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AN EXTENDED STUDY OF A SPACECRAFT ATTITUDE AND ORBIT CONTROL WITH AN INSUFFICIENT NUMBER OF THRUSTERS

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Abstract. An extended study of optimal thruster combination for simultaneous attitude and orbital maneuvers of a jet-controled spacecraft is conducted. In this case, the spacecraft has not enough number of thrusters to control the rotation and translation separately. Therefore, thrusters are employed by combining to eliminate their coupling effects. The combinations are determined to minimize the fuel consumption. The redundancy study for some thruster failure cases is also presented.

1. INTRODUCTION

For a spacecraft equipped with reaction control jets, translation and rotation are coupled by virtue of using the same actuators. In the case where we have enough freedom of actuators, we can control both maneuvers separately,but unfortunately in early design stage of our spacecraft "HOPE", the number of thrusters and the freedom of their possible locations have been quitely limitted. In the case of NASA's Space Schuttle where there is too many redundant thrusters, the thruster combination is obtained by employing the linear programing in real-time, however, in our case we have to develop another algorithm. The proposed method is as follows. First, calculate the optimal selection of thruster combinations and duty cycles for every possible requests of force and torque components. (up to two simultaneous components request) These calculations are implemented by a gradient method where the performance index is selected to minimize the fuel consumption and to minimize the number of thrusters in the combination. These results are summarized to make a decision table, which can control the spacecraft in real-time on the request of the force and torque components from the control system. In the case where more than three components are simultaneously requested, the control system devide the control phase in time sequence according to the importance level of the requests. The tables for some thruster failure cases are also simultaneously made. As the full process of making these decision table is not so time exhausting, we can easily apply the algorithm to satisfy the required specification changes.

2. ALGORITHM

The optimal thruster selection problem, under the requirment of desired force or torque components from the spacecraft control circuit, is expressed as a linear programing (LP) or a nonlinear programing (NLP). As for a former, the simplex method is often employed. Ref.(1) shows an algorithm of real-time implementation for the Space Shuttle, by LP. In the case, there are too many freedom of thruster selections, therefore it has many merits to decide the thrusters in real-time, in regard to control redundancy and fuel efficiency. However, in the case of our spacecraft HOPE, the number of thrusters is far smaller than that of the Space Schuttle, and selecting the thrusters based on the decision tables which are off-line calculated is easier to implement and has a merit, as the exact optimal solution is obtained by the off-line calculation. As the design of HOPE is greatly changed since this study has been conducted, therefore the results are not that of the current actual HOPE, but the method is easily applicable to any modified configuration. The decision tables are made by a gradient method by the following algorithm.

Let the m thrusters are employed, and the duty cycle of the i-th thruster of m, u_i, then

$$0 \le u_i \le 1 \qquad (i = 1 \sim m) \tag{1}$$

These thrusters are fixed to the body. Let the three components of the force and torque in the body frame, produced by the i-th thruster as $F_{xi}, F_{yi}, F_{zi}, T_{xi}, T_{yi}$, and T_{zi}, then they are expressed as follows.

$$(F_{xi}, F_{yi}, F_{zi}, T_{xi}, T_{yi}, T_{zi})^T = (a_{1i}u_i, a_{2i}u_i, \cdots, a_{6i}u_i)^T \quad (2)$$

Let K_j as the penalty coefficient to the sum of the j component of all thrusters.

$$P_j = K_j \sum_{i=1}^{m} a_{ji}u_i \quad (3)$$

Let the fuel consumption rate of i-th thruster as c_i, then, total fuel consumption rate P_o is given by

$$P_o = \sum_{i=1}^{m} c_i u_i \quad (4)$$

The employed thrusters and their duty cycles are calculated following to the next iteration algorithm.

(a) Let select appropriate (producing the desirable components, and effective to cancel the subsidial components) m thrusters and give their duty cycles

$$(0 \le u_i \le 1, \quad i = 1 \sim m) \quad (5)$$

as intial values.

(b)Calculate the value of the performance index z, and their gradient to $\bar{u}$.

$$\partial z / \partial \bar{u} = \left(\frac{\partial z}{\partial u_1}, \frac{\partial z}{\partial u_2}, \cdots \frac{\partial z}{\partial u_m} \right)^T \quad (6)$$

(c) choose the correction term $\Delta\bar{u}$, as follows.

$$\Delta\bar{u} = -W \left(\frac{\partial z}{\partial \bar{u}} \right)^T \quad (7)$$

where W is an arbitrary positive definite m dimensional matrix, however for simplicity, we will select the next diagonal matrix

$$W = \begin{bmatrix} w_1 & & 0 \\ & w_2 & \\ 0 & & w_m \end{bmatrix} \quad (8)$$

$$\left(w_i > 0 \quad i = 1 \sim m \right)$$

Let the new values of $\bar{u}$ as

$$\bar{u}_{new} = \bar{u}_{old} + \Delta\bar{u} \quad (9)$$

If some components of $\bar{u}_{new}$ violate the constraints (5), then the limited values 0 or 1 are selected for these u_i's.

