'94 KACC (1994. 10. 17 ~ 20)

Closed-Loop Predictive Control using Periodic Gain

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Abstract

In this paper a closed-form predictive control which takes the intervalwise receding horizon strategy is presented and its stability properties are investigated. A state-space form output predictor is derived, which is composed of the one-step ahead optimal output prediction, input and output data of the system. A set of feedback gains are obtained using the dynamic programming algorithm so that they minimize a multi-stage quadratic cost function and they are used periodically.

1 Introduction

The stability properties of RHC for state-space models were established in late 1970's[8] for a special case of RHC. From late 1980's, it was shown that stability of RH controllers is closely linked to the monotonicity(in matrix sense) of the solution of the RDE ([3], [1]). The investigation of the stabilizing properties of RHC was extended for periodic time-invariant systems in the state-space framework. It seems that the notion of PRH (periodic receding horizon) control was first introduced in Kwon and Pearson [2] under the name of intervalwise RH control with reference to the periodic stabilization of time-invariant systems. There a special case of PRH was derived with the terminal constraint $x(\tau) = 0$.

More general results on PRH control of discrete systems are very recent. Yan and Bitmead[7] considered periodic stabilization of time invariant systems, whereas De Nicolao[6] considered both periodic and time-invariant systems. Although based on different point of views, both contributions are agree on the fact that cyclomonotonicity of the solution of the Riccati equation is essential in order to guarantee stability of the closed-loop. Very recently, Nicalao[5] showed that cyclomonotonicity is more easily achievable than monotonicity.

Thus, it is desirable to obtain a PRH control law for I/O models to achieve stability more easily and enhance the performance. It seems that, however, there has not been PRH control law for I/O models. Although the standard GPC solution contains the multiple future control inputs, they are open loop control inputs except the first one. In this paper, a periodic GPC which takes the closed-loop intervalwise receding horizon control strategy is derived for I/O models. The concept of intervalwise receding horizon control which is depicted in Fig. 1. At time t, gains $K_0, K_1, \cdots, K_{N-1}$ are calculated and control inputs $u(t), u(t+1), \cdots, u(t+N-1)$ are determined and applied to the system at time $t, t+1, \cdots, t+N-1$ respectively. The control inputs $u(t), u(t+1), \cdots, u(t+N-1)$ are determined based on the data up to time $t, t+1, \cdots, t+N-1$ and the gains $K_1, K_2, \cdots, K_{N-1}$ respectively. The whole procedure is repeated from time $t+N$.

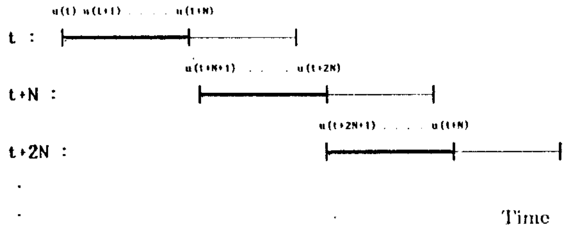

Figure 1: The Periodic Receding Horizon Strategy

In order to obtain the periodic GPC, it is required to solve DRE(difference riccati equation). However, periodic GPC do not require a state estimator and multi-stage output predictors. It must be noted that periodic GPC is different from applying the control inputs which is obtained in the procedure of the standard GPC. A periodic RHPC is also introduced and the equivalence between periodic GPC and periodic RHPC is proved.

2 Cost Function of Periodic GPC

Consider the following CARIMA model:

$$a(q^{-1})y(t) \;=\; b(q^{-1})\Delta u(t) + c(q^{-1})\xi(t) \tag{1}$$

$$\begin{aligned} a(q^{-1}) &= a_c(q^{-1})\Delta = 1 + a_1 q^{-1} + \cdots + a_n q^{-n}, a_n \neq 0 \\ b(q^{-1}) &= b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n} \\ c(q^{-1}) &= 1 + c_1 q^{-1} + \cdots + c_n q^{-n}. \end{aligned}$$

It is well known that the CARIMA model (1) is equivalent to the following SISO state space model:

$$\begin{aligned} x(t+1) &= A_c x(t) + B_c \Delta u(t) + D_c \xi(t) \qquad (2) \\ y(t) &= H_c x(t) + \xi(t), \end{aligned}$$

where

$$A_c = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \cdot & \cdot & 0 \\ -a_2 & 0 & 1 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot & & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot \\ -a_{n-1} & 0 & 0 & 0 & \cdot & \cdot & 1 \\ -a_n & 0 & 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} D_c = \begin{bmatrix} c_1 - a_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ c_n - a_n \end{bmatrix} \qquad (3)$$

$$\begin{aligned} H_c &= [1\ 0 \cdots 0] \\ \xi_1(t) &= \xi_2(t) = \xi(t) \quad with \quad E\{\xi^2(t)\} = \gamma. \end{aligned}$$

Periodic GPC is designed for the system represented in the CARIMA model (1) to minimize the following cost function:

$$J = J_1 + J_2 \qquad (4)$$

where

$$\begin{aligned} J_1 &= E\{\sum_{j=1}^{N} \|y_r(t+j) - y(t+j)\|_Q + \sum_{j=0}^{N} \|\Delta u(t+j)\|_R\} \qquad (5) \\ J_2 &= E\{\sum_{j=N+1}^{N_F} \|y_r(t+j) - y(t+j)\|_{Q_F}\} \qquad (6) \end{aligned}$$

where $\|x\|_Q$ is $\frac{1}{2}x^T Q x$, $Q \in R^{l \times l}$ is a positive semi-definite symmetric matrix, and $Q_F \in R^{l \times l}$ and $R \in R^{m \times m}$ are positive-definite symmetric matrices. We assume that $\Delta u(t+j) = 0$ for $j = N+1, N+2, \cdots, N+N_F$. The control input $\Delta u(t+j)$, $j = 0, 1, \cdots, N$, is determined based on the data available at time $t+j$, respectively. Note that this constraint is different from that of GPC in which the future control inputs are determined based on the data up to the present time t. The optimal control sequence $\Delta u(t)$, $\Delta u(t+1), \cdots \Delta u(t+N)$ are calculated and applied to the plant, At time $t+N+1$, the whole procedure is repeated.

