

# Comparative Performance of Adaptive and Robust Control for Robot Arms

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**Abstract**— The adaptive control and the robust control have been considered as the most influential methods for robotic motion control. The purpose of this paper is to compare control performance between these two strategies in unconstrained motion control of robot manipulator. In order to compare control performance properly, intensive experiments are required and only then can conclusions be drawn on the relative merit and demerit of the controllers. Firstly, the control algorithms for unconstrained motion control are summarized. In adaptive control, the controllers that have been proposed so far are classified according to the signals used for the computed control input. It enables rather easier to compare control performance among the adaptive controllers. In robust control, TDOF (Two Degree of Freedom) robust controller is examined to demonstrate control performance of robust controllers. Finally, The above two approaches, the adaptive and the robust are compared from the viewpoint of robustness to plant uncertainty, which is one of the most demanding properties in robot motion control.

## I Introduction

Robot manipulators have to face uncertainty in many dynamic parameters, in particular the parameters describing the dynamic properties of loads and environments. At this time, Sensitivity to such parameter uncertainty is undesirable and especially severe in high-speed operations.

Two classes of approach have been intensively studied to cope with parameter uncertainty: adaptive control and robust control. We distinguish adaptive control from robust control, although both attempt to accomplish basically the same objects, namely, control under plant uncertainty. Adaptive control incorporates adaptation algorithm to extract unknown parameter informations in the course of operation, while robust control uses a fixed controller designed to satisfy performance specifications over a given range of uncertainty.

Recently, much research has taken place to advance the state of the art in unconstrained motion control of robot manipulators. The adaptive and the robust controls have been considered as the most influential methods for robot motion control. However, the number of comparative studies of adaptive and robust controls is quite few.

In this paper, we present the comparative evaluation of adaptive and robust techniques in robot motion control.

In order to evaluate the effectiveness of the controllers properly, the intensive experimentation is required. Only then can conclusions be drawn on the merit of the controllers. Many computer simulation are often misleading, because they do not adequately reflect the effects of unmodeled dynamics.

## II The Controllers for Robot Motion

First of all, the control algorithms used in this paper are explained briefly.

### The Adaptive Controllers (ACs)

Adaptive robotic control had the turning point of research by introducing the linear parametrization of robot dynamics[2]. The controller could fully utilize the non-linear, time-varying and coupled nature of robot dynamics, based on the possibility of selecting a proper set of equivalent parameters such that the manipulator dynamics depend linearly on these parameters.

Since then, many types of ACs have been actively designed[3, 4, 6, 7]. They are closely related but conceptually and algorithmically distinct ACs. Many look-alikes in appearance make it difficult to understand the structure of ACs explicitly. In addition, the widespread unified notation for control and adaptation laws has not been yet suggested.

Considering these difficulties on ACs, we suggest a unified notation for control laws in ACs. The control law of adaptive scheme for position control consists of a PD regulator plus adaptive model-based feedforward compensation of full dynamics. The model-based part can be defined as

$$\mathbf{Y}(\theta, \hat{\theta}_1, \hat{\theta}_2, \hat{\theta})\dot{\mathbf{a}} = \hat{\mathbf{J}}(\theta)\ddot{\theta} + \hat{\mathbf{D}}(\theta, \hat{\theta}_1)\dot{\theta}_2 + \hat{\mathbf{G}}(\theta) \quad (1)$$

where the quantities with "hats" are computed from estimates of the true parameters,  $\mathbf{J}(\theta)$  is the manipulator mass matrix,  $\mathbf{D}(\theta, \hat{\theta}_1)$  is the centrifugal and Coriolis terms,  $\mathbf{G}(\theta)$  is the gravity term,  $\mathbf{Y}(\theta, \hat{\theta}_1, \hat{\theta}_2, \hat{\theta})$  is the regressor matrix which depends linearly on the unknown parameters,  $\mathbf{a}$  is unknown parameter matrix, the viscous friction terms are ignored for convenience.

With the above unified feedforward terms, The control laws of ACs can be classified into the following three types, according to the signals used for computing control

input.

