'94 KACC (1994. 10. 17 ~ 20)

Design of a New Variable Structure Model Following Control System for Robot Manipulators

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Abstract

In this paper, a new design method of variable structure model following control system(VSMFCS) for robot manipulators is proposed. The proposed controller overcomed reaching phase problem by using function augmenting scheme to the sliding surface. Therefore, it can be guaranteed that the overall system always has a robust property against parameter variations and external disturbances. Furthermore, the proposed controller does not use the model state, x_m, different from other previous works. Regardless of not using the model state, the model following error dynamics, virtual dynamics, is shown to be globally exponentially stable.

The efficiency of the proposed method has been demonstrated by an example.

1 Introduction

The model reference adaptive control(MRAC) is one of the main approaches to adaptive control. The desired performance is expressed in terms of a reference model, A_m, B_m, and model state x_m, which gives the desired response to a command signal r. Actually, however, there is no need to allocate memories to model state x_m, because the desired performance can be defined by the pair (A_m, B_m). Furthermore, for the MRAC scheme, generally one can not say about the decreasing speed of model following error. And for the conventional variable structure model following control system(VSMFCS), system output is sensitive during the overall system is not in the sliding mode, that is, when the system is in reaching phase – it is called reaching phase problem.

To overcome these shortcomings, we propose the new VSMFCS. The proposed controller guarantees the occurrence of sliding mode all the time by using function augmenting scheme to the sliding surface, so we can obtain the robust properties all the time against external disturbances and parameter uncertainties. Furthermore, since the proposed controller does not use the model state x_m, one can save the memory storage. In addition, model following error dynamics is shown to be globally exponentially stable, i.e., one can assign the maximum allowable error decreasing speed, where the model following error is defined by the difference between the system output and the 'virtual' – because we do not use the model state – model output. Though the model following error goes to zero as fast as possible, there is no need to increase the control gain in signum function as in previous works, so excessive chattering is not caused by applying the proposed technique.

2 Modeling of Robotic Manipulator

The dynamic equation of motion for an n degree-of-freedom robot manipulator can be derived using Lagrangian formulation as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + d, \tag{1}$$

where $M(q)$ is an $n \times n$ mass matrix of the manipulator, $C(q,\dot{q})$ is an $n\times 1$ vector of centrifugal and Coriolis terms, $G(q)$ is an $n\times 1$ vector of gravity terms, u is an $n\times 1$ vector of control inputs, and d is an $n \times 1$ vector of bounded disturbances.

Assume that $M = \hat{M} + \Delta M$, $C = \hat{C} + \Delta B$, and $G = \hat{G} + \Delta G$, where "^" denotes the mean value and "Δ" denotes the estimation error. We also assume that the $\Delta M_{ij}, \Delta C_{ij}$ and ΔG_i are bounded by M_{ij}^m, C_{ij}^m and G_i^m as $|\Delta M_{ij}| \le M_{ij}^m$, $|\Delta C_{ij}| \le C_{ij}^m$, $|\Delta G_i| \le G_i^m$, where "m" denotes the maximal absolute estimation error of each element, and $|d_i| \le d_i^m$.

3 Design of the Control System

The reference model is usually described by

$$\frac{d}{dt}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{m1} & A_{m2} \end{bmatrix}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} + \begin{bmatrix} 0 \\ B_{m1} \end{bmatrix} r = A_m x_m + B_m r, \tag{2}$$

where $x_m = (q_m^T \ \ \dot{q}_m^T)^T \in \Re^{2n}$, $A_m \in \Re^{2n\times 2n}$ is a constant stable matrix, $B_m \in \Re^{2n\times n}$ is a constant matrix, and $r \in \Re^n$ is an external input vector.

Let's define a new error vector e as a following form, instead of conventional definition $e = x - x_m$:

$$e = x - \int_0^t (A_m x + B_m r)\, dt, \tag{3}$$

where $x = (q^T \ \ \dot{q}^T)^T \in \Re^{2n}$. Let's define a new sliding surface as a function augmented form:

$$s = Ke - h(t), \tag{4}$$

where $K = (K_1 \ \vdots \ I_n) \in \Re^{n\times 2n}$, I_n is an $n \times n$ identity matrix, $K_1 = diag(k_1, k_2, \cdots, k_n)$, $k_i > 0$ for all $i = 1, 2, \cdots, n$, and each element of the augmenting function vector $h(t) \in \Re^n$ holds the following assumption.

