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Nonlinear Control of an Autonomous Mobile Robot using Nonlinear Observers

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Abstract **— In this paper, we will investigate the position estimation problem for autonomous mobile robots. Formulating this as a state estimation problem for nonlinear SISO system, then we will apply several types of nonlinear observers. Simulation results of observer-based navigation control will be also provided.**

1 Introduction

Recently, navigation problem for mobile robots attracts great attention of control engineers because it is practically important and theoretically interesting. A mobile robot system is a typical example of nonholonomic system, and it is impossible to control its postion by continuous static state feedback. Therefore the path planning is essentially important, and we have already proposed the path tracking controller for multitrailer systems based on time scale transformation and exact linearization[1] .

These schemes require the informations of all state variables, i.e., the position and the orientation of the robot. When we want to measure the whole states, we need some kinds of robot visions like CCD cameras, but they are too exaggerated for a tiny autonomous mobile robot.

In this paper, we will suggest an autonomous vehicle which has only two optical sensors such as PSD (Position Sensitive Devices). Clearly this simple measurement system cannot acquire the whole information on the instant, but it will be shown that we can *dynamically* estimate the state variables. Thus we can compose the state observers for this system, and inject the estimates into state feedback controller.

In Section 2, kinematic model of the vehicle and mesuring configration are presented, then Section 3 describes the state estimation problem and reduces it to an SISO case. Observability will be verified in Section 4, and Section 5 will introduce three types of nonlinear observers applied to this problem. Numerical simulations of state estimation and observer-based nonlinear output feedback controller will be provided in Section 6.

2 System Model

2.1 Kinematics of the Vehicle

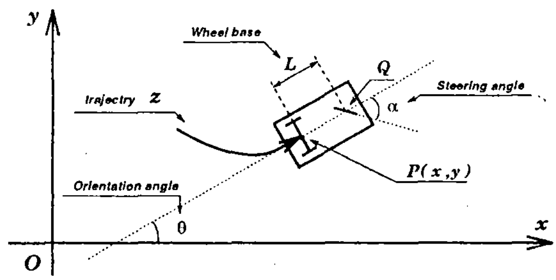

Fig.1: System Model

The vehicle, which we consider in this paper, is a tricycled automobile as shown in Fig.1. This is a two-dimensional object, and $x - y$ is the planar coordinate frame. Configuration of the vehicle is determined by the position (x, y) and orientation θ.

Notations:

$P(x,y)$	:	Center point of the rear axis
θ	:	Absolute orientation of the body
z	:	Trajectory of P
$\dot{z}$	:	Translational velocity of P
L	:	Wheelbase (distance between P and Q)
α	:	Steering angle
Q	:	Center of the front wheel

where we assume that

(1) the vehicle is driven by rear wheels and steered by front wheel,

(2) there are no slide slips in any wheels.

(3) control inputs are $\dot{z}$ and α.

The kinematic model of the vehicle is the following differential equations :

$$\frac{dx}{dz} = \cos\theta \quad \text{(1-a)}$$

$$\frac{dy}{dz} = \sin\theta \quad \text{(1-b)}$$

$$\frac{d\theta}{dz} = u \quad \text{(1-c)}$$

where $u := \frac{\tan\alpha}{L}$.

2.2 Model of the Measurement System

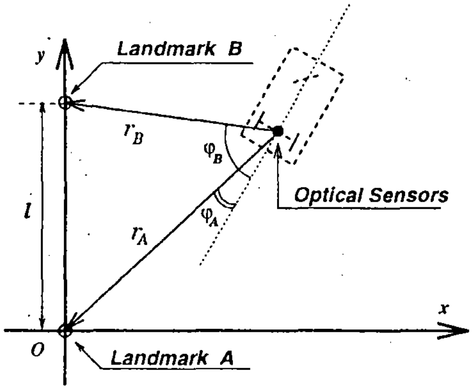

Fig.2: Measuring Configuration

Suppose a quite simple measurement system as shown in Fig.2. Two landmarks are allocated on the field — *landmark A* at $(0,0)$, *landmark B* at $(0,l)$, where *l is an unknown constant.* The vehicle is assumed to have some kind of optical sensors, so as to measure the directions of each towers φ_A, φ_B. The distances between the vehicle and each towers r_A, r_B are unknown.

