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Adaptive Control with Neural Network for a Magnetic Levitation System

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ABSTRACT

This paper presents a nonlinear adaptive control approach to a 4-point attraction magnetic levitation system using the local coordinates transformation and neural network. Based on local coordinates transformations, the magnetic levitation system can be represented in a state space form of a 4-input 4-output. Neural networks which are defined in the new coordinates are used to learn the nonlinear functions of the system which are defined in the new coordinates also. The parameters of the neural networks are updated in an on-line manner according to an augmented tracking error. The simuliation results are reported in this paper.

1. INTRODUCTION

This paper presents a nonlinear adaptive control approach to a 4-point attraction magnetic levitation system using local coordinates transformations and neural network.

Neural network can be considered as general modeling tools for nonlinear functions. Although many related applications and algorithms have been reported, it is difficult to study the stability issue of the neural-network-based control system[(1~3)]. Recently, convergence analysis of neural-network-based control system is reported[(4)]. In this paper, these results will be applied to control a 4-point attraction magnetic levitation system.

Applications of advanced control techniques to magnetic levitation system have received growing attentions, especially by utilizing robust control theories and techniques[(5)]. In controlling a magnetic levitation system, modeling of a magnetic attraction force is very important because of its complex nonlinearity. From the electromagnetic theory, we know the electromagnetic model of the magnetic levitation system is usually strongly nonlinear, depending on the length of the air-gap. Usually, the magnetic force is considered such that it is approximately proportional to the square of the current and inversely proportional to the square of the air gap length between the magnet and the levitated vehicle. However, in our case, the proportional coefficient changes depending on the length of the air gap.

In this paper, the nonlinear system is an unknown linearizable system with relative degree $\{r_1, r_2, r_3, r_4\} = \{3, 3, 3, 3\}$, and the sum $r = r_1 + r_2 + r_3 + r_4$ is exactly equal to the dimension of the state space. Therefore, based on local coordinates transformations[(6)], the magnetic levitation system can be represented in a state space form of a 4-input 4-output. However, the time-varing parameters of the functions in the state equations are unknown. So, a kind of neural network which is defined in the new coordinates is presented to learn the nonlinear functions of the system which is represented in the new coordinates also.

Taking the results of the convergence analysis reported recently of neural-network-based control system into account, the adaptive controller is designed with the neural network. The parameters of the neural networks are updated in an on-line manner according to an augmented tracking error. A local convergence theorem is given on the convergence of the tracking error. The simuliation results are reported in this paper.

2. EXPERIMENTAL EQUIPMENT

2.1 *Modeling of the System*

The movable vehicle and the positions of the magnets are shown as Fig.1. The shape of the levitated vehicle is like a rectangular sheet. There are the electromagnets, the gap sensors and the linear motor in the stator. The mechanical differential equations of the levitated vehicle can be written as:

$$\ddot{\boldsymbol{x}}_{vpr} = \overline{\boldsymbol{A}}\boldsymbol{f} + \overline{\boldsymbol{d}} \qquad (1)$$

where

$$
\left.\begin{aligned}
\bar{\boldsymbol{A}} &= \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ \frac{l}{I_p} & \frac{l}{I_p} & -\frac{h-l}{I_p} & -\frac{h-l}{I_p} \\ \frac{k}{2I_r} & -\frac{k}{2I_r} & \frac{k}{2I_r} & -\frac{k}{2I_r} \end{bmatrix} \\
\bar{\boldsymbol{x}}_{vpr} &= \left[x_v,\ \ x_p,\ \ x_r\right]^{\mathrm{T}} \\
\boldsymbol{f} &= \left[f_1,\ \ f_2,\ \ f_3,\ \ f_4\right]^{\mathrm{T}} \\
\bar{\boldsymbol{d}} &= \left[d_v,\ \ d_p,\ \ d_r\right]^{\mathrm{T}}
\end{aligned}\right\} \quad (2)
$$

where x_v is the vertical position of center of the gravity; x_p is the pitching angle; x_r is the rolling angle; m is the mass of the levitated vehicle; I_p is the moment of inertia in the direction of pitching motion; I_r is the moment of inertia in the direction of rolling motion; d_v, d_p and d_r are disturbances; f_1~f_4 is the electromagnetic force produced by each pair of pulling-up and pulling-down magnets; l is the position of the center of gravity and h, k is the distance between the electromagnets. Since $\bar{\boldsymbol{A}}$ is an unsquare matrix, we introduce the following constraint:

$$
0 = f_1 + f_4 - f_2 - f_3
$$

Therefore, the system (1) can be rewritten as

$$
\ddot{\boldsymbol{x}}_{vpr} = \hat{\boldsymbol{A}}\boldsymbol{f} + \boldsymbol{d} \qquad (3)
$$