◇ **TYPE1** Computed Torque AC (CT-AC)

$$\tau = Y(\theta, \dot{\theta}, \ddot{\theta}_d, \ddot{\theta}_d) \hat{a} + \tau_{pd} \quad (2)$$

◇ **TYPE2** Referenced Trajectory AC (RT-AC)

$$\tau = Y(\theta, \dot{\theta}, \dot{\theta}_r, \ddot{\theta}_r) \hat{a} + \tau_{pd} \quad (3)$$

◇ **TYPE3** Command Input AC (CI-AC)

$$\tau = Y(\theta_d, \dot{\theta}_d, \ddot{\theta}_d, \ddot{\theta}_d) \hat{a} + \tau_{pd} \quad (4)$$

where the feedback PD gains  $\tau_{pd}$  are summarized in Table 1. From this table, the derivative(D) gains of all the

Table 1: Feedback PD Gain of ACs

	feedforward	P-gain	D-gain
CT-AC	$Y(\theta, \dot{\theta}, \ddot{\theta}_d, \ddot{\theta}_d)$	$K_d \Lambda$	$K_d$
RT-AC	$Y(\theta, \dot{\theta}, \dot{\theta}_r, \ddot{\theta}_r)$	$K_d \Lambda$	$K_d$
CI-AC	$Y(\theta_d, \dot{\theta}_d, \ddot{\theta}_d, \ddot{\theta}_d)$	$K_p + K_d \Lambda$	$K_d$

ACs are same and the proportional(P) gains are slightly different. Because the feedforward(FF) parts are not unified, we make the FF parts of ACs same in order to derive effective PD gains. For example, let the FF parts of CT-AC in (5), RT-AC in (6) same and the control laws are rewritten as

$$\tau = Y(\theta, \dot{\theta}, \ddot{\theta}_d, \ddot{\theta}_d) \hat{a} + \underline{K_d \Lambda e} + \underline{K_d \dot{e}} \quad (5)$$

$$\tau = Y(\theta, \dot{\theta}, \ddot{\theta}_d, \ddot{\theta}_d) \hat{a} + (\underline{K_d} + \underline{D}) \Lambda e + (\underline{K_d} + \underline{J \Lambda}) \dot{e} \quad (6)$$

The PD gain of RT-AC is surely greater than that of CT-AC, although the same  $K_d$  and  $\Lambda$  are designated. From deriving effective PD gains of the ACs, it makes clear that the difference of RT-AC and CT-AC is only the difference of the each PD gain. If the effective PD gains of ACs are set to be same, the difference of control performance will be slightly small. This fact will be clarified by the benchmark test between ACs in later section.

## Two-Degree of Freedom (TDOF)

TDOF servosystem has the property to design the command input response and the closed loop characteristics quite independently. Fig. 1 shows the block diagram of TDOF servosystem. In this figure, The Controllers  $C_A(s)$  and  $C_B(s)$  are designed as follows:

$$C_A = \frac{1}{P_n(s)} \frac{Q(s)}{1 - Q(s)} \quad (7)$$

$$C_B = \frac{G_{ry}(s)}{1 - G_{ry}(s)} \frac{1}{P_n(s)} \frac{1}{1 - Q(s)} \quad (8)$$

where  $P_n(s)$  is the nominal model of the plant  $P(s)$  and  $G_{ry}(s)$  is the required command input response.  $Q(s)$  is

the most significant parameter to determine robustness against disturbance.

By the equivalent transformation in Fig. 1, it turns out that TDOF includes disturbance observer, which estimates disturbance and cancel its effects. Fig. 2 illustrates the transfer characteristics of the observer and the whole system, TDOF. In this figure, the cutoff frequency of  $G_{ry}(s)$  is enough low, compared to the frequency of  $Q$ , the transfer function from disturbance to output is not affected so much by the change of command input response,  $G_{ry}$ .  $Q$  determines the closed loop characteristics to suppress disturbance.

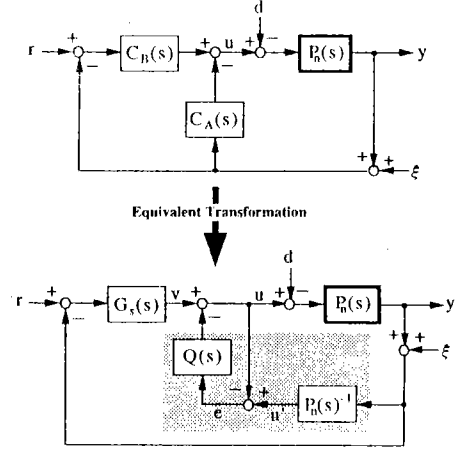


Figure 1: Structure of TDOF

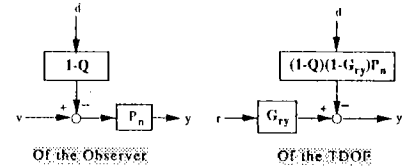


Figure 2: Transfer Characteristics of TDOF

By applying the TDOF servosystem to each joint of robot manipulator, the decentralized position controller can be easily designed[8]. The nominal model of a joint actuator is given as

