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ADAPTIVE CONTROL SYSTEM DESIGN BASED ON CGT APPROACH

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Abstract

Adaptive control systems based upon the command generator tracker (CGT) approach have attracted considerable interest because of the simple structure of its adaptive controller. Some attempts to such improve the adaptive control algorithm, for the sake of the application to broader class of plants, are made. Recently, Su and Sobel (1992) proposed that those schemes can be treated by an unified theory using a metasystem representation with some types of supplementary dynamics. However, in their method, it is difficult to find the dynamic compensator, which is proper and output feedback stabilizable, for the uncertain plant. This paper proposes a new design method of such supplementary dynamics and some parameters of adaptive control system for linear time invariant SISO plants. The method gives a concrete and systematic design method using only a few *priori* knowledge of the plant.

1. Introduction

The command generator tracker (CGT) approach[1] to direct model reference adaptive control (MRAC) was first proposed by Sobel et al.[2] . This idea was extended and refined as the simple adaptive control (SAC)[4] which has attracted considerable interest because of the simple structure of its adaptive controller. Unfortunately, the asymptotic stability of SAC method was guaranteed for the class of † ASPR plants. So, some attempts to extend the method to non-ASPR plants have been made. For example, Sobel et al. proposed an extension which involves augmenting the plant with an inner loop compensator[2]. Further the extension of SAC, which incorporates the feedforward filter into the reference model's output as well as the plant's output, have been proposed by Bar-Kana[5]. Recently, Su and Sobel[7] have shown that these extensions are characterized by the insertion of supplementary dynamics in different locations in the adaptive loop. And it has been emphasized that they can be unified by using the metasystem representation for the adaptive system and introducing adaptive gains for the supplementary dynamics in addition to the adaptive controller gains. However, in the unified method, for the design of the supplementary dynamics, we need to find out the dynamic compensator, which is proper and output feedback stabilizable, for the uncertain plant. Even if we could obtain such a dynamic compensator, the supplementary dynamics located in parallel with the plant may be unstable so that the stability of the adaptive control systems are not necessarily guaranteed. In this paper, a new design method of unified adaptive control system based on CGT approach is proposed for linear time invariant single-input single-output non-ASPR plants which can enable us to solve the above mentioned problems. That is, a systematic and concrete method for the direct design of the parallel supplementary dynamics is discussed and it is proved that such a supplementary dynamics can always be realized in a simple form if one has *priori* informations including the knowledge ;(i) all zeros of the plant lie in the open left half plane, (ii) the upper bound of the relative degree of the plant, and (iii) approximate value of the high-frequency gain. The effectiveness of the parallel supplementary dynamics and adaptive controller designed in this way is examined through the numerical simulation.

†ASPR; an abbreviation of Almost Strictly Positive Real. The plant is said to be ASPR if there exists a static output feedback gain such that the resulting closed loop transfer function is SPR.

2. Basic construction of adaptive control system[7]

A controlled plant is an SISO linear controllable and observable system described as follows.

$$\dot{\boldsymbol{x}}_P(t) = A_P \boldsymbol{x}_P(t) + \boldsymbol{b}_P u_P(t) + \boldsymbol{g}_1(t) \tag{1a}$$

$$y_P(t) = \boldsymbol{c}_P^T \boldsymbol{x}_P(t) + g_2(t) \tag{1b}$$

where, $\boldsymbol{x}_P(t) \in R^n$ is the plant state vector, $y_P(t) \in R$ is the plant output and $u_P(t) \in R$ is the control input, $A_P \in R^{n\times n}$, $\boldsymbol{b}_P \in R^n$ and $c_P \in R^n$ are unknown constant matrix and vectors. $\boldsymbol{g}_1(t) \in R^n$ and $g_2(t) \in R$ are bounded disturbances. The transfer function of the above controlled plant is given as

$$G_P(s) = \boldsymbol{c}_P^T(sI - A_P)^{-1}\boldsymbol{b}_P \tag{2a}$$

$$= k_P + B(s)/A(s) \tag{2b}$$

where $A(s)$ and $B(s)$ are n-th and m-th order monic polynomials, respectively.

