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Near-Resonant Attitude Motion Analysis of a Spinning Satellite via Multiple Scales Method

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ABSTRACT

The attitude stability of a satellite in spin-stabilized injection mode which contains a liquid pool is investigated. The satellite model for investigation is a two-body system consisting of a main body, which is symmetric and rigid, representing the spacecraft, and a spherical pendulum, representing the liquid pool. Assuming that both spacecraft and pendulum are in states of steady spin about the symmetry axis of the spacecraft, the coupled nonlinear equations of motion for the system are simplified. In this paper, by using the multiple scales method, the possible resonance conditions in terms of the system parameters are determined and the corresponding near-resonant solutions are derived .

INTRODUCTION

Usually, nutation of a spinning spacecraft is caused by a single asymmetric impulse due to, for example, separation spring imbalance, rocket motor tailoff, motor side forces, thrust vector misalignment, or principal axis misalignment. Most causes can be minimized or eliminated in design and manufacturing processes. However, if the spacecraft does not maintain its rigid configuration during maneuver, i.e., if it has an internal moving parts such as flexible structure or sloshing liquid, the nutational motion occurs and causes antenna depointing. The nutation instability of the spinning spacecraft carrying the internal moving parts has not been explained well.

Mingori and Yam[1] developed linear stability conditions for a spacecraft consisting of a symmetric rigid body and a planar pendulum, to present the sloshing mass. In their model, the pendulum mass is attached to its pivot by a spring and is free to rotate about the symmetry axis of the spacecraft. Assuming that the spacecraft is in a state of steady spin about its symmetry axis, they obtained linear equations and showed the possibility for unstable coning growth if the thrust magnitude is sufficiently large and the moving mass is aft of the system mass center. In any case, the developed stability criteria are, in author's judgment, not applicable to nonlinear systems such as the spinning powered spacecraft with internal sloshing mass. Since the equations of motion were derived using constant parameters, the model fails to predict the dynamic behavior of the actual system whose physical parameters vary widely with time.

Or[2] also carried out a linear analysis of the stability of a spinning, thrusting spacecraft in a manner similar to that used by Mingori and Yam. To model the liquid pool, he used two types of pendulum models, one being a spherical pendulum model, the other model consisting of two orthogonal plane pendulums. His results showed that both model will produce an unstable motion if "tuned pendulum conditions" are used. The conditions he used are, however, unrealistic, since matching simulated results with flight data using his model requires a pendulum length and pivot location which places the pendulum mass outside the spacecraft. Another drawback of his model is that large amplitude motion of the linear pendulum occurs in contradiction to the assumption of linear motion. This is thought to be due to the fact that his linear equations were obtained by expanding about the vertical equilibrium condition of the pendulum. Apparently, the vertical orientation of the spinning pendulum is always statically unstable.

Cochran and Kang[3] has obtained significant results for describing the attitude motion of the spin-stabilized thrusting spacecraft by using the full nonlinear mathematical models to complement earlier research by others based on linearized pendulum models. By treating the motion of liquid as a perturbation of the motion of the spacecraft as a whole, they obtained nonlinear equations including viscous effects.

Recently, Kang and Shin[4] developed a complete set of the mathematical models, based on the previous study, to characterize the depointing mechanism of the toroidal beam omni antenna and investigated the interrelationship between the pointing instability and the system parameters such as the body configurations, mass variations and energy input due to thrusting. Three basic spacecraft configurations were used for numerical test.

In the present work, by imposing some restrictions on the equations of motion governing a two-body satellite system in spin-stabilized injection mode, resonance conditions are determined and the corresponding solutions are derived.

EQUATIONS OF MOTION

Figure 1 shows a two-body model consisting of a rigid body, representing the spacecraft proper, and a point-mass spherical pendulum, representing a liquid pool. The rigid body, B_1, of mass m_1 with centroidal, principal moments of inertia, $I_1 \geq I_2 > I_3$, about the axes x_1, y_1 and z_1, respectively, and center of mass, C_J. The axes X, Y and Z are a non rotating set. The pendulum consists of a point mass m_2 attached to a rigid massless rod which in turn is attached at a point O to the body B_J. Its motion with respect to the body B_J, is defined by the angles θ and ψ. The length of the pendulum is denoted by $\underline{l}$ and vector from C_J to O is represented by $\underline{r}_0$. Additionally, the angular velocity of the centroidal coordinate system $Cxyz$ which rotates with the rigid body is denoted by $\underline{\omega} = \omega_1 \hat{e}_1 + \omega_2 \hat{e}_2 + \omega_3 \hat{e}_3$ and the attitude of the $Cxyz$ system is defined by the Euler angles θ_3, θ_2 and θ_1 as shown in Fig.2.