The output vector $(\varphi_A, \varphi_B)^T$ are described by the following nonlinear function :

$$\begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix} = \begin{pmatrix} \theta - \operatorname{atan}\frac{y}{x} \\ \theta - \operatorname{atan}\frac{y-l}{x} \end{pmatrix} \tag{2}$$

3 Problem Statement

3.1 Primary Problem

For autonomous navigation control, we have to estimate $x(t), y(t), \theta(t)$ from the set of past outputs $\{\varphi_A(\tau), \varphi_B(\tau) | \tau \leq t\}$. This is the dynamic estimation problem.

At first we derive a system representation. We take the augmented state vector as $(x, y, \theta, l)^T$, since l is assumed to be unknown. From (1), the state equation is

$$\frac{d}{dz}\begin{pmatrix} x \\ y \\ \theta \\ l \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} u \tag{3}$$

Remark: Note that (3) does not contain the time t, but the P's trajectry z is used as a proxy of t. Since $\dot{z}$ is one of the control inputs, we can control the time scale of (3) in a sense. Thus (3) is recognized as a single input system.

Our purpose here is to design the *state observer(s)* for the system (2)(3) so as to estimate the states of the system dynamically.

3.2 Reduced Problem

In this subsection, we intend to simplify the problem to the SISO case which is much easier to handle than the primary one. The following procedure shows that we can split the system into two SISO subsystems, only by coordinate transformation.

First, we take the new coordinates (state vector) consists of the output values φ_A and φ_B, and the distances from each landmarks r_A, r_B.

$$x = (r_A, \varphi_A, r_B, \varphi_B)^T \tag{4}$$

where

$$\begin{aligned} r_A &= \sqrt{p_x^2 + p_y^2} \quad , \quad \varphi_A = \theta - \operatorname{atan}\frac{p_y}{p_x} \\ r_B &= \sqrt{p_x^2 + (p_y - l)^2} \quad , \quad \varphi_B = \theta - \operatorname{atan}\frac{p_y - l}{p_x} \end{aligned} \tag{5}$$

Then the state equation is translated into

$$\frac{d}{dz}\begin{pmatrix} r_A \\ \varphi_A \\ r_B \\ \varphi_B \end{pmatrix} = \begin{pmatrix} \cos\varphi_A \\ -\frac{\sin\varphi_A}{r_A} \\ \cos\varphi_B \\ -\frac{\sin\varphi_B}{r_B} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} u \tag{6}$$

There are no interaction between (r_A, φ_A) and (r_B, φ_B), so we can split the system into a pair of single-output system. Moreover, we need not to distinct these two systems because they are exactly similar. Finally the problem is reduced to estimate the state of

$$\frac{d}{dz}\begin{pmatrix} r \\ \varphi \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ -\frac{\sin\varphi}{r} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \tag{7}$$

where φ is directly measurable (i.e., output equation is linear).

Once we obtain the estimates $\hat{r}_A, \hat{r}_B$, our principal purpose is acheived by this inverse transformation:

$$\hat{l} = \sqrt{\hat{r}_A^2 + \hat{r}_B - 2\hat{r}_A\hat{r}_B\cos(\varphi_B - \varphi_A)} \tag{8-a}$$

$$\hat{x} = \frac{\hat{r}_A\hat{r}_B\sin(\varphi_B - \varphi_A)}{\hat{l}} \tag{8-b}$$

$$\hat{y} = \frac{\hat{r}_A^2 - \hat{r}_A\hat{r}_B\cos(\varphi_B - \varphi_A)}{\hat{l}} \tag{8-c}$$

$$\hat{\theta} = \varphi_A + \operatorname{atan}\frac{\hat{y}}{\hat{x}} \tag{8-d}$$

Remark: This result can be intuitively derived in terms of surveying — the principle of triangulation. When a surveyer wants to know the distance between a pair of mountaintops, he will measure the directions to each mountaintops at distinct two points. In other words, he will determine l from (φ_A, φ_B) and (φ'_A, φ'_B), and incidentally x, y, θ are acquired. The next section supports this fact systematically.

4 Observability

4.1 Definitions

The notion of nonlinear observability has been proposed in various forms, while we are going to adopt *locally weakly observability* according to Hermann-Krener[2].