where

$$
\boldsymbol{x}_{vpr} = \left[x_v,\ x_p,\ x_r,\ 0\right]^{\mathrm{T}}
$$

$$
\boldsymbol{d} = \left[d_v,\ d_p,\ d_r,\ 0\right]^{\mathrm{T}}
$$

$$
\hat{\boldsymbol{A}} = \left(\bar{\boldsymbol{A}}\,\boldsymbol{m}\right)^{-1}
$$

$$
\hat{\boldsymbol{A}} = \begin{bmatrix} \frac{(h-l)}{2h} & \frac{1}{2h} & \frac{1}{2k} & \frac{1}{4} \\ \frac{(h-l)}{2h} & \frac{1}{2h} & -\frac{1}{2k} & -\frac{1}{4} \\ \frac{l}{2h} & -\frac{1}{2h} & \frac{1}{2k} & -\frac{1}{4} \\ \frac{l}{2h} & -\frac{1}{2h} & -\frac{1}{2k} & \frac{1}{4} \end{bmatrix}
$$

$$
\boldsymbol{m} = \mathrm{diag}\left(m,\ \ I_p,\ \ I_r,\ \ 1\right)
$$

The electromagnetic force is given by (j=1,2,3,4):

$$
f_j = K_j\left(g_j\right)\frac{i_j^2}{g_j^2} \qquad (4)
$$

where, K_1~K_4 changes depending on the air gap length g_1~g_4, respectivly. The electrical equation of a coil can be written as:

$$
v_j = R_j i_j + \frac{d}{dt}\left(L_j\left(g_j\right)i_j\right) \qquad (5)
$$

Now, define

$$
\boldsymbol{x} = \left[x_1, x_2, \cdots, x_{12}\right]^{\mathrm{T}}
$$

$$
\boldsymbol{u} = \left[u_1 \ \ u_2 \ \ u_3 \ \ u_4\right]^{\mathrm{T}}
$$

as new states, where, x_1, x_4, x_7 and x_{10} is each air gap length, respectivly; x_2, x_5, x_8 and x_{11} is the speed for each magnet in air gap length, respectivly; x_3, x_6, x_9 and x_{12} is the current value of each coil; u_1, u_2, u_3 and u_4 is the voltage value of each coil. $\boldsymbol{x}_{vpr}$ can be obtained by the air gap length x_1, x_4, x_7 and x_{10} with the following equation.

$$
\left.\begin{aligned}
\boldsymbol{x}_{vpr} &= \boldsymbol{B}\left[x_1,\ \ x_4,\ \ x_7,\ \ x_{10}\right]^{\mathrm{T}} \\
\boldsymbol{B} &= \begin{bmatrix} \frac{h-l}{2h} & \frac{h-l}{2h} & \frac{l}{2h} & \frac{l}{2h} \\ \frac{1}{2h} & \frac{1}{2h} & \frac{-1}{2h} & \frac{-1}{2h} \\ \frac{1}{2k} & \frac{-1}{2k} & \frac{1}{2k} & \frac{-1}{2k} \\ 1 & -1 & -1 & 1 \end{bmatrix}
\end{aligned}\right\} \quad (6)
$$

From (3)~(6), the system can be modeled into a 4-inputs and 4-outputs system as following:

$$
\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= H_1 + d_v + l d_p + \frac{k}{2}d_r \\
\dot{x}_3 &= -\frac{R_1(x_1)}{L_1(x_1)}x_3 + \frac{1}{L_1(x_1)}u_1 \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= H_2 + d_v + l d_p - \frac{k}{2}d_r \\
\dot{x}_6 &= -\frac{R_2(x_4)}{L_2(x_4)}x_6 + \frac{1}{L_2(x_4)}u_2 \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= H_3 + d_v + (l-h)d_p + \frac{k}{2}d_r \\
\dot{x}_9 &= -\frac{R_3(x_7)}{L_3(x_7)}x_9 + \frac{1}{L_3(x_7)}u_3 \\
\dot{x}_{10} &= x_{11} \\
\dot{x}_{11} &= H_4 + d_v + (l-h)d_p - \frac{k}{2}d_r \\
\dot{x}_{12} &= -\frac{R_4(x_{10})}{L_4(x_{10})}x_{12} + \frac{1}{L_4(x_{10})}u_4
\end{aligned}\right\} \quad (7)
$$

where,

$$
\boldsymbol{H} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \mathbf{C}\begin{bmatrix} K_1(x_1)\frac{x_3^2}{x_1^2} \\ K_2(x_4)\frac{x_6^2}{x_4^2} \\ K_3(x_4)\frac{x_9^2}{x_7^2} \\ K_4(x_{10})\frac{x_{12}^2}{x_{10}^2} \end{bmatrix}
$$

$$
\mathbf{C} = \boldsymbol{B}^{-1}\hat{\boldsymbol{A}}
$$

The multivariate nonlinear system (7) we consider can be described in state space form as follows