Let us consider the following asymptotically stable reference model which the plant output is required to follow

$$\dot{x}_m(t) = A_m x_m(t) + b_m u_m(t) \quad (3a)$$
$$y_m(t) = c_m^T x_m(t) \quad (3b)$$

where $A_m \in R^{n_m \times n_m}$, b_m, $c_m \in R^{n_m}$, $x_m(t) \in R^{n_m}$, $y_m(t)$, $u_m(t) \in R$. It is stressed that the order n_m of the reference model is given regardless of the order n of the plant.

The adaptive control algorithm will utilize supplementary dynamics which will be inserted into parallel path with a plant. The state space representation for the parallel dynamics is given by

$$\dot{x}_f(t) = A_f x_f(t) + b_f u_P(t) \quad (4a)$$
$$y_f(t) = c_f^T x_f(t) \quad (4b)$$

where, $x_f(t) \in R^{n_f}$ is the state vector and $y_f(t) \in R$ is the output of parallel supplementary dynamics, respectively, $A_f \in R^{n_f \times n_f}$, $b_f \in R^{n_f}$ and $c_f \in R^{n_f}$ are the constant matrix and vectors, respectively. Then, the plant and supplementary dynamics are concatenated to form an $(n + n_f)$ order metasystem ;

$$\dot{x}(t) = Ax(t) + bu_P(t) \quad (5a)$$
$$y(t) = Cx(t) \quad (5b)$$

where

$$x = \begin{bmatrix} x_P \\ x_f \end{bmatrix}, A = \begin{bmatrix} A_P & 0 \\ 0 & A_f \end{bmatrix}$$
$$b = \begin{bmatrix} b_P \\ b_f \end{bmatrix}, C = \begin{bmatrix} c_P^T & 0^T \\ 0^T & c_f^T \end{bmatrix}.$$

Now, we make the following assumptions.

[Assumption 1]

(1) There exists a constant vector k_e^* such that the following matrix A_c;
$$A_c = A + bk_e^{*T}C \quad (6)$$
is stable.

(2) There exist the constant vector q and constants γ and d $(\gamma > d > 0)$ such that the following transfer function $H(s)$;
$$H(s) = d + (q^T + \gamma k_e^{*T})C(sI - A_c)^{-1}b \quad (7)$$
is SPR (Strictly Positive Real).

(3) $u_m(t)$ is an input of the reference model in which the output of a linear constant coefficient stable system with input $\dot{u}_m(t)$ is continuous and uniformly bounded.

Under the above assumptions, we define the control input $u_P(t)$ as

$$u_P(t) = k(t)^T z(t) \quad (8a)$$
$$k(t) = [k_e(t)^T, k_x(t)^T, k_u(t)]^T \quad (8b)$$
$$z(t) = [e_v(t), x_m(t)^T, u_m(t)]^T \quad (8c)$$
$$e_v(t) = [e_y(t), -y_f(t)]^T \quad (8d)$$
$$k_e(t) = [k_{ey}(t), k_{ef}(t)]^T \quad (8e)$$

where $e_y(t) = y_P(t) - y_m(t)$ denotes the output following error signal and $k(t)$ is adaptively adjusted by the following conventional parameter adjusting law.

$$k(t) = k_I(t) + k_P(t) \quad (9a)$$
$$\dot{k}_I(t) = -\Gamma_I z(t)v(t) - \sigma k_I(t) \quad (9b)$$
$$k_P(t) = -\Gamma_P z(t)v(t) \quad (9c)$$
$$v(t) = q^T e_v(t) + \gamma k(t)^T z(t) \quad (9d)$$
$$q = [q_P, q_f]^T \quad (9e)$$
$$\Gamma_I = \Gamma_I^T > 0, \Gamma_P = \Gamma_P^T > 0, \sigma > 0$$

Applying the control input (8) to the metasystem (5), it is verified by Su and Sobel[7] that the all of the signals contained in the adaptive control system are ultimately uniformly bounded under the assumption 1.