Equations of motion may be derived for the system of two bodies by using various methods. We have chosen a Newton-projection method in which we write the equations of motion of the system and the equations of relative motion for the pendulum by projection.

Equations governing the motion for the system about its center of mass may be derived by first writing the angular momentum of the system about C in the form

$$\underline{H} = \underline{\underline{I}} \cdot \underline{\omega} + \frac{m_1 m_2}{M} \underline{r}_2 \times \{(\underline{\omega} \times \underline{r}_2) + \dot{\underline{r}}_2\} \quad (1)$$

where $\underline{\underline{I}}$ is the centroidal inertia dyadic of the system, $M = m_1 + m_2$, $\underline{r}_2 = \underline{r}_0 + \underline{l}$ and $\dot{\underline{r}}_2$ is the time rate of change of $\underline{r}_2$ due to motion of the pendulum relative to B_J. By taking components in the $Cxyz$ system we can write the following matrix form of $\underline{H}$:

$$\underline{H} = \underline{\underline{J}}\underline{\omega} + \underline{h} \quad (2)$$

where $\underline{\underline{J}} = \underline{\underline{I}} - m_1 \mu \tilde{\underline{r}}_2 \tilde{\underline{r}}_2$ and $\underline{h} = m_1 \mu \tilde{\underline{r}}_2 \dot{\underline{l}}$ and we have introduced the symbol $\mu = m_2 / M$. The tilde over $\underline{r}_2$ denotes the skew-symmetric matrix used to form components of cross-products of vectors and $\underline{l} = l\{-\sin\theta \sin\psi \quad \sin\theta\cos\psi \quad -\cos\theta\}^T$. The time derivative of the vector $\underline{H}$ and Eq.(2) may be used to get the matrix equations,

$$\dot{\underline{H}} = \tilde{\underline{H}} \underline{\underline{J}}^{-1} (\underline{H} - \underline{h}) + \underline{T} \quad (3)$$

where $\underline{T}$ is the external torque about the C. To model $\underline{T}$, we assume that the thrust vector $\underline{F}$ passes through the point C_J. Then, $\underline{T} = -\mu \underline{r}_2 \times \underline{F}$. Or, in matrix form

$$\underline{T} = -\mu \tilde{\underline{r}}_2 \underline{F} \quad (4)$$

An equation for relative motion of the pendulum is more difficult to find. One way to obtain a suitable equation is to write the absolute acceleration of m_2, equate it to the force on m_2 and cross $\underline{l}$ into both sides of the resulting equation. The equation is

$$\underline{T}_0 = m_1 \mu l^2 \tilde{\underline{u}}_l \left\{ \ddot{\underline{u}}_l - \tilde{\underline{u}}_r \dot{\underline{\omega}} - 2\tilde{\underline{\omega}} \dot{\underline{u}}_l - \tilde{\underline{\omega}} \tilde{\underline{u}}_r \underline{\omega} + \frac{\underline{F}}{m_1 l} \right\} \quad (5)$$

where $\underline{u}_l = \underline{l}/l$, $\underline{u}_r = \underline{u}_l + (0 \;\; 0 \;\; -\underline{r}_0/l)^T$ and $\underline{T}_0$ is the torque about O.

Because we have $\dot{\underline{\omega}}$ in Eq.(5) instead $\dot{\underline{H}}$, there is some measure of complication in solving Eq.(5) for $\ddot{\theta}$ and $\ddot{\psi}$. However, the necessary substitutions may be carried out fairly easily in a digital computer program. Then we may find the necessary equations by using the projection matrix

$$\underline{\underline{B}} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

The first row of $\underline{\underline{B}}$ is a unit vector perpendicular to the plain in which θ is measured. The second row is a unit vector orthogonal to the first column. Equations (3) and (5), along with suitable kinematic equations for θ_1, θ_2 and θ_3, constitute a mathematical model of the systems, they do not provide any insight into the possibility of resonances.