The z value changes by $\Delta\bar{u}$ given in (7) becomes

$$\Delta z = \frac{\partial z}{\partial \bar{u}} \Delta\bar{u} = -\sum_{i=1}^{m} w_i \left(\frac{\partial z}{\partial u_i} \right)^2 \le 0 \quad (10)$$

therefore so far as there is a room for improvement, z decreases. The $\bar{u}_{new}$ componens given by (9) is applied as new u_i's in (5), and from step (a)to (c) are repeated until the z value can not be decreased any more.

The employed performance index is as follows.

$$z = koPo - \sum_{j \in L_1} k_j p_j + \sum_{j \in L_2} k_j p_j^2 \quad (11)$$

$$L = \{1, 2, \cdots, 6\}, \quad L_1 = \{l_1, \cdots, l_s\}, \quad L_2 = L - L_1 \quad (12)$$

$$\left(\bar{F}, \bar{T}\right)^T = \left(F_X, F_Y, F_Z, T_X, T_Y, T_Z\right) \quad (13)$$

where L_1 is the numbers of the required force or torque components and L_2 is the other (undesired) components.

The first term of (11) is to minimize the total fuel consumption. While the second term is to maximize the required force or torque conponents where the signs of K_j's are determined following to the signs of above components. The third term is introduced to minimize undesirable force or torque components.

3. RESULTS AND DISCUSSIONS

In the early design of HOPE, the total of fourteen thrusters, namely F1 through F6 and A1 through A8 are provided. The symbols "F" and "A" are derived from their location (Front or Aft of C.G.) Table 1 shows the thrust and torque component produced by the thrusters per unit fuel consumption. Table 2 shows the basic thruster combinations obtained from section 2, for corresponding to required force or torque components. As for discribed efficiency values, the value 1.0 for force means that there is no loss. For torques, the torque armlength are multiplied to the force, therefore the values exceed 1.0, however, the larger values mean the higher efficiency. From Table 2 , six thrusters have to be employed to produce +Fx force, which is a little complicated. Decision tables are also made for simultaneous control of arbitrary two components of force and torque, which show higher efficiency than control those components separately based on Table 2. Table 3 shows a part of decision table for simultaneously controlling two components. For example, two candidates exist for controlling +Fx and +Fy simultaneously. The described efficiency shows the produced +Fx and +Fy per 1N thrust, therefor their sum is consideres as the total efficiency. Although both candidates produce very small undesirable torque, however the total efficiency is higher than controling +Fx and +Fy separately. Considering the redundancy, the substitution tables are also made for arbitrary one thruster failure cases. Table 4 shows a part of decision tables for these substitution tables. For example, nominal combination to produce +Fx requires thrusters F3,F4,A3,A4,A7 and A8. The table shows the substitutional combination in the lack of one of these thrusters. In this case the failure of any thruster is substituted by another combination without employing the failed thruster, although it brings about a small reduction of the efficiency, or produce a small undesirable subsidiary torque component. However, there are some cases where these substitution are not possible. The study shows that, if F5 failes, -Fx control become impossible, and if F6 fails, +Fz and -Ty control become impossible. A1 failure brings about very low efficiency for +Fy and-Tz controls, and A2 failure also prduces the same problem for -Fy and +Tz controls. As the current thruster design of HOPE is perfectly changed from that of this study, but the design method expressed in this paper is easily applicable and can be employed to improve the spacecraft jet control system.

4. CONCLUSIONS

A spacecraft optimal thruster combination study with an insufficient number of thruster is conducted. The study employed a gradient method which can calculate optimal thruster combinations and their duty cycles. The results are summarized to make decision tables, and the spacecraft is controlled in real-time based on these tables. The algorithm and an example is shown in the paper.

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REFERENCE

Redding, D. C. and Persson, B. A., " Combined Solution of Spacecraft Rotational and Translational Maneuvers ", AIAA paper 86-2106, (1986).