Remark 1: It is important to emphasize that only the existence of the vector k_e^* is required for the adaptive control algorithm. Neither its numerical value nor its implementation is required.

Remark 2: The adaptive control algorithm described by (8) and (9) yields an asymptotically vanishing output error $e_y(t)$ if the following conditions are satisfied[7];

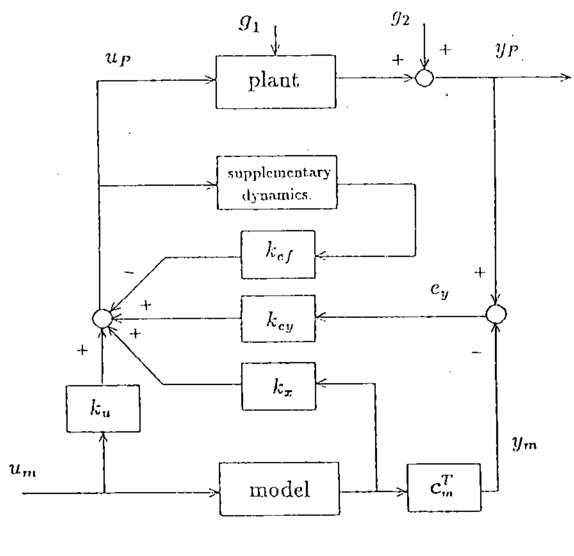

Fig.1 Block diagram of adaptive control system

[Condition 1]

(1) $u_m(t)$ is constant for $t \geq t_1 > 0$.

(2) $\sigma > 0$

(3) γ is chosen to be

$$\gamma = q_f c_f^T A_f^{-1} b_f \tag{10}$$

(4) $g_1(t) = 0$ and $g_2(t) = 0$

(5) There exists the matrix Ω which is a solution of the matrix equation

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_3 & \Omega_4 \end{bmatrix} \tag{11a}$$

$$I = \begin{bmatrix} A & b \\ \begin{bmatrix} c_P^T & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_3 & \Omega_4 \end{bmatrix} \tag{11b}$$

where no eigenvalue of Ω_1 is equal to the inverse of an eigenvalue of A_m.

3. Main Results

In the preceding section, we have shown the basic formulation of the CGT based adaptive control system with parallel supplementary dynamics for plants that satisfy the assumption 1. Although the assumption 1 is very important, because which is imposed to guarantee the stability of the adaptive control system, it is difficult to design the parallel supplementary dynamics and the other control parameters in just the form. Related to this problem, Su and Sobel proposed an adaptive control system design method using a dynamic output feedback compensator which stabilizes the plant [7]. However, their method requires a *priori* knowledge of plant parameters A_P, b_P and c_P.

In this section, we will give a concrete and systematic design scheme of adaptive control system using a *priori* few knowledge of the plant.

Now, we impose the following assumptions on the plant (1).

[Assumption 2]

(1) An upper bound γ^* of the relative degree $\gamma = n - m$ of the plant (1) is known.

(2) Plant (1) is strictly minimum phase, i.e. the zero polynomial denoted as $B(s)$ (monic) is a Hurwitz polynomial.

(3) There exist known positive constants δ_1 and δ_2 such that

$$k_P \geq \delta_1 > 0 \tag{12a}$$

$$|\, G_P(j0) \,| \geq \delta_2 > 0 \tag{12b}$$

Under the above assumptions, we can obtain the following theorem to establish the design method for the parallel supplementary dynamics (3) and the constant vector q which is the coefficients of $v(t)$.