ATTITUDE RESONANCE ANALYSIS

The angular momentum variables are $\underline{H}$; $a = H\sin\Theta$ where Θ is the torque-free nutation angle; Φ, the angle of proper rotation; and Ψ, the angle of precession of the z-axis about $\underline{H}$[5][6]. Because $\underline{H}$ is not constant, the orientation is not, in general, fixed. The angles Θ_H and Ψ_H may be used to define the direction of $\underline{H}$. The new variables a, H and Φ, may be used to write

$$H_1 = a\sin\Phi$$

$$H_2 = a\cos\Phi \quad (7)$$

$$H_3 = \sqrt{H^2 - a^2}$$

Equations (7) may be used in Eqs.(3) and (5) to get the approximate equations through first order in $\varepsilon = \mu m_1 l^2 / I_1$. This approximate equation set consists of highly coupled, nonlinear equations, some of which have quasi-linear form and some that do not. Apparently, neither linear solutions, nor approximate nonlinear solutions for the given equations can be obtained without further restrictions or assumptions. In fact, the problems arise from the relatively complex nature of the motion of the spherical pendulum coupled to the dynamics of the spinning main body. Thus, if we make the pendulum equations somewhat more simple and find the solutions to them, we may solve the entire problem. For this purpose, we can make the further assumptions that both spacecraft and pendulum are in states of quasi-steady spin about the symmetry axis of the spacecraft. Justification of the assumption of a steady spinning spacecraft is based on using constant mass moments of inertia. Then, one can assume that the pendulum also spins in quasi-steady state with small periodic motion. Numerical simulation results from the previous work[3] showed that this assumption is valid.

Using the above assumptions and selecting dominant terms only, the approximate equations of motion for the spacecraft and the pendulum are;

$$\dot{a} = \varepsilon \left[\left\{ H_3 P \left(\frac{r_0}{l} + \cos\theta \right) - \frac{F I_1}{m_1 l} \right\} \sin\theta \sin w - H_3 \dot{\theta} \left(1 + \frac{r_0}{l} \cos\theta \right) \cos w - \frac{a H_3}{2 I_1} \sin^2\theta \sin 2w \right] \quad (8)$$

$$\dot{w} = P - \frac{H_3}{I_1} + \varepsilon\frac{1}{a}\left[\left\{H_3\left(P - \frac{a^2}{I_3 H_3}\right)\left(\frac{r_0}{l} + \cos\theta\right) - \frac{F_3 I_1}{m_1 l}\right\}\sin\theta\cos w + \left\{\frac{aH_3}{2I_1}(1-\cos 2w) - \frac{I_1}{I_3}aP\right\}\sin^2\theta \right. \quad (9)$$
$$\left. + H_3\dot{\theta}(1 + \frac{r_0}{l}\cos\theta)\sin w + \frac{aH_3}{I_1}\left(\frac{r_0}{l} + \cos\theta\right)^2\right]$$

$$\ddot{\theta} = \left(P^2\cos\theta - \frac{F_3}{m_1 l}\right)\sin\theta + \frac{aH_3}{I_1^2}\left(1 + \frac{r_0}{l}\cos\theta\right)\cos w - \frac{2aP}{I_1}\sin^2\theta\cos w + \frac{1}{\mu m_1 l^2}\left(\dot{T}_{0_1}\cos\psi + T_{0_2}\sin\psi\right) \quad (10)$$

where

$P = \left(\dot{\psi} + \frac{H_3}{I_3}\right) + O(\varepsilon)$

a = magnitude of lateral angular momentum

$w = \Phi + \psi$

H_3 = angular momentum along with the spin axis, z_1 (11)

θ = pendulum rotation angle about y_1

ψ = pendulum rotation angle about z_1

$\mu = m_2/(m_1 + m_2)$.