$$\left.\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} &= [y_1,\ y_2,\ y_3,\ y_4]^T = \boldsymbol{h}(\boldsymbol{x})\end{aligned}\right\} \quad (8)$$

where

$$\left.\begin{aligned}\boldsymbol{x} &= [x_1,\ x_2,\ \bullet\bullet\bullet,\ x_{12}]^T \\ \boldsymbol{u} &= [u_1,\ u_2,\ u_3,\ u_4]^T \\ \boldsymbol{F}(\boldsymbol{x}) &= [f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ \bullet\bullet\bullet,\ f_{12}(\boldsymbol{x})]^T \\ \boldsymbol{h}(\boldsymbol{x}) &= [x_1\ \ x_4\ \ x_7\ \ x_{10}]^T \\ \boldsymbol{G}(\boldsymbol{x}) &= [\boldsymbol{g}_1(\boldsymbol{x})\ \ \boldsymbol{g}_2(\boldsymbol{x})\ \ \boldsymbol{g}_3(\boldsymbol{x})\ \ \boldsymbol{g}_4(\boldsymbol{x})]\end{aligned}\right\} \quad (9)$$

in which $\boldsymbol{F}(\boldsymbol{x})$, $\boldsymbol{g}_1(\boldsymbol{x})$~$\boldsymbol{g}_4(\boldsymbol{x})$ are smooth vector fields as shown in following

$$\left.\begin{aligned}\boldsymbol{g}_1(\boldsymbol{x}) &= \left[0\ 0\ \frac{1}{L_1(x_1)}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]^T \\ \boldsymbol{g}_2(\boldsymbol{x}) &= \left[0\ 0\ 0\ 0\ 0\ \frac{1}{L_2(x_4)}\ 0\ 0\ 0\ 0\ 0\ 0\right]^T \\ \boldsymbol{g}_3(\boldsymbol{x}) &= \left[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{L_3(x_7)}\ 0\ 0\ 0\right]^T \\ \boldsymbol{g}_4(\boldsymbol{x}) &= \left[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{L_4(x_{10})}\right]^T\end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned}f_1 &= x_2 \\ f_2 &= H_1 + d_v + l d_p + \frac{k}{2} d_r \\ f_3 &= -\frac{R_1(x_1)}{L_1(x_1)} x_3 \\ f_4 &= x_5 \\ f_5 &= H_2 + d_v + l d_p - \frac{k}{2} d_r \\ f_6 &= -\frac{R_2(x_4)}{L_2(x_4)} x_6 \\ f_7 &= x_8 \\ f_8 &= H_3 + d_v + (l-h) d_p + \frac{k}{2} d_r \\ f_9 &= -\frac{R_3(x_7)}{L_3(x_7)} x_9 \\ f_{10} &= x_{11} \\ f_{11} &= H_4 + d_v + (l-h) d_p - \frac{k}{2} d_r \\ f_{12} &= -\frac{R_4(x_{10})}{L_4(x_{10})} x_{12}\end{aligned}\right\} \quad (11)$$

where $\boldsymbol{x} \in \mathcal{R}^n$ (n=12).

Assumption 1: $\boldsymbol{x}$ will stay in a set $\mathcal{R}_0 = \{\boldsymbol{x}: x_3 x_6 x_9 x_{12} \neq 0\}$.

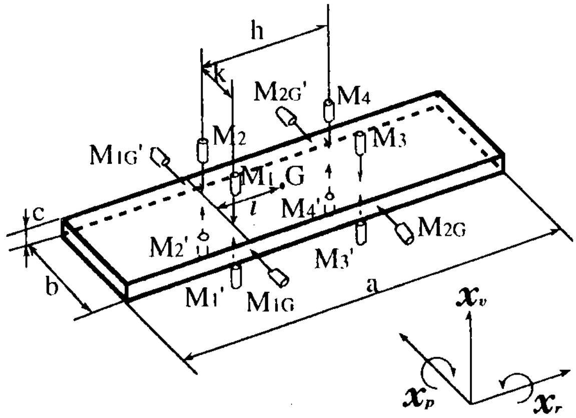

Fig.1 Movable vehicle and positions of the magnets.

For the system, the following condition is satisfied.

$$K_1(x_1)K_2(x_4)K_3(x_7)K_4(x_{10}) \neq 0$$

From the equations (8)~(11), the following results can be obtained:

(1)for all $1 \le j \le 4$, for all $1 \le i \le 4$ for all $k < 2$, and for all $\boldsymbol{x} \in \mathcal{R}^{12}$

$$L_{g_j} L_f^k h_i(\boldsymbol{x}) = 0$$

(2) when the assumption 1 is satisfied, a 4×4 matrix

$$\bar{\boldsymbol{B}}(\boldsymbol{x}) = \begin{bmatrix} L_{g_1}L_f^2h_1 & L_{g_2}L_f^2h_1 & L_{g_3}L_f^2h_1 & L_{g_4}L_f^2h_1 \\ L_{g_1}L_f^2h_2 & L_{g_2}L_f^2h_2 & L_{g_3}L_f^2h_2 & L_{g_4}L_f^2h_2 \\ L_{g_1}L_f^2h_3 & L_{g_2}L_f^2h_3 & L_{g_3}L_f^2h_3 & L_{g_4}L_f^2h_3 \\ L_{g_1}L_f^2h_4 & L_{g_2}L_f^2h_4 & L_{g_3}L_f^2h_4 & L_{g_4}L_f^2h_4 \end{bmatrix}$$

$$= \boldsymbol{C}\,\mathrm{diag}\left(\frac{x_3K_1(x_1)}{x_1^2L_1(x_1)},\ \frac{x_6K_2(x_4)}{x_4^2L_2(x_4)},\ \frac{x_9K_3(x_7)}{x_7^2L_3(x_7)},\ \frac{x_{12}K_4(x_{10})}{x_{10}^2L_4(x_{10})}\right)$$

is nonsingular. Therefore, the multivariate nonlinear system shown in (8) has a relative degree

$$\{r_1,\ r_2,\ r_3,\ r_4\} = \{3,\ 3,\ 3,\ 3\}$$

for all $\boldsymbol{x}=\boldsymbol{x}_0 \in \mathcal{R}_0$ and

$$r = r_1 + r_2 + r_3 + r_4 = 12$$

is exactly equal to the dimension n=12 of the state space. In this case, the following state transformation can be suggested[6]

$$\boldsymbol{z}(\boldsymbol{x}) = \Phi(\boldsymbol{x})$$

$$\boldsymbol{z}(\boldsymbol{x}) = [z_1(\boldsymbol{x}),\ z_2(\boldsymbol{x}),\ \ldots,\ z_{12}(\boldsymbol{x})]^T$$

$$\Phi(\boldsymbol{x}) = [\varphi_1(\boldsymbol{x}),\ \varphi_2(\boldsymbol{x}),\ \ldots,\ \varphi_{12}(\boldsymbol{x})]^T$$

$$\begin{aligned}
\varphi(\boldsymbol{x})_1 &= h_1(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_2 &= L_f h_1(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_3 &= L_f^2 h_1(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_4 &= h_2(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_5 &= L_f h_2(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_6 &= L_f^2 h_2(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_7 &= h_3(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_8 &= L_f h_3(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_9 &= L_f^2 h_3(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_{10} &= h_4(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_{11} &= L_f h_4(\boldsymbol{x}) \\
\varphi(\boldsymbol{x})_{12} &= L_f^2 h_4(\boldsymbol{x})
\end{aligned}$$

Now, put

$$\begin{aligned}
\boldsymbol{G} &= \mathrm{diag}\left(\frac{1}{L_1},\ \frac{1}{L_2},\ \frac{1}{L_3},\ \frac{1}{L_4}\right) \\
&= \mathrm{diag}(G_{11},\ G_{22},\ G_{33},\ G_{44})
\end{aligned}$$

$$\Lambda(\mathbf{z}) = \begin{bmatrix} \lambda_1(\mathbf{z}) \\ \lambda_2(\mathbf{z}) \\ \lambda_3(\mathbf{z}) \\ \lambda_4(\mathbf{z}) \end{bmatrix} = \begin{bmatrix} \frac{R_1}{L_1}\sqrt{\frac{z_1^2}{K_1}(E_{11}(\mathbf{z}) - E_{21})} \\ \frac{R_2}{L_2}\sqrt{\frac{z_4^2}{K_2}(E_{12}(\mathbf{z}) - E_{22})} \\ \frac{R_3}{L_3}\sqrt{\frac{z_7^2}{K_3}(E_{13}(\mathbf{z}) - E_{23})} \\ \frac{R_4}{L_4}\sqrt{\frac{z_{10}^2}{K_4}(E_{14}(\mathbf{z}) - E_{24})} \end{bmatrix}$$

$$\begin{aligned}
\boldsymbol{E}_1 &= \left[E_{11}(\mathbf{z})\ \ E_{12}(\mathbf{z})\ \ E_{13}(\mathbf{z})\ \ E_{14}(\mathbf{z})\right]^{\mathrm{T}} \\
&= \boldsymbol{D}\left[z_3\ \ z_6\ \ z_9\ \ z_{12}\right]^{\mathrm{T}} \\
\boldsymbol{E}_2 &= \left[E_{21}(\mathbf{z})\ \ E_{22}(\mathbf{z})\ \ E_{23}(\mathbf{z})\ \ E_{24}(\mathbf{z})\right]^{\mathrm{T}} \\
&= \bar{\boldsymbol{A}}\,\boldsymbol{m}\,\boldsymbol{d} \\
\boldsymbol{D} &= \bar{\boldsymbol{A}}\,\boldsymbol{m}\,\boldsymbol{B}
\end{aligned}$$

then, in the new coordinates the system (8) is descirbed by the form

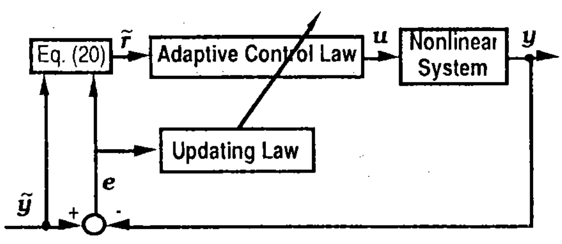

Fig.2 Block diagram of the controller.