[Theorem 1]

Suppose that the parallel supplementary dynamics (3) is constructed as

$$c_f^T(sI - A_f)^{-1} b_f = F(s) + f_{\gamma^*} \tag{13a}$$

$$F(s) = \sum_{i=1}^{\gamma^*} F_i(s) \tag{13b}$$

$$F_i(s) = -f_i \cdot N_i(s)/D_i(s) \tag{13c}$$

where, $i = 1, \cdots, \gamma^*$, $\gamma^* \geq 2$, $D_i(s)$: ζ_i-th monic Hurwitz polynomial, $N_i(s)$: ν_i-th monic polynomial and

$$\zeta_i - \nu_i = \gamma^* - i \tag{14}$$

$$\delta_1 \gg f_1 \gg \cdots \gg f_{\gamma^*} > 0 \tag{15}$$

$$\delta_2 \gg |\, F(j0) \,| \tag{16}$$

And let the constant vector q ,which is the coefficient of $v(t)$, to be

$$q = \begin{bmatrix} q_P & q_f \end{bmatrix}^T = \begin{bmatrix} \gamma/f_{\gamma^*} & -\gamma/f_{\gamma^*} \end{bmatrix} \tag{17}$$

Then the assumption 1 is satisfied.

Proof;

Consider the augmented plant $G_a(s)$;

$$G_a(s) = G_P(s) + F(s) \tag{18}$$

where $F(s)$ is given by (13*b*). According to Iwai et. al.[6], $G_a(s)$ can be ASPR under the conditions (14) $\sim$ (16). Therefore, the plant (1) can be stabilized by the output feedback compensation with the inverted system of $F(s)$ [3].

Suppose that the state space representation of inverted system of $F(s)$ is given as follows.

$$\dot{x}_c(t) = A_{c1} x(t) + b_c u_c(t) \tag{19a}$$

$$y_c(t) = c_c^T x_c(t) + d_c u_c(t) \tag{19b}$$

where $u_c(t) = y_P(t), u_P(t) = y_c(t)$. Then we have the following relations.

$$A_{c1} = A_f + b_f c_c^T \tag{20a}$$

$$b_c = d_c b_f \tag{20b}$$

$$c_c = d_c c_f^T \tag{20c}$$

$$d_c = -1/f_{\gamma^*} \tag{20d}$$

and the closed loop matrix $\tilde{A}_c$:

$$\tilde{A}_c = \begin{bmatrix} A_P + d_c b_P c_P^T & b_P c_c^T \\ b_c c_P^T & A_{c1} \end{bmatrix} \tag{21}$$

Next, we observe from the assumption 1 (1) that the A_c;

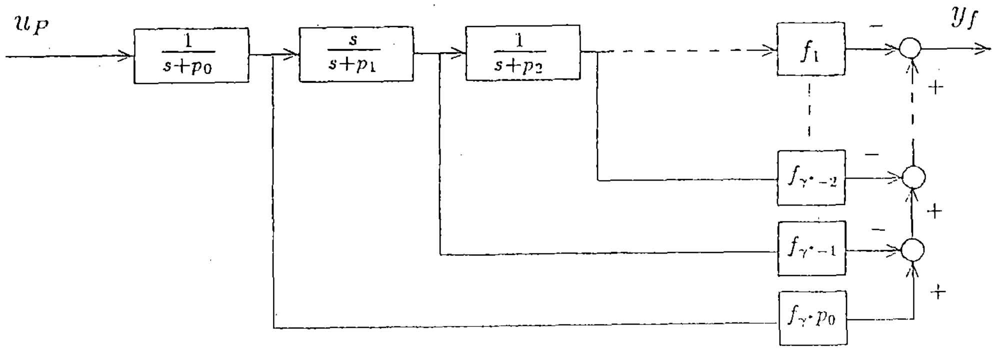

Fig.2 Block diagram of supplementary dynamics

$$A_c = A + bk_e^{*T}C \tag{22a}$$

$$= \begin{bmatrix} A_P + k_{pe}^* b_P c_P^T & k_{pf}^* b_P c_f^T \\ k_{pe}^* b_f c_P^T & A_f + k_{pf}^* b_f c_f^T \end{bmatrix} \tag{22b}$$

is required to be a stable matrix for some gain vector $k_e^* = \left[k_{pe}^*, k_{pf}^*\right]^T$. Thus, by comparing (21) and (22b) we observe that the choice

$$k_{pe}^* = -k_{pf}^* = d_c \tag{23}$$

will result in A_c being a stable matrix. Furthermore, $q^T = -\gamma k_e^{*T}$ is held from (17). Hence (7) reduces to $H(s) = d > 0$, the assumption 1 (2) is also satisfied. □