Equation (10) can be expanded about a particular value of θ, say θ_0 and when the friction torques are dropped, the following equation is obtained

$$\ddot{\vartheta} = \left(P^2\cos\theta_0 - \frac{F_3}{m_1 l}\right)\sin\theta_0 + \left(\frac{F_3}{m_1 l}\cos\theta_0 - P^2\cos 2\theta_0 + c_1 a\cos w\right)\vartheta + c_2 a\cos w + h.o.t \quad (12)$$

where as usual $h.o.t$ denotes higher order terms in ε and

$$c_1 = \frac{H_3 r_0\sin\theta_0}{I_1^2 l} + \frac{2P\sin 2\theta_0}{I_1} \quad (13a)$$

$$c_2 = \frac{H_3 r_0\cos\theta_0}{I_1^2 l} + \frac{P(\cos 2\theta_0 - 1)}{I_1} + \frac{H_3}{I_1^2} \quad (13b)$$

In a non resonance case, a is relatively small so that $c_1 a$ is much smaller. Then, an approximate stationary solution can be found by setting

$$P^2\cos\theta_0 - \frac{F_3}{m_1 l} \approx 0. \quad (14)$$

By using this solution, Eq.(12) can be written

$$\ddot{\vartheta} \approx -\omega_0^2\left(1 + \frac{c_1}{\omega_0^2}a\cos w\right)\vartheta + c_2 a\cos w + h.o.t. \quad (15)$$

where

$$\omega_0^2 = P^2\sin\theta_0 \quad (16)$$

As it can be observed in Eq.(15), the pendulum is subject to parametric and "external" excitations. The "external" excitation is not truly external to the system, but may be treated as such to zeroth order in ε. If the motions are not small, the effect of nonlinearity on the stability must be considered. In this paper, the motion of the pendulum with respect to equilibrium is assumed small so that the higher order terms in ϑ are dropped from the equation. For convenience, Eq.(15) can be rewritten in a form similar to Mathieu's equation with forcing functions. To this end, let

$$\begin{aligned} &w = \Omega t + w_0 \\ &t = (2/\Omega)\tau \\ &\delta = 4\omega_0^2/\Omega^2 \\ &\alpha_1 = 2c_1 a\cos w_0/(m_1\mu l^2\Omega^2) \\ &\alpha_2 = -2c_1 a\cos w_0/(m_1\mu l^2\Omega^2) \\ &k_1 = 4c_2 a\cos w_0/(m_1\mu l^2\Omega^2) \\ &\text{and} \\ &k_2 = -4c_2 a\sin w_0/(m_1\mu l^2\Omega^2) \end{aligned} \quad (17)$$

Then, the solution of Eq.(15) may be written in the form

$$\vartheta'' + (\delta + 2\varepsilon\alpha_1\cos 2\tau + 2\varepsilon\alpha_2\sin 2\tau)\vartheta = \varepsilon(k_1\cos 2\tau + k_2\sin 2\tau) \quad (18)$$

where the double prime denotes $d^2\cdot/d\tau^2$. To find approximate solutions for Eq.(18), one may employ the method of multiple scales[7] defined by

$$T_n = \varepsilon^n\tau \qquad \text{for } n = 0,\ 1,\ 2,\ \ldots \quad (19)$$

Then, the solution of Eq.(18) can be represented by an expansion having the form

$$\vartheta(\tau;\varepsilon) = \vartheta_0(T_0, T_1, T_2, ..) + \varepsilon\vartheta_1(T_0, T_1, T_2, ..) + \varepsilon^2\vartheta_2(T_0, T_1, T_2, ..) + \ \ldots \quad (20)$$

For convenience, let

$$\frac{\partial}{\partial T_0} = D_0,\quad \frac{\partial}{\partial T_1} = D_1,\quad \frac{\partial}{\partial T_2} = D_2,\ \ldots\ \frac{\partial}{\partial T_i} = D_i\ \ldots \quad (21)$$

Carrying out the expansion to the order of ε^2 and equating the coefficients of equal powers of ε, one may find the following equations:

$$D_0^2\vartheta_0 + \delta\vartheta_0 = 0 \quad (22)$$

$$D_0^2\vartheta_1 + \delta\vartheta_1 = -2D_0 D_1\vartheta_0 - 2\alpha_1\vartheta_0\cos 2T_0 - 2\alpha_2\vartheta_0\sin 2T_0 + k_1\cos 2T_0 + k_2\sin 2T_0 \quad (23)$$

$$D_0^2\vartheta_2 + \delta\vartheta_2 = -2D_0 D_2\vartheta_0 - D_1^2\vartheta_0 - 2D_0 D_1\vartheta_1 - 2\alpha_1\vartheta_1\cos 2T_0 - 2\alpha_2\vartheta_1\sin 2T_0 \quad (24)$$

