

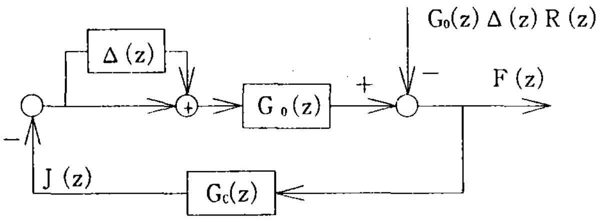

Fig.2 A considered feedback system

We can construct the feedback system shown in Fig.2 by regarding *J(z)* as the input, *F(z)* as the output and $G_0(z)\Delta(z)R(z)$ as the disturbance.

The pulse transfer matrix from the disturbance $G_0(z)\Delta(z)R(z)$ to the output *F(z)* can be obtained as

$$\left[I + G_0(z)\{I + \Delta(z)\}G_c(z)\right]^{-1} \quad (2)$$

So we define the sensitivity function matrix as

$$S(z) = \left[I + G_0(z)G_c(z)\right]^{-1} \quad (3)$$

On the other hand, the complementary sensitivity function matrix *T(z)* can be described as

$$\begin{aligned} T(z) &= I - S(z) \\ &= \left[I + G_0(z)G_c(z)\right]^{-1} G_0(z)G_c(z) \end{aligned} \quad (4)$$

It is assumed that uncertainty $\Delta(z)$ satisfies the following inequality

$$\|\Delta(\varepsilon^{j\omega T})\| \le \|\ell(\varepsilon^{j\omega T})\| \qquad 0 \le \omega \le \frac{2\pi}{T} \quad (5)$$

Then it is known [1] that if Eq.(6) is satisfied, stability of the feedback system in Fig.2 is guaranteed in spite of existence of uncertainty $\Delta(z)$.

$$\|T(\varepsilon^{j\omega T})\| < \|\ell(\varepsilon^{j\omega T})^{-1}\| \qquad 0 \le \omega \le \frac{2\pi}{T} \quad (6)$$

In other words, the system is of robust stability. Therefore, when we obtain the compensator $G_c(z)$ as a solution of the mixed sensitivity problem in H infinity control theory [2],[3] under the performance index of Eq.(7) with appropriate weighting functions $W_S(z)$ and $W_T(z)$, low sensitivity and robust stability are guaranteed in the system shown in Fig.3, which is called a basic robust model following system.

$$\left\| \begin{matrix} W_S(z)S(z) \\ W_T(z)T(z) \end{matrix} \right\|_\infty \quad (7)$$

Consequently, we can regard the pulse transfer matrix from *R(z)* to *Y(z)* as $G_0(z)$ in Fig.3 because of low sensitivity.

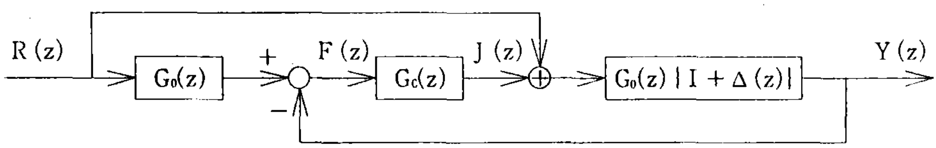

Fig.3 A basic robust model following system

3 Robust compensation controller

A general system configuration which is designed by paying attention to only input-output properties and steady robustness and no attention to uncertainty in the plant can be shown in Fig.4. We consider this system as a reference model of which input -output pulse transfer matrix is $G_m(z)$.

We can construct the system shown in Fig.5 by uniting the two systems in Fig.3 and Fig.4, which is called a robust model following system[**RMFS**]. It follows easily that the **RMFS** is of low sensitivity and robust stability because we can obtain Eqs.(3) and (4) as before from the **RMFS** and the input-output pulse transfer matrix from $V(z)$ to $Y(z)$ is $G_m(z)$ because the pulse transfer matrix from $R(z)$ to $Y(z)$ can be regarded as $G_0(z)$. The elements enclosed by the dotted line in the **RMFS** shown in Fig.5 is constructed in a digital computer and called a robust compensation controller.