Assumption 1 *$h_i : \Re_+ \to \Re$, $h_i \in C^1[0,\infty)$, $\dot{h}_i \in L^\infty$, the support of h_i is the bounded interval $[0,T] \subset \Re_+$ for some $T > 0$, and $h_i(0) = \dot{q}(0) + k_i q(0)$, where $C^1[0,\infty)$ represents the set of all first differentiable continuous functions defined on $[0,\infty)$ and $i = 1, 2, \cdots, n$.*

Consider a Lyapunov function candidate as $V = \frac{1}{2}s^T M s$. Differentiating V with respect to time and adopting the skew-symmetricity of $\dot{M}(q) - 2C(q,\dot{q})$, we have

$$\begin{aligned}\dot{V} &= s^T C s + s^T M \dot{s} = s^T\left[Cs + M\left(K\dot{e} - \dot{h}\right)\right] \\ &= s^T\left[Cs + M\left(\ddot{q} + K_1\dot{q} - KA_m x - KB_m r - \dot{h}\right)\right] \\ &= s^T\left[Cs + u + d - C\dot{q} - G\right. \\ &\qquad \left. + M\left(K_1\dot{q} - KA_m x - KB_m r - \dot{h}\right)\right].\end{aligned} \tag{5}$$

Therefore, the equivalent control input can be got as

$$\begin{aligned} u_{eq} &= -\hat{C}(s - \dot{q}) + \hat{G} \\ &\quad -\hat{M}\left(K_1\dot{q} - KA_m x - KB_m r - \dot{h}\right).\end{aligned} \tag{6}$$

Now, we introduce the control input u such as

$$u = u_{eq} - N \bullet sgn(s), \tag{7}$$

where "$\bullet$" means the element-by-element multiplication of two vectors, and

$$\begin{aligned} N &= M^m \mid K_1\dot{q} - KA_m x - KB_m r - \dot{h} \mid \\ &\quad + C^m \mid s - \dot{q} \mid + G^m + d^m + \eta, \\ \eta &= [\,\eta_1, \eta_2, \cdots, \eta_n\,]^T, \qquad \eta_i > 0, \\ sgn(s) &= [\,sgn(s_1), sgn(s_2), \cdots, sgn(s_n)\,]^T, \\ sgn(s_i) &= \begin{cases} 1 & \text{if } s_i > 0 \\ 0 & \text{if } s_i = 0 \\ -1 & \text{if } s_i < 0 \end{cases}, \qquad i = 1, 2, \cdots, n, \end{aligned}$$

and the absolute of a vector denotes the vector whose elements have their absolute values, i.e., $|x| = [\,|x_1|, |x_2|, \cdots, |x_n|\,]^T$.

Lemma 1 *For the robot manipulator (1), the control law (7) guarantees the occurrence of the sliding mode all the time.*

Proof By Inserting (7) in (5), we can easily derive the following inequality:

$$\dot{V} \leq -\sum_{i=1}^{n} \eta_i |s_i|. \tag{8}$$

Therefore, V is really a Lyapunov function. From (8), it can be known that $\dot{V} \leq 0$, and $\dot{V} = 0$ if and only if $s = 0$. Furthermore, it is easy to know from (4) that $s(0) = 0$. Therefore, $V(t) = 0 \quad \forall\ t \geq 0$. This also implies that $s = 0\ \forall\, t \geq 0$. Thus, the system is forced to stay in the sliding mode all the time.

□

Though we did not use the model state vector, x_m, we can state about the stability of the model following error dynamics as in the following theorem.

Theorem 1 *For the robot manipulator (1) with the control law (7), the model following error dynamics is globally exponentially stable.*

Proof Since the system is in the sliding mode all the time, we obtain the following equation:

$$\dot{s} = K\dot{e} - \dot{h} = \ddot{q} - A_{m2}\dot{q} - A_{m1}q - B_{m1}r - \dot{h} = 0. \tag{9}$$

Since $\dot{h} = 0 \quad \forall\ t > T$ by Assumption 1, (9) can be rewritten as following for $t > T$:

$$\ddot{q} - A_{m2}\dot{q} - A_{m1}q - B_{m1}r = 0. \tag{10}$$

On the other hand, the model dynamics can be derived from (2) as following:

$$\ddot{q}_m - A_{m2}\dot{q}_m - A_{m1}q_m - B_{m1}r = 0. \tag{11}$$

Subtracting (11) from (10), we get, with model following error $\epsilon = q - q_m$

$$\ddot{\epsilon} - A_{m2}\dot{\epsilon} - A_{m1}\epsilon = 0, \qquad \text{i.e.,} \qquad \frac{d}{dt}\begin{bmatrix} \epsilon \\ \dot{\epsilon} \end{bmatrix} = A_m \begin{bmatrix} \epsilon \\ \dot{\epsilon} \end{bmatrix}. \tag{12}$$

Since A_m was assumed to be a stable matrix and T is a finite value, the model following error dynamics is globally exponentially stable.