The general solution to Eq.(22) can be written in the form

$$\vartheta_0 = A(T_1, T_2)e^{i\beta T_0} + \overline{A}(T_1, T_2)e^{-i\beta T_0} \quad (25)$$

where $\beta = \delta^{1/2}$ and $\overline{A}$ is the complex conjugate of A. Then, Eq.(23) may be written as

$$D_0^2\vartheta_1+\delta\vartheta_1=-2i\beta D_1 Ae^{i\beta T_0}-(\alpha_1-i\alpha_2)Ae^{i(2+\beta)T_0} -(\alpha_1-i\alpha_2)\overline{A}e^{i(2-\beta)T_0}+\frac{1}{2}(k_1-ik_2)e^{i2T_0} + complex\ conjugate \tag{26}$$

Since the behavior of the motion near resonance is of concern, a detuning parameter σ can be introduced such that

$$\beta_r=\beta+\varepsilon\sigma \tag{27}$$

The parameter σ allows one to describe the nearness of the excitation frequency to the system frequency and helps one recognize the terms in the governing equation for ϑ_1 that lead to secular, or nearly secular terms. In analyzing the particular solution of Eq.(26), only two cases of the resonance will be considered. Then, approximate solutions to the pendulum equations for each case are derived.

Resonance Case 1: $\beta\approx 1$ ($\dot{w}\approx 2P\sin\theta_0$)

This is the case when the natural frequency of the system is close to one half of the frequency of the parametric excitation. The instability of the system occurs when

$$-\frac{\alpha}{2}+\varepsilon\frac{\alpha^2}{4}-\varepsilon^2\frac{\alpha^3}{16}+\ldots<\sigma<\frac{\alpha}{2}+\varepsilon\frac{\alpha^2}{4}+\varepsilon^2\frac{\alpha^3}{16}+\ldots \tag{28}$$

where

$$\alpha=\sqrt{\alpha_1^2+\alpha_2^2}. \tag{29}$$

Hence the transition curves emanating from $\delta=1$ are given by

$$\delta=1-\varepsilon\alpha-\varepsilon^2\frac{1}{2}\alpha^2-\varepsilon^3\frac{1}{8}\alpha^3+O(\varepsilon^4) \tag{30}$$

and

$$\delta=1+\varepsilon\alpha-\varepsilon^2\frac{1}{2}\alpha^2+\varepsilon^3\frac{1}{8}\alpha^3+O(\varepsilon^4) \tag{31}$$

The approximate solution near $\beta=1$ is

$$\begin{aligned}\vartheta&=a_1e^{\varepsilon\lambda\frac{\Omega}{2}t}(\cos\frac{\Omega}{2}t-\nu_1\sin\frac{\Omega}{2}t)\\&+a_2e^{-\varepsilon\lambda\frac{\Omega}{2}t}(\cos\frac{\Omega}{2}t+\nu_2\sin\frac{\Omega}{2}t)\\&+\varepsilon\frac{1}{8}a_1e^{\varepsilon\lambda\frac{\Omega}{2}t}\left\{(\alpha_1+\nu_1\alpha_2)\cos\frac{3\Omega}{2}t-(\nu_1\alpha_1-\alpha_2)\sin\frac{3\Omega}{2}t\right\}\\&+\varepsilon\frac{1}{8}a_2e^{-\varepsilon\lambda\frac{\Omega}{2}t}\left\{(\alpha_1-\nu_2\alpha_2)\cos\frac{3\Omega}{2}t+(\nu_2\alpha_1+\alpha_2)\sin\frac{3\Omega}{2}t\right\}\\&-\varepsilon\frac{1}{3}(k_1\cos\Omega t+k_2\sin\Omega t)+O(\varepsilon^2)\end{aligned} \tag{32}$$

where

$$\lambda=\frac{1}{2}\sqrt{\alpha^2-4\sigma^2}\qquad \nu_1=\frac{2\lambda-\alpha_2}{\alpha_1+2\sigma}\qquad \nu_2=-\frac{2\lambda+\alpha_2}{\alpha_2+2\sigma}$$

a_1 and a_2 are integration constants.