$$\left.\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= \lambda_1(\mathbf{z}) + G_{11}u_1 \\
\dot{z}_4 &= z_5 \\
\dot{z}_5 &= z_6 \\
\dot{z}_6 &= \lambda_2(\mathbf{z}) + G_{22}u_2 \\
\dot{z}_7 &= z_8 \\
\dot{z}_8 &= z_9 \\
\dot{z}_9 &= \lambda_3(\mathbf{z}) + G_{33}u_3 \\
\dot{z}_{10} &= z_{11} \\
\dot{z}_{11} &= z_{12} \\
\dot{z}_{12} &= \lambda_4(\mathbf{z}) + G_{44}u_3 \\
\boldsymbol{y} &= \left[z_1\ \ z_4\ \ z_7\ \ z_{10}\right]^{\mathrm{T}}
\end{aligned}\right\} \qquad (12)$$

3. ADAPTIVE CONTROL FOR THE MIMO SYSTEM

For the 4-input/4-output system shown in (12), the states $\mathbf{z} \in \mathcal{R}_1$ are assumed available. Differentiating with respect to time for r_1, r_2, r_3, r_4 times, respectively, until the input appear, one obtains the input/output form of (12) as

$$\boldsymbol{y}^{(3)} = \Lambda(\mathbf{z}) + \boldsymbol{G}(\mathbf{z})\boldsymbol{U} \qquad (13)$$

It is clear that, since the inductances L_1~L_4 are nonzero and finite value, $\mathbf{G}(\mathbf{z})^{-1}$ exists and

$$\sigma(\mathbf{G}(\mathbf{z})) \geq b_1 > 0 \qquad (14)$$

where $\sigma(\mathbf{G}(\mathbf{z}))$ represents the smallest singular value of the matrix $\mathbf{G}(\mathbf{z})$.

3.1 *Structure of the Neural Network*

From the electromagnetic theory, we know the parameters L_1~L_4 and K_1~K_4 of $\Lambda(\mathbf{z})$ and $\mathbf{G}(\mathbf{z})$ are the functions of the air-gap length, therefore we view them as time-varing parameters. Define the parameter vector of the neural network as

$$\Theta = \left[\hat{\boldsymbol{w}}^{\mathrm{T}}\ \ \hat{\boldsymbol{w}}_b^{\mathrm{T}}\right]^{\mathrm{T}} \qquad (15)$$

where

$$\begin{aligned}
\hat{\boldsymbol{w}}_b &= \left[\hat{w}_{b1}\ \ \hat{w}_{b2}\ \ \hat{w}_{b3}\ \ \hat{w}_{b4}\right]^{\mathrm{T}} \\
\hat{\boldsymbol{w}} &= \left[\hat{\boldsymbol{w}}_k^{\mathrm{T}}\ \ \hat{\boldsymbol{w}}_m^{\mathrm{T}}\ \ \hat{\boldsymbol{w}}_d^{\mathrm{T}}\right]^{\mathrm{T}} \\
\hat{\boldsymbol{w}}_k &= \left[\hat{w}_{k1}\ \ \hat{w}_{k2}\ \ \hat{w}_{k3}\ \ \hat{w}_{k4}\right]^{\mathrm{T}} \\
\hat{\boldsymbol{w}}_m &= \left[\hat{w}_{mm}\ \ \hat{w}_{mp}\ \ \hat{w}_{mr}\right]^{\mathrm{T}} \\
\hat{\boldsymbol{w}}_d &= \left[\hat{w}_{dv}\ \ \hat{w}_{dp}\ \ \hat{w}_{dr}\right]^{\mathrm{T}}
\end{aligned}$$

and put

$$\hat{\boldsymbol{D}} = \tilde{\boldsymbol{A}}\,\mathrm{diag}\left(\hat{w}_m,\ \hat{w}_{I_p},\ \hat{w}_{I_r},\ 1\right)\boldsymbol{B}$$