□

4 Simulation Results

A 2-link robotic manipulator model used by Young [4] was used for the simulation as shown in Figure 1. Parameter values are also the same as those of [4]. In order to avoid the chattering phenomenon, the function $sgn(s)$ in the controller (7) has been replaced by saturation function $sat(s)$.

Figure 2 shows that system output successfully tracks the desired model output regardless of not using model stata x_m, and model following error decreases exponentially to zero. And Figure 3 shows that the sliding function s_1 and s_2 were confined to the predetermined boundary $\delta = 0.05$, and the fact means that the system state is always in the sliding mode.

In the Leung's simulation results, Figure 12 and 13 have shown that the sliding function started from zero ($s_1 = 0,\ s_2 = 0$) was initially away from zero. In addition, the sliding function s_1 has gone away from the predetermined boundary layer $\delta_1 = 0.05$ as shown in Figure 12 of Leung's paper [3]. However, for the proposed controller designed by the concept that the overall system ensures the stability of the intersection of the sliding surfaces without necessarily stabilizing each individual one, all sliding function s_i is always confined to stay on the hyperplane, $s_1 = s_2 = 0$. Therefore, error transient can be predetermined in advance for all time interval under consideration.

5 Conclusions

In this paper, a new design method of VSMFCS is proposed in order to remove the reaching phase problem and to control the system without using model state x_m. The proposed control system always has a robust property against parameter variations and external disturbances, and the model following error dynamics was shown to be globally exponentially stable. So, we can assign the model following error decreasing rate, though MFAC method can generally guarantee only the asymptotic stability. Chattering during the transient phase can be reduced by using the boundary layer technique. Simulation results have shown the good performance of the overall system.

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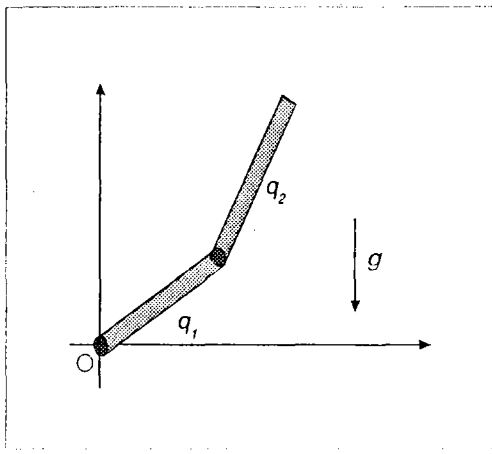

Figure 1. 2-link Robot Manipulator

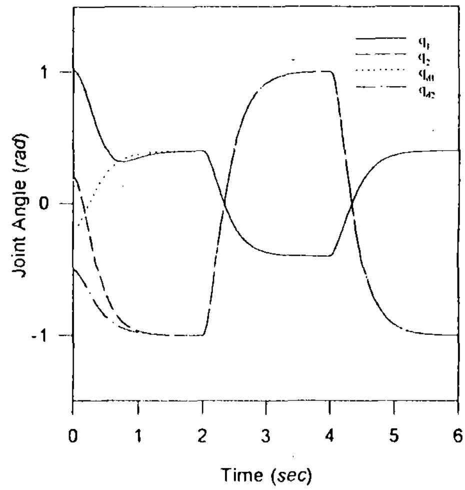

Figure 2. Actual and Desired Joint Angles

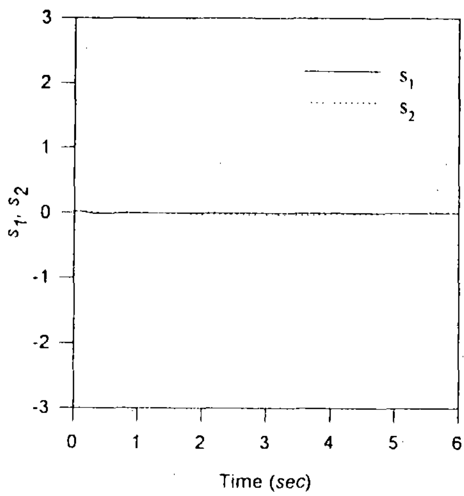

Figure 3. Sliding Function Values