$$P_n(s) = \frac{K_{tn}}{J_n s^2} \quad (9)$$

where  $J_n$  and  $K_{tn}$  are the nominal values of rotor inertia moment and torque constant. Next, we specify the desired position response  $G_{ry}(s)$  and free parameter  $Q(s)$  as follows:

$$G_{ry}(s) = \frac{1}{(\tau_r s)^2 + 2\tau_r s + 1} \quad (10)$$

$$Q(s) = \frac{1 + \sum_{k=1}^{N-2} a_k (s\tau_N)^k}{1 + \sum_{k=1}^N a_k (s\tau_N)^k} \quad (11)$$

It is very important to investigate the stability of this fixed controller. Our previous paper[9] shows that TDof servosystem is enough robust against the changes of inertia or Coriolis term not to worry about its unstable behavior as long as the commonly used trajectories are applied.

### III The Experimental Results

The intensive experiments are done in two axis Direct Drive(DD) robot, for which no gear reduction is available to mask effective inertia variations.

#### The Benchmark Test of AC

In the previous section, the control laws of ACs were classified into three types, according to the signals used for computing control input. Based on the classification, it was argued that

- The main difference between the control laws of ACs is laid on the different PD gains consequently.
- The tracking performance is expected to be all the same, if effective PD gains are set evenly.

The experiments will clarify the above facts. The  $L^2$  norms of positional tracking errors are calculated at the interval  $[t_0, t]$ .

$$L^2[e(t)] = \sqrt{\frac{1}{t-t_0} \int_{t_0}^t \|e(t)\|^2 dt} \quad (12)$$

Fig. 3 illustrates  $L^2$  norms of positional tracking errors among three typed ACs. Although the difference of tracking errors becomes more evident by loading, it is slightly small. Moreover, the difference mainly results from the different PD gains. If the PD gains of the ACs are set evenly, even small difference in control performance is expected to disappear.

#### The Control Performance of ACs

After the benchmark test is done, the control performance of ACs is examined more in detail by implementing RT-AC.

Fig. 4 shows that the tracking performance of adaptive model based controllers is much superior to conventional PD controllers. The difference is much more obvious when unpredicted loads are induced. Notice that P-gain and D-gain in PD controllers are adjusted in the same manner with the ACs.

In Fig. 5, the tracking performance is investigated when the frequency of the command input is altered. As the frequency of the command input increases, the effect of disturbance by centrifugal and Coriolis term also becomes stronger. The tracking error increases as its frequency of the command input increases during its initial time interval, while the error becomes smaller as motion is repeated. It has a lot to do with the accuracy of robot model that the tracking error never converges to zero. In