We suppose the nonlinear system

$$\dot{x} = f(x) + gu \qquad (9\text{-a})$$
$$y = h(x) \qquad (9\text{-b})$$

where $x \in \mathfrak{M}$ (state space; n-dimensional manifold), $f : \Re^n \mapsto \Re^n, g \in \Re^{n\times 1}$ and $h : \Re^n \mapsto \Re$ are sufficiently smooth mapping (Note that g is supposed to be constant vector for simplicity). Let *Lie derivative* of h by f be also a mapping defined by

$$\mathrm{L}^f h(x) := \frac{\partial h}{\partial x} f(x)$$

Definition 4.1 ($\mathfrak{U}$-indistingushablity) *Let $\mathfrak{U}$ be a subset of $\mathfrak{M}$ and $x_0, x_1 \in \mathfrak{U}$. If for every input $(u(t), t \in [t_0, t_1])$ whose trajectories $(x_0(t), t \in [t_0, t_1])$ and $(x_1(t), t \in [t_0, t_1])$ lie in $\mathfrak{U}$, $y(x_0(t)) = y(x_1(t)), t \in [t_0, t_1]$ is satisfied, then x_0 is called $\mathfrak{U}$-indistinguishable from x_1.*

Definition 4.2 *System (3) is called locally weakly observable at x_0 if there exists an open neighborhood $\mathfrak{U}$ of x_0 such that for every open neighborhood $\mathfrak{V}$ of x_0 contained in $\mathfrak{U}$, the only $\mathfrak{V}$-indistinguishable point in $\mathfrak{V}$ is x_0.*

We can easily verify this property by computing the following observability rank condition.

Theorem 4.3 (Observability Rank Condition) *Assume $\frac{\partial u}{\partial x} = 0$. Then system (3) is locally weakly observable at x, if the following observability rank condition:*

$$\dim \mathrm{span} \left\{ \frac{\partial}{\partial x} h(x), \cdots, \frac{\partial}{\partial x} \mathrm{L}_f^n h(x) \right\} = n \qquad (10)$$

is satisfied.

According to this theorem 4.3, system (7) is locally weakly observable if

$$\det \begin{pmatrix} 0 & 1 \\ \frac{\sin\varphi}{r^2} & -\frac{\cos\varphi}{r} \end{pmatrix} = -\frac{\sin\varphi}{r^2} \neq 0 \qquad (11)$$

or, equivalently,

$$0 < r < \infty, 0 < |\varphi| < \pi \qquad (12)$$

Remark: Then the observability of system (7) has been verified, but remember that the z is a proxy of time t in system (7). This means that we can estimate the state vector of system (7) as long as z *increases monotonously.*

5 Nonlinear Observer Design

The general form of nonlinear observers for system (3) are described as follows:

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + gu + k(h(x) - h(\hat{x})) \qquad (13)$$

The terms $f(\hat{x}(t))$ and gu are employed to follow the true system (9). $k(\cdot)$ is a function of the output error $h(x) - h(\hat{x})$, which corrects the current estimate (assume that $k(0) = 0$). Nonlinear observer design is to find an appropriate function $k(\cdot)$, so that $\tilde{x} := x - \hat{x}$ converges to zero as $t \to \infty$ (corresponds to $z \to \infty$, in our case).

Now we will design three types of nonlinear observers for system (3).

5.1 Lyapunov's Direct Method

This observer has the form

$$\frac{d}{dz}\begin{pmatrix} \hat{r} \\ \hat{\varphi} \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ -\frac{\sin\varphi}{\hat{r}} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + k(\tilde{\varphi}) \qquad (14)$$

Then the error system is

$$\frac{d}{dz}\begin{pmatrix} \tilde{r} \\ \tilde{\varphi} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\tilde{r}\sin\varphi}{r\hat{r}} \end{pmatrix} - k(\tilde{\varphi}) \qquad (15)$$

Suppose a candidate of Lyapunov function:

$$V(\tilde{x}) = \frac{1}{2}\tilde{r}^2 - \epsilon_1 \tilde{r}\tilde{\varphi} + \epsilon_2 \tilde{\varphi}^2, \quad \epsilon_1, \epsilon_2 \in \Re_+ \qquad (16)$$

and then find an appropriate $k(\tilde{x})$ so that $V(\tilde{x})$ will be positive definite and $\frac{d}{dz}V(\tilde{x})$ will be negative definite. One of such function is

$$k(\tilde{x}) = \begin{pmatrix} \frac{k\tilde{\varphi}}{4(\epsilon_2 - \epsilon_1^2)\hat{r}^2} \\ \frac{k\epsilon_2\tilde{\varphi}}{4\epsilon_1(\epsilon_2 - \epsilon_1^2)\hat{r}^2} \end{pmatrix} \qquad (17)$$

5.2 Extended Kalman Filter

Neglecting higher order terms in taylor expansion of (9) arround the current estimate $\hat{x}(t)$, we obtain the approximatly linearized system

$$\frac{dx}{dt} = f(\hat{x}) + F\tilde{x} + gu \qquad (18\text{-a})$$
$$y = h(\hat{x}) + H\tilde{x} \qquad (18\text{-b})$$

where $F = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}}, H = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}}$.