Resonance Case 2: $\beta\approx 2$ ($\dot{w}\approx P\sin\theta_0$)

This is the case of combined primary resonance, i.e., when the frequency of the excitation is the same as the natural frequency of the system. When $\varepsilon\sigma$ is small enough, the approximate solution near $\beta=2$ is given by

$$\begin{aligned}\vartheta&=(2x_0-\frac{k_1}{4\sigma})\cos\Omega t-(2y_0+\frac{k_2}{4\sigma})\sin\Omega t\\&+\varepsilon\frac{1}{6}\left\{(\alpha_1x_0+\alpha_2y_0)-\frac{1}{8\sigma}(\alpha_1k_1-\alpha_2k_2)\right\}\cos2\Omega t\\&-\varepsilon\frac{1}{6}\left\{(\alpha_1y_0-\alpha_2x_0)+\frac{1}{8\sigma}(\alpha_1k_2+\alpha_2k_1)\right\}\sin2\Omega t\\&-\varepsilon\frac{1}{2}\left\{(\alpha_1x_0-\alpha_2y_0)-\frac{1}{8\sigma}(\alpha_1k_1+\alpha_2k_2)\right\}+O(\varepsilon^2)\end{aligned} \tag{33}$$

where x_0 and y_0 are integration constants. As observed in Eq.(33), as σ approaches zero, the theoretical motion becomes unbounded; when σ is nonzero, but small, the motion is bounded with large amplitudes. The first two terms of the solution are due to external excitation and the rest, additive terms, to both external and parametric excitations.

RIGID BODY SOLUTIONS

Now, what is to be done is to analyze the motion of the main body subject to the perturbational motion of the pendulum. For simplicity, the variables a and w can be transformed into new variables η and ζ defined by

$$\eta=a\sin w \quad\text{and}\quad \zeta=a\cos w \tag{34}$$

These are projections of the length a onto the pendulum plane. By differentiating these variables once with respect to time and substituting Eqs.(8) and (9) into them, the following first-order differential equations are obtained

$$\begin{aligned}\dot{\eta}&=\left[\Omega+\varepsilon\left\{\frac{H_3}{I_1}\left(\frac{r_0}{l}+\cos\theta\right)^2-\frac{I_1}{I_3}P\sin^2\theta\right\}\right]\zeta\\&-\varepsilon\left[\frac{1}{I_3}\left(\frac{r_0}{l}+\cos\theta\right)\zeta^2-H_3P\left(\frac{r_0}{l}+\cos\theta\right)+\frac{F_3I_1}{m_1l}\right]\sin\theta\end{aligned} \tag{35a}$$

$$\begin{aligned}\dot{\zeta}&=-\left[\Omega+\varepsilon\left\{\frac{H_3}{I_1}\left(\frac{r_0}{l}+\cos\theta\right)^2+\left(\frac{H_3}{I_1}-\frac{I_1}{I_3}P\right)\sin^2\theta\right\}\right]\eta\\&+\varepsilon\left[\frac{1}{I_3}\left(\frac{r_0}{l}+\cos\theta\right)\sin\theta\ \ \eta\zeta-H_3\dot{\theta}\left(1+\frac{r_0}{l}\cos\theta\right)\right]\end{aligned} \tag{35b}$$

By dropping relatively small terms and expanding the trigonometric functions about θ_0, Eq. (35) reduce to the form

$$\underline{\dot{\xi}}=\underline{\underline{\Lambda}}\,\underline{\xi}+\varepsilon\underline{g} \tag{36}$$

where

$$\underline{\xi}=\begin{bmatrix}\eta\\ \zeta\end{bmatrix} \tag{37}$$

$$\underline{\underline{\Lambda}} = \begin{pmatrix} 0 & \Omega + O(\varepsilon) \\ -\Omega - O(\varepsilon) & 0 \end{pmatrix} \tag{38}$$

and

$$\underline{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{39}$$

where

$$g_1 = H_3\Omega\left(\frac{r_0}{l}\sin\theta_0 + \frac{1}{2}\sin 2\theta_0\right) - \frac{F_3 l_1}{m_1 l}\sin\theta_0 + \left\{H_3\Omega\left(\frac{r_0}{l}\cos\theta_0 + \cos 2\theta_0\right) - \frac{F_3 l_1}{m_1 l}\cos\theta_0\right\}\vartheta$$

$$g_2 = -H_3\dot{\vartheta}\left(1 + \frac{r_0}{l}\cos\theta_0\right) + H_3\dot{\vartheta}\vartheta\frac{r_0}{l}\sin\theta_0$$