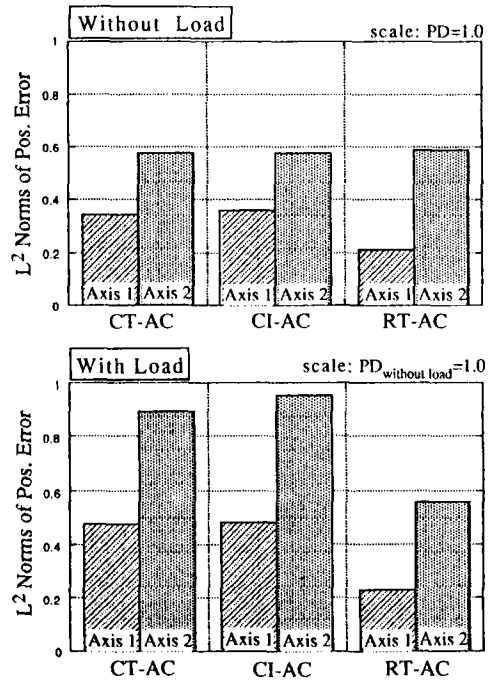


Figure 3: Comparison of  $L^2$  norm of AC

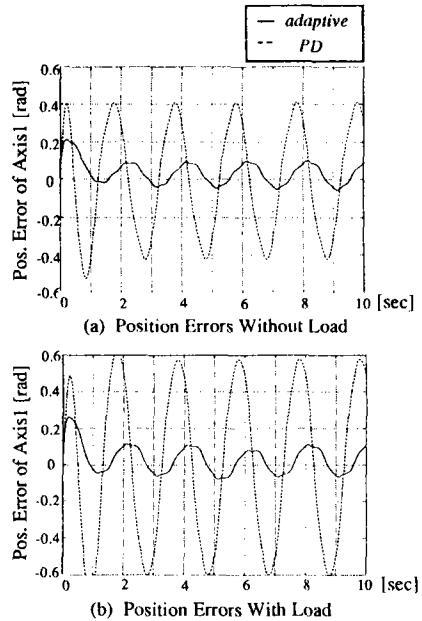


Figure 4: Comparison of AC and PD controller (with load (2.3kg), command input:  $\sin 2\pi ft$ , AC:  $K_d = \text{diag}[10, 10]$ ,  $\Lambda = \text{diag}[10, 10]$ ,  $\Gamma = \text{diag}[0.01, 0.01, 0.01]$ , PD: P-gain(100,100), D-gain(10,10))

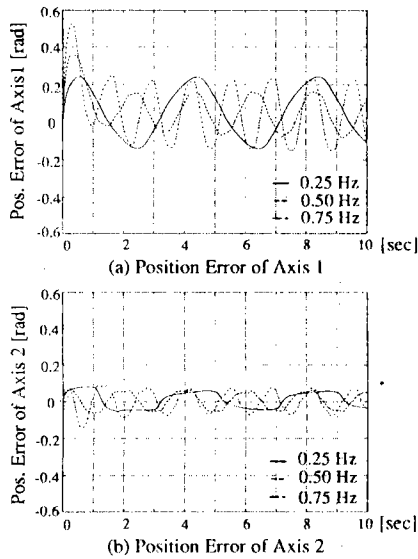


Figure 5: Tracking Error Varying the Frequency of Command Input (AC, command input:  $\sin 2\pi ft$ ,  $f=0.25, 0.5, 0.75$  [Hz],  $K_d=\text{diag}[10, 10]$ ,  $\Lambda=\text{diag}[5, 5]$ ,  $\Gamma=\text{diag}[0.01, 0.01, 0.01]$ )

our experiment, the viscous friction is entirely ignored in both of ACs and TDOF servosystem.

The two gains,  $\Lambda$  and  $K_d$ , must be designated adequately to perform good tracking with the specified trajectory. Fig. 6 and Fig. 7 indicate that high gain improves the tracking performance. However, it becomes clear in Fig. 7 that high gain may occur high frequency vibration or even unstable behavior.

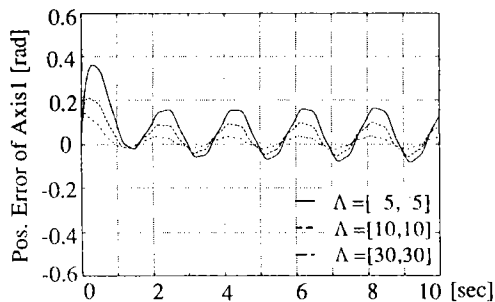


Figure 6: Tracking Error Varying Gain  $\Lambda$  (AC, command input:  $\sin \pi t$ ,  $\Lambda=5, 10, 30$ ,  $K_d=\text{diag}[10, 10]$ ,  $\Gamma=\text{diag}[0.01, 0.01, 0.01]$ )

Fig. 8 shows the tracking error when varying the initial value of the estimated parameter. Seeing from this figure, the adaptation seems to be well performed regardless of the initial value. Fig. 9, however, illustrates that the estimated parameter never reaches the real value. Instead,

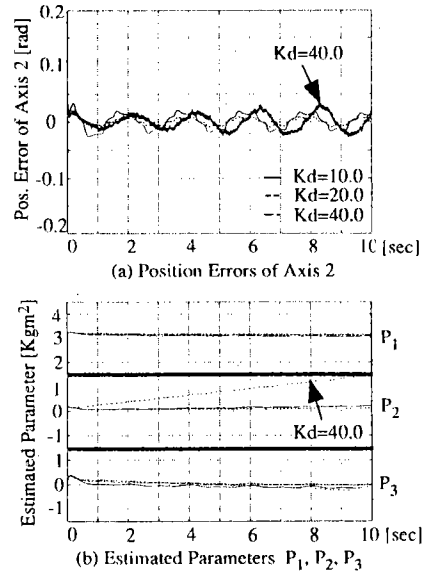


Figure 7: Tracking Error varying with Gain  $K_d$  (AC, command input:  $\sin \pi t$ ,  $K_d=10, 20, 40$ ,  $\Lambda=\text{diag}[20, 20]$ ,  $\Gamma=\text{diag}[0.01, 0.01, 0.01]$ )

the other parameters, which was not altered, change their values in order to perform good tracking.