Remark 3: It is clear that the choice of f_{γ^*} such that

$$f_{\gamma^*} = -c_f^T A_f^{-1} b_f \tag{24}$$

means the satisfaction of the condition 1 (3), because we have

$$(f_{\gamma^*} + c_f^T A_f^{-1} b_f)\gamma = 0 \tag{25}$$

from (10) and (17) immediately. And we can recognize that (24) is held by the choice of $N_i(s)$ in (13) such as $N_i(0) = 0$.

4. Practical Design Procedure

In the case of an actual implementation, the parallel supplementary dynamics is constructed in the control loop as a kind of filter. In this case it is convenient to reduce the order of the parallel supplementary dynamics.

The simplest form of parallel supplementary dynamics based on (13) and (24) is given by the following polynomials. (See Fig. 2)

$$N_i(s) = s \tag{26a}$$

$$D_{\gamma^*-j}(s) = (s + p_j) D_{\gamma^*-j+1}(s) \tag{26b}$$

$$D_{\gamma^*+1} = 1 \tag{26c}$$

$$p_j > 0, i = 0, \cdots, \gamma^*, j = 0, \cdots, \gamma^* - 1$$

While (26a) signifies that $N_i(0) = 0$ is held, if the others of condition 1 are held then the output error $e_y(t)$ will be asymptotically vanished.

5. Simulation Results

The effectiveness of the design method of adaptive control based on the CGT approach developed in the previous section will be demonstrated by the following numerical simulation.

Suppose that the controlled plant is illustrated in Figure 3, which is the one-link direct drive (D.D.) arm, whose output is denoted as the hub rotatory angle $\theta(t)$ and input is denoted as the command voltage $v(t)$ for the servo drive amplifier unit. The output torque of the motor $\tau(t)$ is related to the command voltage $v(t)$ such as approximated as

$$\tau(t) = \eta v(t) \tag{27}$$

$$\eta = 60/8$$

This plant can be approximated by the following state equation for small $\theta(t)$.

$$\begin{bmatrix} \dot\theta(t) \\ \ddot\theta(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot\theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} v(t) + \begin{bmatrix} 0 \\ g(t) \end{bmatrix} \tag{28a}$$

$$y_P(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot\theta(t) \end{bmatrix} \tag{28b}$$

$$g(t) = \alpha \sin\theta(t) \ , \ \alpha = mgr/J \ , \beta = \eta/J$$

where m is a total mass of the arm and the payload, r is a distance from the center of the motor hub to the center of gravity of the arm, g is an acceleration of gravity and J is a total moment of the inertia of arm and the rotor of D.D. motor.

The relative degree of this plant is two. Thus, for implementation of the CGT based adaptive control method, we have to introduce a parallel supplementary dynamics. Here, it is constructed as

$$F(s) = -\frac{f_1 s}{(s+p_0)^2} + \frac{f_2 p_0}{s+p_0} \tag{29}$$

based upon (26), where $f_1, f_2 > 0$, $(f_1 \gg f_2)$ *and* , $p_0 > 0$ are chosen as the following values, respectively.

$$f_1 = 0.4\ ,\ f_2 = 0.002\ ,\ p_0 = 200 \tag{30}$$

The reference model is given as

$$G_m(s) = \frac{100}{s+10} \tag{31}$$

and the reference model input $u_m(t)$ is given as

$$u_m(t) = (\dot{y}_r(t) + 10 y_r(t))/100 \tag{32}$$

where $\dot{y}_r(t)$ is given as Fig. 4 and $y_r(t)$ is calculated from $\dot{y}_r(t)$.