The extended Kalman filter[3][4] is a kind of non-steady Kalman filter, proceeds by updating the estimates of state vector and estimation error covariance matrix P, and by correcting P and the filter gain K whenever a new outputs are given.

Updating process:

$$\frac{d\hat{x}}{dt} = f(\hat{x}) + gu + K(h(x) - h(\hat{x}))$$

$$\frac{dP}{dt} = FP + PF^T + Q$$

Correcting process:

$$K = PH^T(HPH^T + R)^{-1}$$

$$P = (I - KH)P$$

where Q, R are covariance matrix of states and output disturbances.

In our case, the following are F, g, H are substituted into this procedure.

$$F = \begin{pmatrix} 0 & -\sin\varphi \\ \frac{\sin\varphi}{r^2} & -\frac{\cos}{\varphi} r \end{pmatrix}, g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, H = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{19}$$

5.3 Extended Luenberger Observer

A special class of nonlinear error systems are known to be exactly linearized by coordinate transformation. Although (7) does not satisfy the necessary condition for this linearization, there is an approximate linearization method proposed by Zeitz[5].

For an observable SISO system, there is the observable canonical form derived by coordinate transformation $x^* = P(x)$. Then the error system in new coordinates, or dynamics of $\tilde{x}^*$, is linearized by 1st-order approximation, and the observer which estimates x^* is designed. By transforming this observer to the primary coordinate, we can obtain the observer which estimates x (See [5] for detail).

The extended Luenberger observer for system (7) is :

$$\frac{d}{dz}\begin{pmatrix} \hat{r} \\ \hat{\varphi} \end{pmatrix} = \begin{pmatrix} \cos\hat{\varphi} \\ -\frac{\sin\hat{\varphi}}{\hat{r}} \end{pmatrix} + \begin{pmatrix} \frac{\hat{r}^2}{\sin\hat{\varphi}} p_1 - \sin\hat{\varphi} \\ p_2 + \frac{2\cos\hat{\varphi}}{\hat{r}} \end{pmatrix} \tilde{\varphi} \tag{20}$$

where the characteristic polynomial of the dyanmics of $\tilde{x}^*$ is $s^2 + p_2 s + p_1$. In other words, we can specify poles of the convergence.

6 Simulations

6.1 State Estimation

Fig.3 and Fig.4 are simulation results of state estimation, under the following conditions:

Initial states :

$$(x, y, \theta, l) = (1.0[m], 2.0[m], -0.5[rad], 1.0[m])$$

Initial estimates :

$$(\hat{r}_A, \hat{r}_B) = (1.0[m], \sqrt{2.0}[m])$$

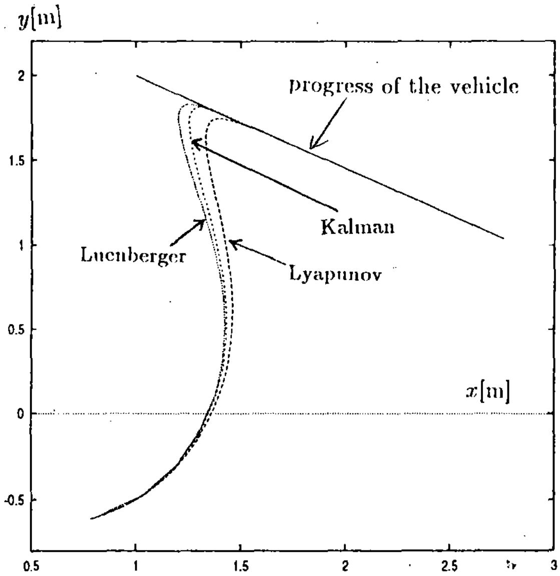

Fig.3: Estimation of x, y

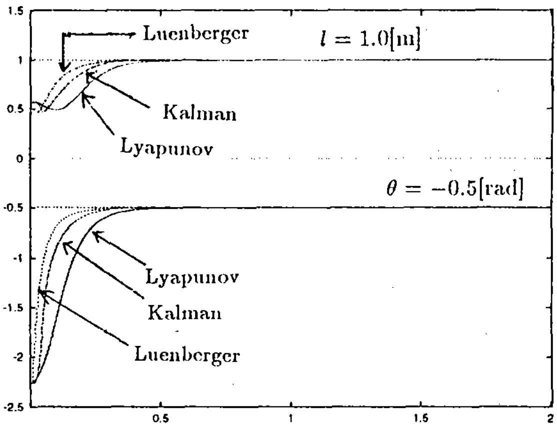

Fig.4: Estimation of θ, l

Steering angle is identically zero, so θ is constant (the vehicle moves starightforward). On the other hand, observers' configurations are