Then, the vector solution of Eq.(36) has the form

$$\underline{\xi}(t) = e^{\underline{\Lambda}t}\,\underline{\xi}(0) + \varepsilon\int_0^t e^{\underline{\Lambda}(t-T)}\,\underline{g}(T)dT \tag{40}$$

where $e^{\underline{\Lambda}t}$ is the state transition matrix of the linear and constant part of Eq.(36). Finally, the following approximate solutions are obtained

$$\begin{aligned}\eta(t) = {} & \cos\Omega t\left\{\eta(0) + \varepsilon\int_0^t (g_1\cos\Omega t - g_2\sin\Omega t)dT\right\} \\ & + \sin\Omega t\left\{\zeta(0) + \varepsilon\int_0^t (g_1\sin\Omega t + g_2\cos\Omega t)dT\right\}\end{aligned} \tag{41a}$$

$$\begin{aligned}\zeta(t) = {} & -\sin\Omega t\left\{\eta(0) + \varepsilon\int_0^t (g_1\cos\Omega t - g_2\sin\Omega t)dT\right\} \\ & + \cos\Omega t\left\{\zeta(0) + \varepsilon\int_0^t (g_1\sin\Omega t + g_2\cos\Omega t)dT\right\}\end{aligned} \tag{41b}$$

The evaluation of the integrals in Eq.(41) is straightforward. By substituting the solutions for ϑ into Eq.(41), one can find explicitly the corresponding resonance solution to the motion of the main body. As shown in the solutions, the boundness of the responses totally depends on whether the integral parts are bounded or not. In order words, the stability of the motion of the main body depends on the type of resonance of the pendulum motion. When $\beta \approx 1$, the coning motion of the main body always grows with that of the pendulum. When $\beta \approx 2$, the coning motion of the main body does not always grow with θ. In this case, the coning growth depends on the constants x_0 and y_0, and the phase of the motions of w and θ.

CONCLUSIONS

By assuming that both main body(spacecraft) and pendulum (liquid pool) were in states of the steady spin about the symmetry axis of the satellite and the mass moments of inertia were constant, the coupled nonlinear equations of motion for the two-body satellite system are simplified and two possible resonance conditions for slosh motion were found in terms of the system parameters. The first resonance case occurs when the natural frequency of the liquid is close to one half of the frequency of the parametric excitation by main body and the second case when the frequency of the external-type excitation by the main body approaches to the natural frequency of the liquid. The corresponding near-resonant solutions showed that the near-resonant motion induced by the parametric excitation may occur slowly while the near-resonant motion induced by the external excitation may have large but bounded amplitude.

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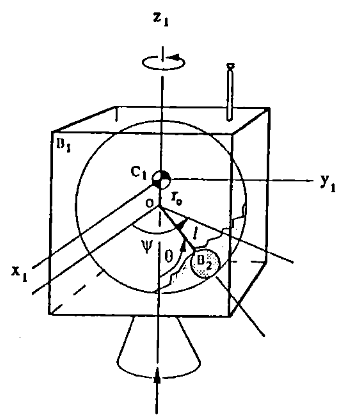

Fig. 1 Two-Body Model of the Spacecraft and Liquid Pool.

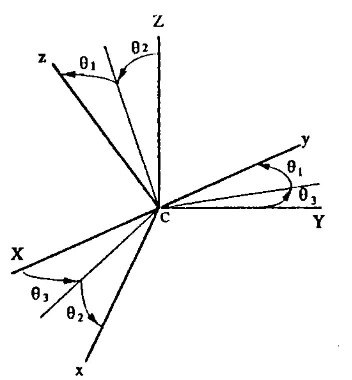

Fig.2 Conventional Euler Angles(3-2-1 sequence)