The design parameters for the adaptive control algorithm are chosen as follows.

$$\Gamma_I = diag[3\times 10^6, 3000, 3000, 3000] \tag{33a}$$

$$\Gamma_P = diag[3\times 10^5, 300, 300, 300] \tag{33b}$$

$$\sigma = 0.1 \tag{33c}$$

$$\gamma = 50 \tag{33d}$$

Suppose that we can neglect the influence of the disturbance, $g(t) = 0$, then the plant and model output are shown in Fig.5 for period of 5 seconds. We observe that the output error is driven to zero in approximately 0.5 seconds albeit with a maximum overshoot of nearly 6% .

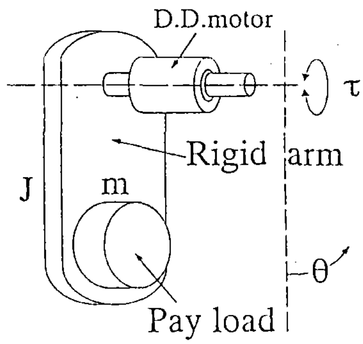

Fig.3 One-link D.D. arm

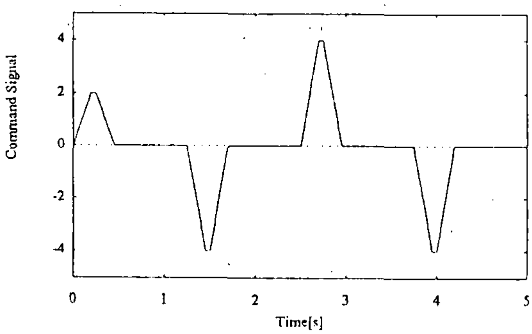

Fig.4 Command signal $\dot{y}_r(t)$ of reference model input

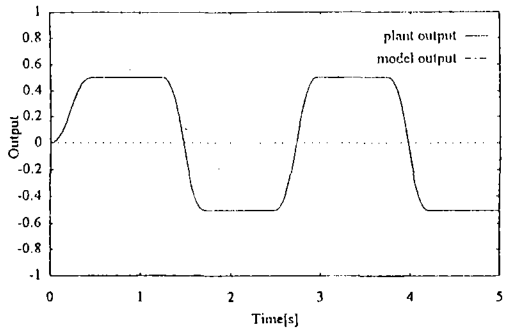

Fig.5 Plant output and model output ($g(t) = 0$)

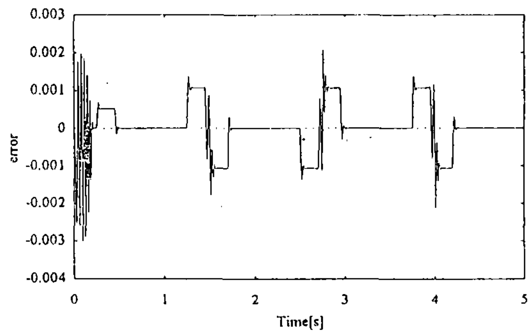

Fig.6 Output tracking error ($g(t) = 0$)