Lyapunov's Direct Method :

$$\epsilon_1 = 1.0 \times 10^{-2}, \epsilon_2 = 2.0 \times 10^{-3}, k = 4.0$$

Extended Kalman Filter :

$$Q = \mathrm{diag}(1.0 \times 10^3, 0.2), R = 1.0 \times 10^{-3}$$

Extended Luenberger Observer :

$$p_1 = 64, p_2 = 16(\text{polesare} - 8, -8)$$

We can see that every estimation error converges sufficiently fast.

6.2 Observer-based Control

Finally, we will show an example of observer-based navigation control in Fig.5. The control law is the following nonlinear state feedback :

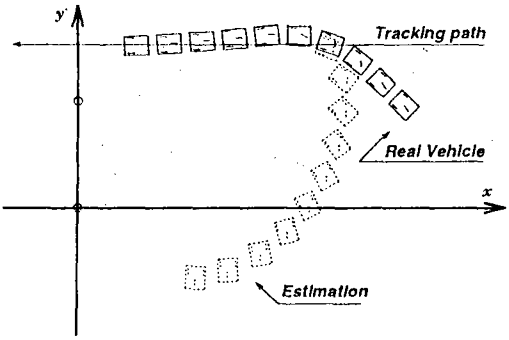

Fig.5: Observer-based path following control (using Extended Luenberger Observer)

$$u = \cos^3\theta\,\{F_1(y-\bar{y}) + F_2\tan\theta\} \qquad (21)$$

where F_1, F_2 is feedback gain, and $y = \bar{y}$ is the reference path. (See [6] for detail).

Fig.5 is the result of observer-based path following control. (In this case the vehicle moves backward, so z decreases monotonously). Estimates by the extended Luenberger observers are injected into the nonlinear state feedback controller.

Reference path : $y = 1.5[m]$

State feedback gain : $F_1 = -4, F_2 = -4$

(decided by pole assignment to $-2, -2$)

Initial state :

$$(x, y, \theta, l) = (3.0[m], 1.0[m], -0.8[rad], 1.0[m])$$

Initial estimate :

$$(\hat{r}_A, \hat{r}_B) = (1.0[m], \sqrt{2.0}[m])$$

And the configurations of extended Luenberger observer are completely same as the previous example.

We can see that the stability of observer-based feedback control system is conserved in this case, and the vehicle follows the reference path autonomously.

7 Conclusions

In this paper, we formulated the state estimation problem for autonomous mobile robots, and cleared systematically that nonlinear observers are applicable. Moreover, observer-based output feedback control was acheived and verified by numerical simulations. We found that the extended Luenberger observer is especially useful for output feedback control because we can decide the convergence rate of estimation by specifying the poles of the error system.

References

[1] M.Sampei, T.Tamura, T.Itoh, and M.Nakamichi. Path tracking control of a trailer-like mobile robot moving backward. *KACC '91*, pages 1282 – 1285, 1991.

[2] R.Hermann and A.J.Krener. Nonlinear controllability and observability. *IEEE Trans. on Automatic Control*, AC-22(5):728–740, 1977.

[3] B.D.O.Anderson and J.B.Moore. *Optimal Filtering.* Prentice-Hall, 1979.

[4] T.Katayama. *Applied Kalman Filter.* Asakura Shoten, 1983(in Japanese).

[5] M.Zeitz. The extended luenberger observer for nonlinear systems. *Systems & Control Letters*, 9:149–156, 1987.

[6] M.Sampei and K.Furuta. Path tracking control of an mobile robot. *Proc. of 30th Annual Conference on Automatic Control*, pages 371–372, 1987 (in Japanese).

[7] M.Sampei and M.Ishikawa. State estimation of mobile robot using nonlinear observers. *Proc. of 23rd SICE Symposium on Control Theory*, pages 297 – 300, 1994(in Japanese).

[8] A.Isidori. *Nonlinear Control Systems.* Springer-Verlag, 1989.

[9] J.-J.E.Slotine and W.Li. *Applied Nonlinear Control.* Prentice Hall, 1991.