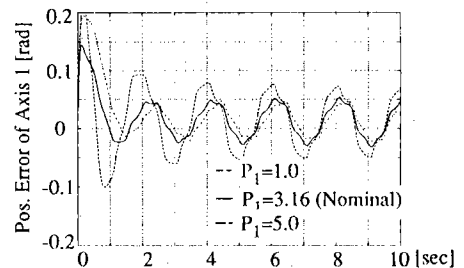


Figure 8: Tracking Error Varying the Initial Value of  $P_1$  (AC, command input:  $\sin \pi t$ ,  $P_1=1, 3.16$  (nominal),  $5$ ,  $\Gamma=\text{diag}[0.01, 0.01, 0.01]$ ,  $K_d=\text{diag}[10, 10]$ ,  $\Lambda=\text{diag}[20, 20, 20]$ )

## The Control Performance of TDOF

An adaptive controller can learn and adapt from repetitive motions in the sense that parameters are changed on-line, whereas a robust controller, for example, TDOF servosystem does not usually learn from past performance. Fig. 10 shows that the tracking errors produced by the fixed TDOF controller tend to be repetitive, while Fig. 5 reveals quite different result done by ACs.

In the previous section, we mentioned that the free parameter  $Q(s)$  determines the closed loop characteristics,

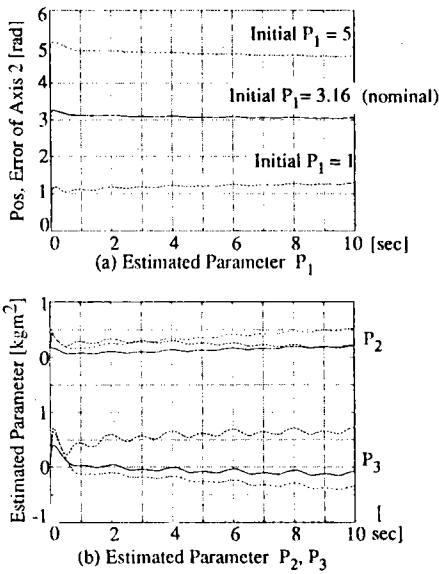


Figure 9: Tracking Error Varying the Initial Value of  $P_1$  (AC, command input:  $\sin \pi t$ ,  $P_1=1, 3.16(\text{nominal}), 5$ ,  $\Gamma=\text{diag}[0.01, 0.01, 0.01]$ ,  $K_d=\text{diag}[10, 10]$ ,  $\Lambda=\text{diag}[20, 20, 20]$ )

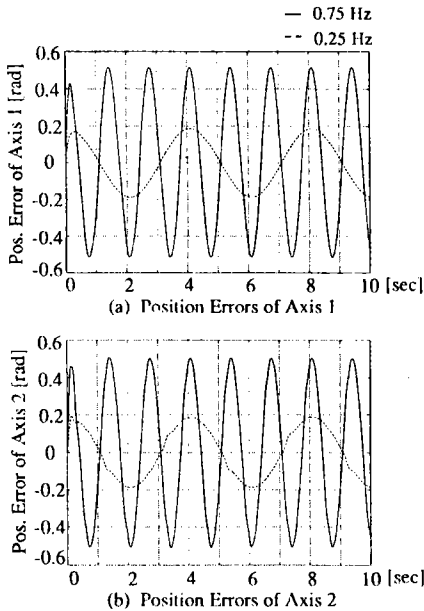


Figure 10: Tracking Error Varying the Frequency of Command Input (TDOF,  $\tau_r = 60\text{msec}$ ,  $\tau_n = 10\text{msec}$ )

that is, disturbance rejection and noise cancellation. In general,  $Q(s)$  is designed as low pass filter to suppress the effect of low-frequency disturbances. Fig. 11 indicates that the tracking errors are almost same regardless of the value  $\tau_n$ . The result means that most disturbances caused by this robot motion is enough suppressed, although no much attention is paid on the selection of the value  $\tau_n$ .