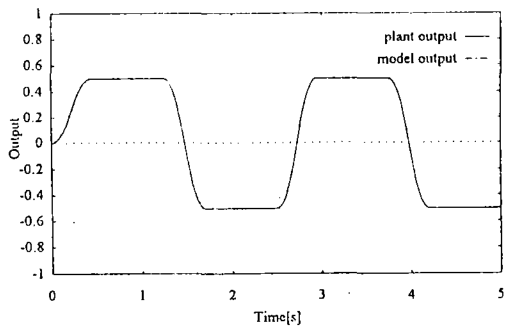

Fig.7 Plant output and model output

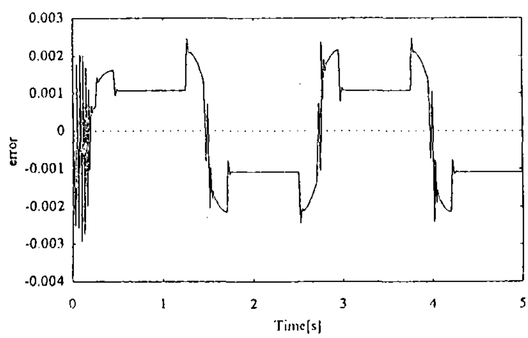

Fig.8 Output tracking error

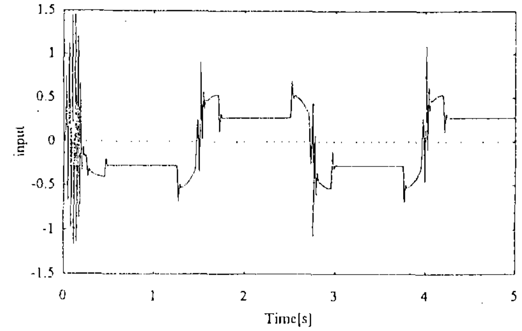

Fig.9 Adaptive control input

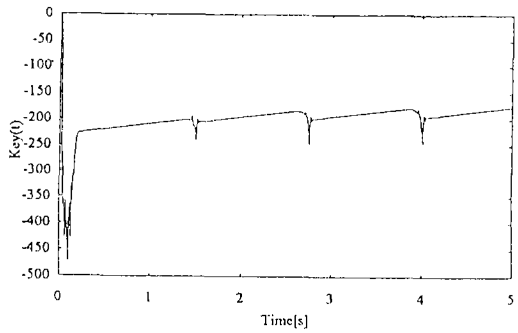

Fig.10a Adaptive gain $k_{ey}(t)$

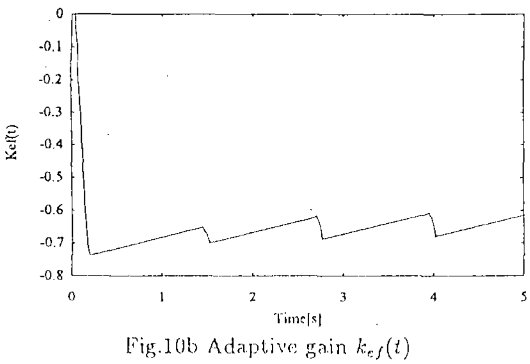

Fig.10b Adaptive gain $k_{ef}(t)$

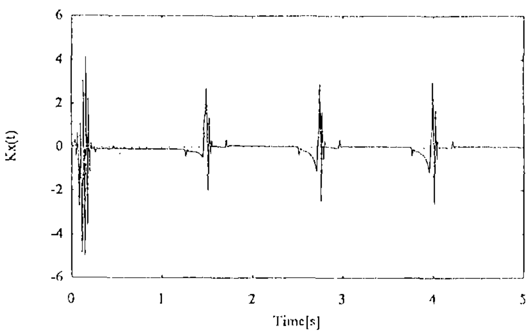

Fig.10c Adaptive gain $k_x(t)$

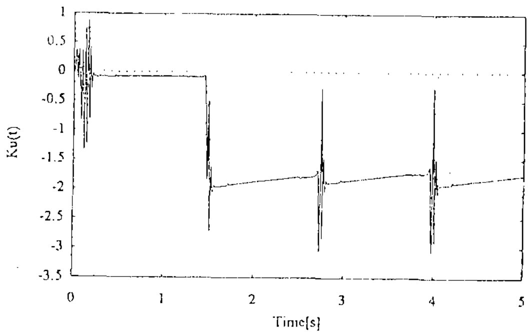

Fig.10d Adaptive gain $k_u(t)$

Fig. 7 and 8 show the plant output and model output and the output error in the case of the presence of disturbance. We observe that there exist the steady state error of approximately 2%. The adaptive gains $k_{ey}(t), k_{ef}(t), k_x(t)$ and $k_u(t)$ are shown in Fig. 10a $\sim$ Fig. 10d, respectively. We observe that the gains become momentarily large at the time when the reference model output rapidly changes. It is in order to force the output error to zero.

5. Conclusions

A concrete and systematic design scheme of the unified adaptive control system based on CGT approach using the parallel supplementary dynamics are proposed. According to the design method, the asymptotically vanishing of the output error is obtained when the model input is constant for $t \geq t_1 > 0$ in a disturbance free environment. The effectiveness of the proposed design method is confirmed through the simulation of the position control of the one-link D.D. arm.

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