$$\begin{aligned}\hat{\boldsymbol{E}}_1(\boldsymbol{z},\hat{\boldsymbol{w}}) &= \left[\hat{E}_{11}(\boldsymbol{z},\hat{\boldsymbol{w}})\ \hat{E}_{12}(\boldsymbol{z},\hat{\boldsymbol{w}})\ \hat{E}_{13}(\boldsymbol{z},\hat{\boldsymbol{w}})\ \hat{E}_{14}(\boldsymbol{z},\hat{\boldsymbol{w}})\right]^{\mathrm{T}} \\ &= \hat{\boldsymbol{D}}\left[z_3\ z_6\ z_9\ z_{12}\right]^{\mathrm{T}}\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{E}}_2(\hat{\boldsymbol{w}}_d) &= \left[\hat{E}_{21}(\hat{\boldsymbol{w}}_d)\ \hat{E}_{22}(\hat{\boldsymbol{w}}_d)\ \hat{E}_{23}(\hat{\boldsymbol{w}}_d)\ \hat{E}_{24}(\hat{\boldsymbol{w}}_d)\right]^{\mathrm{T}} \\ &= \boldsymbol{A}\,\hat{\boldsymbol{w}}_d\end{aligned}$$

then, the mappings of $\Lambda(\boldsymbol{z})$ and $\boldsymbol{G}(\boldsymbol{z})$ can be formed as

$$\left.\begin{aligned}\hat{\Lambda}(\boldsymbol{z},\hat{\boldsymbol{w}}) &= \begin{bmatrix}\hat{\lambda}_1(\boldsymbol{z},\hat{\boldsymbol{w}})\\ \hat{\lambda}_2(\boldsymbol{z},\hat{\boldsymbol{w}})\\ \hat{\lambda}_3(\boldsymbol{z},\hat{\boldsymbol{w}})\\ \hat{\lambda}_4(\boldsymbol{z},\hat{\boldsymbol{w}})\end{bmatrix} \\ &= \begin{bmatrix}\sqrt{\hat{w}_{k1} z_1^2\left(\hat{E}_{11}(\boldsymbol{z},\hat{\boldsymbol{w}}_m) - \hat{E}_{21}(\hat{\boldsymbol{w}}_d)\right)}\\ \sqrt{\hat{w}_{k2} z_4^2\left(\hat{E}_{12}(\boldsymbol{z},\hat{\boldsymbol{w}}_m) - \hat{E}_{22}(\hat{\boldsymbol{w}}_d)\right)}\\ \sqrt{\hat{w}_{k3} z_7^2\left(\hat{E}_{13}(\boldsymbol{z},\hat{\boldsymbol{w}}_m) - \hat{E}_{23}(\hat{\boldsymbol{w}}_d)\right)}\\ \sqrt{\hat{w}_{k4} z_{10}^2\left(\hat{E}_{14}(\boldsymbol{z},\hat{\boldsymbol{w}}_m) - \hat{E}_{24}(\hat{\boldsymbol{w}}_d)\right)}\end{bmatrix} \\ \hat{\boldsymbol{G}}(\hat{\boldsymbol{w}}_b) &= \mathrm{diag}\left(\hat{w}_{b1},\ \hat{w}_{b2},\ \hat{w}_{b3},\ \hat{w}_{b4}\right)\end{aligned}\right\} \quad (16)$$

3.2 *Desgin of the controller*

It is clear that, if the parameters of the system are constants, there exist Θ such that the $\hat{\Lambda}(\boldsymbol{z},\hat{\boldsymbol{w}})$ and $\hat{\boldsymbol{G}}(\hat{\boldsymbol{w}}_b)$ becomes the exact mapping of $\Lambda(\boldsymbol{z})$ and $\boldsymbol{G}(\boldsymbol{z})$, respevtivly.

Assumption 2:. In a finite time interval T_i, the parameters of the system are constants and there exist $\Theta(\mathrm{T}_i)$ such that

$$\left.\begin{aligned}\max\left\|\hat{\Lambda}(\boldsymbol{z},\hat{\boldsymbol{w}}(\mathrm{T}_i)) - \Lambda(\mathrm{T}_{i+1})\right\| &\le \varepsilon\\ \max\left\|\hat{\boldsymbol{G}}(\hat{\boldsymbol{w}}_b(\mathrm{T}_i)) - \boldsymbol{G}(\mathrm{T}_{i+1})\right\| &\le \varepsilon\\ \forall \boldsymbol{z} &\in \mathcal{R}_0\end{aligned}\right\} \quad (17)$$

where ε is small enough.

Now, let Θ_t ($\Theta_t = \left[\hat{\boldsymbol{w}}_t^{\mathrm{T}}\ \ \hat{\boldsymbol{w}}_{bt}^{\mathrm{T}}\right]^{\mathrm{T}}$) denot the estimates of Θ at the time t and let

$$\tilde{\Theta}(t) = \Theta_t - \Theta \quad (18)$$

denot the parameter error vector. The control law is defined as follows.