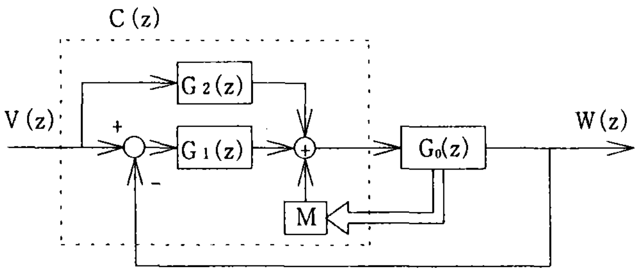

Fig.4 A general system configuration in disregard of robustness

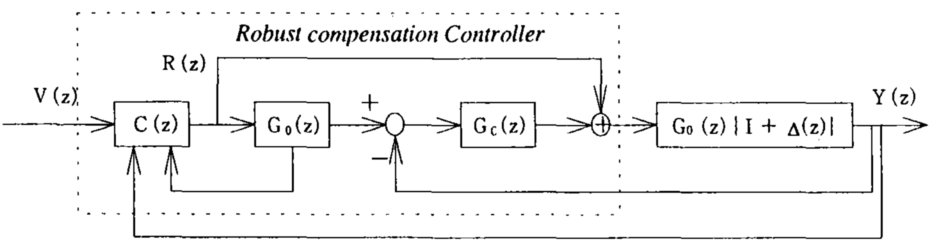

Fig.5 A robust model following system

4 Simulation

We show simulation results of the **RMFS**, where

$$G_0(s) = \frac{s+1}{s^2+0.4s+1} \qquad (8)$$

$$C(s) = \begin{cases} M = (-0.2 \quad -1.82) \\ G_1(s) = 0 \\ G_2(s) = \dfrac{1}{s(s+1)} \end{cases} \qquad (9).$$

We give the plant with uncertainty which has parameter variations of 20% in comparison of Eq.(8). The plant with uncertainty as following.

$$G_0(s)\{I + \Delta(s)\} = \frac{s+0.8}{s^2+0.48s+0.8} \qquad (10)$$

In this case the uncertainty $\Delta(s)$ is

$$\Delta(s) = \frac{-0.28s^2+0.04s}{s^3+1.48s^2+1.28s+0.8} \qquad (11)$$

and the input-output transfer function of the reference model is

$$G_m(s) = \frac{1}{s^3+2.3s^2+1.2s+1} \qquad (12).$$

To inspect the effect of the weighting functions $W_s(s)$ for the sensitivity function, we designed the two compensators.

First, we chose Eq.(13) and Eq.(14) as the weighting functions to design the compensator.

$$W_S(s) = \frac{0.5(s+3)}{s+0.03} \qquad (13)$$

$$W_T(s) = \frac{50(s+100)}{s+10000} \qquad (14)$$

By using **MATLAB** [4], we obtained the following compensator .

$$G_{c1}(s) = \frac{35.8922s^3+358979s^2}{s^4+863.267s^3+81727.6s^2} * \frac{+568456s+393932}{+78068.9s+2268.54} \qquad (15)$$

In this case, frequency characteristics of the sensitivity and the complementary sensitivity functions are shown in Fig. 6.

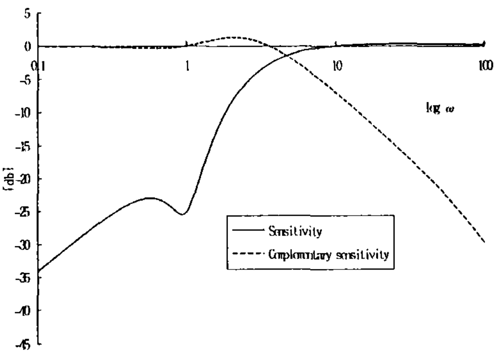

Fig.6 Frequency characteristics of the Sensitivity and Complementary sensitivity functions.