Table 1. Normarized thruster force and torque components

Thruster No.	Thrust (N)			Torque (Nm)		
	x	y	z	x	y	z
F1	0	1	0	-0.15	0	3
F2	0	-1	0	0.15	0	-3
F3	0	0.7071	-0.7071	0.3536	2.1232	2.1232
F4	0	-0.7071	-0.7071	-0.3536	2.1232	-2.1232
F5	-1	0	0	0	0.45	0
F6	0	0	1	0	-3	0
A1	0	1	0	0.25	0	-2.8
A2	0	-1	0	-0.25	0	2.8
A3	0	0	1	-0.8	2.8	0
A4	0	0	1	0.8	2.8	0
A5	0	0	-1	0.8	-2.8	0
A6	0	0	-1	-0.8	-2.8	0
A7	1	0	0	0	-0.25	0.8
A8	1	0	0	0	-0.25	-0.8

Table 2. Basic thruster combinations and their duty cycles for arbitrary force or torque components

Required force and torque	Thruster combinations (Duty cycle)	Efficiency (N) (Nm)	Thrust (N)			Torque (Nm)		
			FX	FY	FZ	TX	TY	TZ
+FX	F3 + F4 + A3 + A4 + A7 + A8 .061 .061 .043 .043 1.0 1.0	0.960	2.000					
-FX	F5 + F6 + A5 + A6 1.0 .078 .039 .039	0.865	-1.000					
+FY	F1 + A1 + A3 + A6 .933 1.0 .069 .069	0.934		1.933				
-FY	F2 + A2 + A4 + A5 .933 1.0 .069 .069	0.934		-1.933				
+FZ	F6 + A3 + A4 1.0 .536 .536	1.000			2.072			
-FZ	F3 + F4 + A5 + A6 1.0 1.0 .758 .758	0.833			-2.930			
+TX	A4 + A5 1.0 1.0	0.800				1.6000		
-TX	A3 + A6 1.0 1.0	0.800				-1.6000		
+TY	F3 + F4 + A3 + A4 1.0 1.0 .707 .707	2.404					8.208	
-TY	F6 + A5 + A6 1.0 .500 .500	2.900					-5.802	
+TZ	F1 + A2 + A4 + A5 1.0 1.0 .250 .250	2.320						5.800
-TZ	F2 + A1 + A3 +A6 1.0 1.0 .250 .250	2.320						-5.800

Table 3. Simultaneous controlling two components

Required force and torque	Thruster combinations (Duty cycle)	Efficiency (N),(Nm)	Thrust (N)			Torque (Nm)		
			FX	FY	FZ	TX	TY	TZ
+FX,+FY NO.1	F1+ F3+ A1+ A3+ A7+ A8 .848 .122 1.0 .087 1.0 1.0	0.493 0.477	2.000	1.934				
+FX,+FY NO.2	F1+ F3+ A1+ A3+ A6+ A7+ A8 .847 .122 1.0 .146 .060 1.0 1.0	0.479 0.463	2.000	1.934				
+FX,-FY NO.1	F2+ F4+ A2+ A4+ A7+ A8 .912 .031 1.0 .155 1.0 1.0	0.488 0.472	2.000	-1.934	0.133			
+FX,-FY NO.2	F2+ F4+ A2+ A4+ A5+ A7+ A8 .847 .122 1.0 .147 .060 1.0 1.0	0.479 0.463	2.000	-1.933				
+FX,+FZ	F6+ A3+ A4+A7 +A8 1.0 .625 .625 1.0 1.0	0.471 0.529	2.000		2.250			
+FX,-FZ	F3+ F4+ A5+ A6+ A7 +A8 1.0 1.0 .669 .669 1.0 1.0	0.375 0.516	2.000		-2.753			
+FX,+TX	F2+ F3+ A4+ A5+ A7+ A8 .086 .121 1.0 .914 1.0 1.0	0.485 0.385	2.000			1.587		
+FX,-TX	F1+ F4+ A3+ A6+ A7+ A8 .086 .121 1.0 .914 1.0 1.0	0.485 0.385	2.000			-1.587		
+FX,+TY	F3+ F4+ A3+ A4+ A7 +A8 1.0 1.0 .707 .707 1.0 1.0	0.369 1.423	2.000				7.706	
+FX,-TY	F6+ A5+ A6+ A7+ A8 1.0 0.5 0.5 1.0 1.0	0.500 1.575	2.000				-6.300	
+FX,+TZ NO.1	F1+ F3+ D6+ A2+ A7 .385 .870 .533 1.0 1.0	0.264 1.743	1.000		-0.082			6.603
+FX,+TZ NO.2	F1+ F3+ F6+ A2+ A4+ A7 .438 .795 .519 1.0 .043 1.0	0.263 1.740	1.000					6.603
+FX,-TZ NO.1	F2+ F4+ F6+ A1+ A7+ A8 .385 .870 .449 1.0 1.0 1.0	0.425 1.233	2.000		-0.166			-5.802
+FX,-TZ NO.2	F2+ F4+ F6+ A1+ A3+ A7+ A8 .491 .720 .423 1.0 .086 1.0 1.0	0.424 1.229	2.000					-5.801
-FX,+FY NO.1	F1+ F5+ F6+A1+ A6 .934 1.0 .077 1.0 .078	0.324 0.626	-1.000	1.934		0.048		
-FX,+FY NO.2	F1+ F5+ F6+ A1+ A3+ A6 .934 1.0 .077 1.0 .030 .108	0.318 0.614	-1.000	1.934				
-FX,-FY NO.1	F2+ F5+ F6+ A2+ A5 .934 1.0 .078 1.0 .077	0.324 0.626	-1.000	-1.934		-0.048		
-FX,-FY NO.2	F2+ F5+ F6+ A2+ A4+ A5 .933 1.0 .078 1.0 .029 .108	0.318 0.614	-1.000	-1.933				
-FX,+FZ	F5+ F6+ A3+ A4 1.0 1.0 .455 .455	0.344 0.656	-1.000		1.911			
-FX,-FZ	F3+ F4+ F5+ A5+ A6 1.0 1.0 1.0 .839 .839	0.214 0.661	-1.000		-3.092			
-FX,+TX	F5+ F6+ A4+ A5 1.0 .077 .922 1.0	0.333 0.513	-1.000			1.538		
-FX,-TX	F5+ F6+ A3+ A6 1.0 .077 .922 1.0	0.333 0.513	-1.000			-1.538		
-FX,+TY NO.1	F3+ F4+ F5+ A3+ A4 1.0 1.0 1.0 .708 .708	0.226 1.961	-1.000				8.659	
-FX,+TY NO.2	F5 1.0	1.000 0.450	-1.000				0.450	