Control law:.

$$\boldsymbol{u} = \hat{\boldsymbol{G}}^{-1}\left(-\hat{\Lambda}(\boldsymbol{z},\hat{\boldsymbol{w}}) + \bar{\boldsymbol{r}}\right) \quad (19)$$

where the control input $\bar{\boldsymbol{r}}$ is defined as

$$\left.\begin{aligned}\bar{\boldsymbol{r}} &= \left[\bar{r}_1\ \ \bar{r}_2\ \ \bar{r}_3\ \ \bar{r}_4\right]^{\mathrm{T}}\\ \bar{r}_i &= \bar{y}_i^{(3)} + a_{i3}\left(\bar{y}_i^{(2)} - y_i^{(2)}\right) + a_{i2}\left(\bar{y}_i^{(1)} - y_i^{(1)}\right)\\ &\quad + a_{i1}\left(\bar{y}_i - y_i\right)\\ &\quad (i = 1 \sim 4)\end{aligned}\right\} \quad (20)$$

and $\bar{y}_1,\cdots,\bar{y}_4$ are the reference trajectory.

Define the tracking error vector as

$$\begin{aligned}\boldsymbol{e} &= \left[e_1\ \ e_2\ \ e_3\ \ e_4\right]^{\mathrm{T}}\\ &= \left[\bar{y}_1 - y_1\ \ \bar{y}_2 - y_2\ \ \bar{y}_3 - y_3\ \ \bar{y}_4 - y_4\right]^{\mathrm{T}}\end{aligned}$$

With the control input $\bar{\boldsymbol{r}}$, the system (13) can be rewritten as

$$\left.\begin{aligned}\begin{bmatrix}e_1^{(3)} + a_{13}e_1^{(2)} + a_{12}e_1^{(1)} + a_{11}e_1\\ e_2^{(3)} + a_{23}e_2^{(2)} + a_{22}e_2^{(1)} + a_{21}e_2\\ e_3^{(3)} + a_{33}e_3^{(2)} + a_{32}e_3^{(1)} + a_{31}e_3\\ e_4^{(3)} + a_{43}e_4^{(2)} + a_{42}e_4^{(1)} + a_{41}e_4\end{bmatrix} &= -\tilde{\Theta}(t)\boldsymbol{J} + \Xi\\ &= \Lambda(\boldsymbol{z}) - \hat{\Lambda}(\boldsymbol{z},\hat{\boldsymbol{w}}) + \left(\boldsymbol{G} - \hat{\boldsymbol{G}}\right)\boldsymbol{u}\end{aligned}\right\} \quad (21)$$

where

$$\Xi = \left(O\left(\left|\tilde{\Theta}(t)\right|^2\right) + O(\varepsilon)\right)\left[1\ \ 1\ \ 1\ \ 1\right]^{\mathrm{T}}$$

$$\boldsymbol{J}^{\mathrm{T}} = \left[\boldsymbol{J}_1\ \ \boldsymbol{J}_2\ \ \boldsymbol{J}_3\ \ \boldsymbol{J}_4\right]$$

$$\boldsymbol{J}_1 = \mathrm{col}\left(\left.\frac{\partial\hat{\lambda}_1}{\partial\hat{\boldsymbol{w}}}\right|_{\hat{\boldsymbol{w}}_t},\ u_1,\ 0,\ 0,\ 0\right)$$

$$\boldsymbol{J}_2 = \mathrm{col}\left(\left.\frac{\partial\hat{\lambda}_2}{\partial\hat{\boldsymbol{w}}}\right|_{\hat{\boldsymbol{w}}_t},\ 0,\ u_2,\ 0,\ 0\right)$$

$$\boldsymbol{J}_3 = \mathrm{col}\left(\left.\frac{\partial\hat{\lambda}_3}{\partial\hat{\boldsymbol{w}}}\right|_{\hat{\boldsymbol{w}}_t},\ 0,\ 0,\ u_3,\ 0\right)$$

$$\boldsymbol{J}_4 = \mathrm{col}\left(\left.\frac{\partial\hat{\lambda}_4}{\partial\hat{\boldsymbol{w}}}\right|_{\hat{\boldsymbol{w}}_t},\ 0,\ 0,\ 0,\ u_4\right)$$

Define the augmented error as:

$$\left.\begin{aligned}\boldsymbol{e}_s &= \left[e_{1s}\ \ e_{2s}\ \ e_{3s}\ \ e_{4s}\right]^{\mathrm{T}}\\ e_{is} &= c_{i3}e_i^{(2)} + c_{i2}e_i^{(1)} + c_{i1}e_i\\ &\quad (i = 1 \sim 4)\end{aligned}\right\} \quad (22)$$