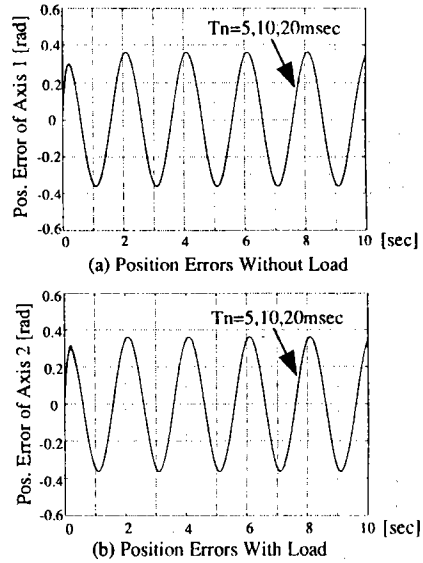


Figure 11: Tracking Error Varying the Cutoff Frequency of  $Q$  (TDOF,  $\tau_r = 60\text{msec}$ ,  $\tau_n = 5, 10, 20\text{msec}$ )

It may be thought that selecting the initial value of parameters is more important in a fixed controller rather than an adaptive one. Seeing from Fig. 12, it turns out to be less critical. The worse choice of the initial value does not deteriorate the stability of system. The control output is, however, corrupted by noise, because bad choice of the initial values makes system weaken its robustness to noise.

#### IV The Adaptive VS. The Robust

So far, we have mentioned ACs and TDOF, one of fixed robust controllers, through experiments. They differ from the method of research, that is, adaptation in AC and robustness in TDOF. Despite mutual difference, both of them attempt to accomplish basically the same objects, namely, control under plant uncertainty. This can enable us to compare the both.

The advantages of ACs result mainly from their property of adaptation. In our experiments, the ACs showed good adaptation, regardless of the initial values and the disturbances acted on robot joints. The adaptation and thus the tracking performance, however, depend on the accuracy of the adopted model. Suppose that the implemented model is not valid and good tracking cannot be

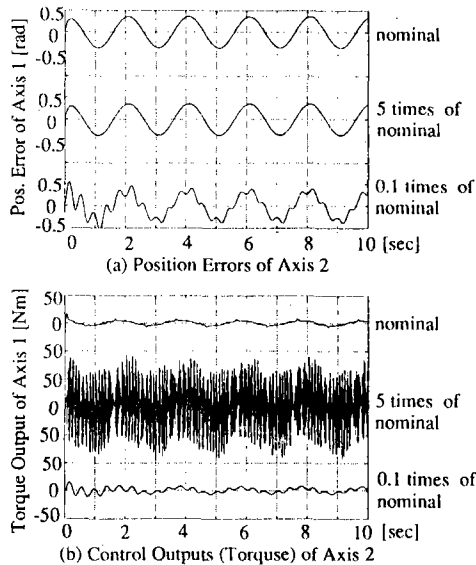


Figure 12: Tracking Error of Initial Value (TDOF,  $\tau_r = 60msec$ ,  $\tau_n = 10msec$ )

expected any more.

The disadvantages of ACs come from their model based structure. It is much more complicated compared to the TDOF servosystem. There is no effective experimental results so much in more than two axis robot. In addition, the selection of gains are troublesome and even adjusted gains may not be valid any more with another trajectory.

The Robust controller showed good tracking performance in our experiments using TDOF servosystem. Although it showed worse performance compared to optimally adjusted ACs, TDOF had enough merit, considering its simple structure and easy parameter adjustment. Especially, the surprising robustness against the change of plant parameters, is very attractive merit practically.

The potential disadvantages of TDOF-like robust controllers result from their non-adaptation. Even though the controller is misleadingly designed or inadequate, they cannot learn from repetitive motions and the tracking error is not decreasing. However, TDOF servosystem turns out to be very effective controller in our experiments, as long as the robot manipulator has rigid bodied structure.

## V Conclusion

In this paper, we have evaluated the tracking performance between adaptive controllers and one of robust controller, TDOF servosystem. The intensive experiments have been done in two axis DD robot and then the relative merits of each control were remarked.

In ACs, it has been shown that the control law of ACs can be divided into three types according to the signal for

the computed control input. This classification enables us to make proper comparison between ACs. According to the result, the main difference between the control law of ACs results from the different PD gains. The same tracking performance can be drawn only if PD gains are set to be same.

Next, we have suggested TDOF servosystem as one of the robust controllers for robot motion. With simple structure of the controller and easy adjustment of controller parameters, it outperforms the conventional PD and shows almost same degrees of tracking accuracy in our experiments.

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