Table 4. Substitution table for thruster failure cases

Required force and torque	Thruster combinations (Duty cycle)	Efficiency (N),(Nm)	Thrust (N)			Torque (Nm)		
			FX	FY	FZ	TX	TY	TZ
+FX nominal	F3+ F4+ A3+ A4+ A7+ A8 .061 .061 .043 .043 .1.0 1.0	0.906	2.000					
+FX F3 fail.1	F1+ F4+ A4+ A7+ A8 .086 .121 .086 1.0 1.0	0.872	2.000			0.013		
+FX F3 fail.2	F1+ F4+ A3+ A4+ A7+ A8 .085 .121 .008 .078 1.0 1.0	0.872	2.000					
+FX F4 fail.1	F2+ F3+ A3+ A7+ A8 .086 .121 .086 1.0 1.0	0.872	2.000			-0.013		
+FX F4 fail.2	F2+ F3+ A3+ A4+ A7+ A8 .086 .121 .078 .008 1.0 1.0	0.872	2.000					
+FX A3 fail	same as F3 fail.1							
+FX A4 fail.1	same as F4 fail.1							
+FX A4 fail.2	F2+ F3+ F6+ A3+ A7+ A8 .105 .149 .020 .085 1.0 1.0	0.848	2.000				-0.004	
+FX A7 fail	F1+ F3+ A2+ A4+ A8 .094 .061 .137 .043 1.0	0.749				0.007		
+FX A8 fail	F2+ F4+ A1+ A3+ A7 .094 .061 .137 .043 1.0	0.749				-0.007		
-FX nominal	F5+ F6+ A5+ A6 1.0 .078 .039 .039	0.865	-1.000					

Required force and torque	Thruster combinations (Duty cycle)	Efficiency (N),(Nm)	Thrust (N)			Torque (Nm)		
			FX	FY	FZ	TX	TY	TZ
-FX F5 fail	no alternative substitute exists							
-FX F6 fail	F5+ A5+ A6 1.0 .080 .080	0.862	-1.000		-0.161			
-FX A5 fail	F2+ F3+ F5+ F6+ A6 .095 .134 1.0 .172 .077	0.677	-1.000				0.005	
-FX A6 fail	F1+ F4+ F5+ F6+ A5 .095 .134 1.0 .172 .077	0.677	-1.000				-0.005	
+FY nominal	F1+ A1+ A3+ A6 .935 .1.0 .069 .069	0.934		1.933				
+FY F1 fail	F3+ F6+ A1+ A3+ A6 1.0 .708 .758 .340 .339	0.466		1.465				
+FY A1 fail	F1+ F5+ F6+ A5+ A8 .267 1.0 .035 .034 1.0	0.114		0.267		-0.012		
+FY A3 fail.1	F1+ F4+ F6+ A1 1.0 .217 .153 .907	0.770		1.753				
+FY A3 fail.2	F1+ A1 .933 1.0	1.000		1.933		0.110		
+FY A6 fail	same as A3 fail.1 and A3 fail.2							