The parameters c_{i1}, c_{i2}, c_{i3} in (22) and a_{i1}, a_{i2}, a_{i3} in (20) are chosen such that

$$M_i(s) = \frac{c_{i3}s^2 + c_{i2}s + c_{i1}}{s^3 + a_{i3}s^2 + a_{i2}s + a_{i1}} = \frac{N_i(s)}{D_i(s)} \quad (23)$$

are SPR (Strictly Positive Real) transfer functions and

$N_i(s)$ and $D_i(s)$ are coprime (i=1~4). Define the states as:

$$\left.\begin{array}{l} \boldsymbol{e}_m = \left[\boldsymbol{e}_{1m}^{\mathrm{T}} \quad \boldsymbol{e}_{2m}^{\mathrm{T}} \quad \boldsymbol{e}_{3m}^{\mathrm{T}} \quad \boldsymbol{e}_{4m}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \boldsymbol{e}_{im} = \left[e_i, e_i^{(1)}, e_i^{(2)}\right]^{\mathrm{T}} \\ (i = 1 \sim 4) \end{array}\right\} \quad (24)$$

then (23) can be realized as

$$\left.\begin{array}{l} \dot{\boldsymbol{e}}_m(t) = \boldsymbol{A}\boldsymbol{e}_m(t) + \boldsymbol{b}\left[-\tilde{\Theta}(t)\boldsymbol{J} + \Xi\right] \\ \boldsymbol{e}_s(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{e}_m(t) \end{array}\right\} \quad (25)$$

where

$$\boldsymbol{A} = \text{block diag}(\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3, \boldsymbol{A}_4)$$
$$\boldsymbol{b} = \text{block diag}(\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3, \boldsymbol{b}_4)$$
$$\boldsymbol{c} = \text{block diag}(\boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3, \boldsymbol{c}_4)$$

and

$$\boldsymbol{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{i1} & -a_{i2} & -a_{i3} \end{bmatrix}$$
$$\boldsymbol{b}_i = [0 \quad 0 \quad 1]^{\mathrm{T}}$$
$$\boldsymbol{c}_i^{\mathrm{T}} = [c_{i1} \quad c_{i2} \quad c_{i3}]$$

Then, there exist symmetric and positive definite matrices $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ (i=1~4)such that

$$\left.\begin{array}{r} \boldsymbol{PA} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} = -\boldsymbol{Q} \\ \boldsymbol{Pb} = \boldsymbol{c} \end{array}\right\} \quad (26)$$

where

$$\boldsymbol{P} = \text{block diag}(\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{P}_4)$$
$$\boldsymbol{Q} = \text{block diag}(\boldsymbol{Q}_1 \quad \boldsymbol{Q}_2 \quad \boldsymbol{Q}_3 \quad \boldsymbol{Q}_4)$$

The parameters of the neural network are updated according to the following law.

Updating law:

$$\dot{\Theta} = \begin{cases} 0 & \text{if} \quad \boldsymbol{e}_m^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}_m \le d_0^2 \\ \mu\boldsymbol{J}^{\mathrm{T}}\boldsymbol{e}_s & \text{if} \quad \boldsymbol{e}_m^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}_m > d_0^2 \end{cases} \quad (27)$$

where μ is a positive number representing the learning rate, d_0 is the size of the dead-zone.

Assumption 3: For $\boldsymbol{x} \in \mathcal{R}_0$,

$$\left.\begin{array}{r} |\boldsymbol{e}_m| \le \xi \\ |\tilde{\Theta}| \le \delta \end{array}\right\} \quad (28)$$

is satisfied, where δ is small enough and ξ is large enough.

If the assumptions 1~3 are satistfied, applying the results of Liu and Chen[4], the following results can be readily verified.

Theorem 1:

(A) The tracking error will converge to the ellipsoid $\boldsymbol{e}_m^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}_m \le d_0^2$ as $t \to \infty$.

(B) $|\tilde{\Theta}(t)|$ will converge to a constant.

Fig. 3 show one of the simulation resulats of the vertical position control.

4. CONCLUSION

The adaptive control with neural networks for a magnetic levitation system has been presented in this paper. The parameters of the neural networks were updated in an on-line manner according to an augmented tracking error. The simulation resulats showns that the tracking error between the plant outputs and the reference trajectories has converged

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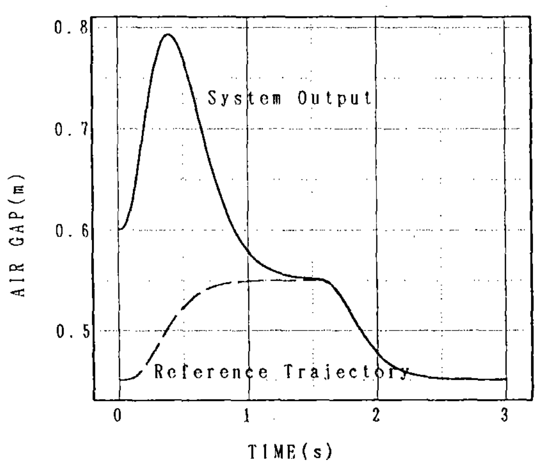

Fig.3 System Output and Reference Trajectory.