Second, we chose Eq.(16) and Eq.(17) as the weighting functions.

$$W_S(s) = \frac{0.25(s+3)}{s+0.03} \qquad (16)$$

$$W_T(s) = \frac{50(s+100)}{s+10000} \qquad (17)$$

And we obtained the following compensator.

$$G_{c2}(s) = \frac{1.01162s^3+10117s^2}{s^4+151.366s^3+5283.49s^2} * \frac{+8410.77s+7547.4}{+3899.19s+112.224} \qquad (18)$$

In this case, frequency characteristics of the sensitivity and the complementary sensitivity functions are shown in Fig.7.

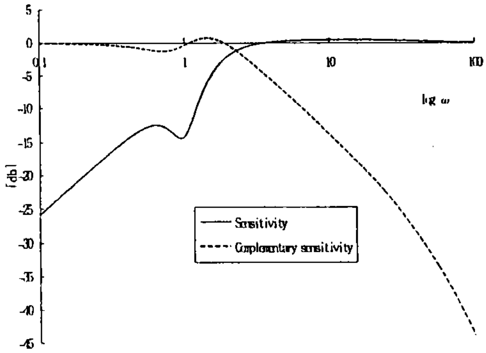

Fig.7 Frequency characteristics of the Sensitivity and Complementary sensitivity functions.

Each element of the total system is discretized by using a sampler of which sampling period is *10 msec* and a zero-order hold device.

The simulation results of the step and disturbance responses of the **RMFS** are shown in Fig.8 and Fig.9.

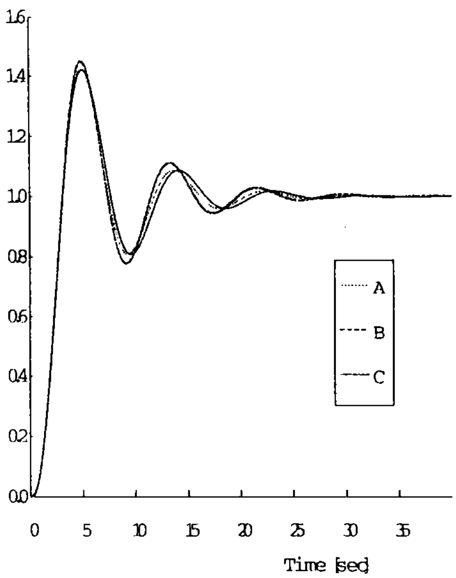

Fig.8 The step responses of the robust model following system

In Fig.8, ***'A'*** and ***'B'*** show the step responses of the **RMFS** with $G_{c1}(z)$ and $G_{c2}(z)$, respectively. And ***'C'*** shows the step response of the reference model.

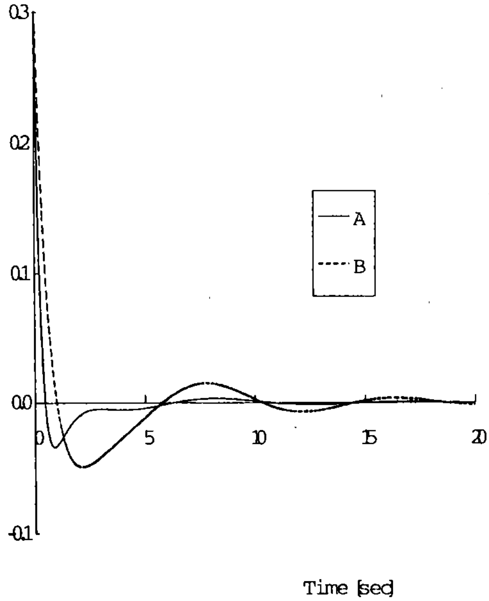

Fig.9 The disturbance responses of the robust model following system

In Fig.9, ***'A'*** and ***'B'*** show the disturbance responses of the **RMFS** with $G_{c1}(z)$ and $G_{c2}(z)$, respectively, in the case where the step disturbance with magnitude 0.3 was applied to the output of the plant.

It follows from these results that the **RMFS** with $G_{c1}(z)$ has lower sensitivity for parameter variations and disturbances than with $G_{c2}(z)$ as was expected.

5 Conclusion

We proposed a controller called the robust compensation controller. In the **RMFS** which is constructed by using the robust compensation controller, the input-output properties can be specified independently of sensitivity and robust stability.

6 Reference

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