3 Periodic GPC

Consider an one-step ahead optimal output predictor as follows:

$$c(q^{-1})\hat{y}(t+1) = P_1(q^{-1})y(t) + b(q^{-1})\Delta u(t) \qquad (7)$$

where

$$\begin{aligned} P_1(q^{-1}) &= (c_1 - a_1) + (c_2 - a_2)q^{-1} + \cdots + (c_n - a_n)q^{-n+1} \\ &= p_1^1 + p_2^1 q^{-1} + \cdots + p_n^1 q^{-n+1} \end{aligned}$$

Thus we get:

$$\begin{aligned} \hat{y}(t+1) &= \sum_{j=1}^{n} p_j^1 y(t+1-j) - c_j \hat{y}(t+1-j) + b_j \Delta u(t+1-j) \\ \hat{y}(t+2) &= \sum_{j=1}^{n} p_j^1 y(t+2-j) - c_j \hat{y}(t+2-j) + b_j \Delta u(t+2-j) \\ &\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (8) \\ \hat{y}(t+N+N_F) &= \sum_{j=1}^{n} p_j^1 y(t+N+N_F-j) - c_j \hat{y}(t+N+N_F-j) \\ &\quad + b_j \Delta u(t+N+N_F-j) \end{aligned}$$

where $\hat{y}(t+i)$, $i = 1, 2, \cdots, N+N_F$ are the optimal one-step ahead output prediction which is the expected value of the output $y(t+i)$ based on the data up to time $t+i-1$.
If we define:

$$\bar{x}(t+1) = \begin{bmatrix} \sum_{j=1}^{n} p_j^1 y(t+1-j) - c_j \hat{y}(t+1-j) + b_j \Delta u(t+1-j) \\ \sum_{j=2}^{n} p_j^1 y(t+2-j) - c_j \hat{y}(t+2-j) + b_j \Delta u(t+2-j) \\ \vdots \\ p_n^1 y(t) - c_n \hat{y}(t) + b_n \Delta u(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\bar{x}(t+2) = \begin{bmatrix} 0 \\ p_1^1 y(t+1) - c_1 \hat{y}(t+1) + b_1 \Delta u(t+1) \\ p_2^1 y(t+1) - c_2 \hat{y}(t+1) + b_2 \Delta u(t+1) \\ \vdots \\ p_n^1 y(t+1) - c_n \hat{y}(t+1) + b_n \Delta u(t+1) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\vdots$$

$$\vdots$$

$$\bar{x}(t+N+N_F) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ p_1^1 y(t+N+N_F-1) - c_1 \hat{y}(t+N+N_F-1) \\ + b_1 \Delta u(t+N+N_F-1) \end{bmatrix}$$

and

$$\tilde{X}(t+i) = \sum_{j=1}^{i} \tilde{x}(t+i), \qquad (9)$$

then we can get the following recursions:

$$\begin{aligned} \tilde{x}(t+i) &= \tilde{A}_i \tilde{X}(t+i-1) + B_i \Delta u(t+i-1) \qquad (10) \\ &\quad + D_i e_c(t+i-1) \end{aligned}$$

where

$$\begin{aligned} e_c(t+i) &= y(t+i) - \hat{y}(t+i) \\ \tilde{A}_i &= H^T \tilde{A}_{i-1} H, \quad i \geq 3 \\ \tilde{A}_1 &= \left.\begin{bmatrix} -a_1 & 0 & \cdots & 0 \\ -a_2 & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ -a_n & \vdots & & \vdots \\ 0 & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}\right\} N + N_F \\ \tilde{A}_2 &= H^T \tilde{A}_1 \\ H &\equiv \begin{bmatrix} 0 & & \\ \vdots & I_{N+N_F-1 \times N+N_F-1} & \\ 0 & \cdots \quad \cdots & 0 \end{bmatrix}. \qquad (11) \\ B_i &= H^T B_{i-1}, \quad B_1 = [b_1\ b_2\ \cdots b_n\ 0\ \cdots 0] \\ D_i &= H^T D_{i-1}, \quad D_1 = [p_1\ p_2\ \cdots p_n\ 0\ \cdots 0] \end{aligned}$$

From Equations (9) and (10), we get:

$$\begin{aligned} \tilde{X}(t+i) &= A_i \tilde{X}(t+i-1) + B_i \Delta u(t+i-1) \qquad (12) \\ &\quad + D_i e_c(t+i-1) \\ \hat{y}(t+i) &= H_N H^{i-1} \tilde{X}(t+i). \end{aligned}$$

It is easy to check that system equation (12) is equivalent to the following Model:

$$\begin{aligned} X(t+i) &= A_c X(t+i-1) + B_c \Delta u(t+i-1) \qquad (13) \\ &\quad + D_c e_c(t+i-1) \\ \hat{y}(t+i) &= H_c X(t+i) \end{aligned}$$

where $X(t+i) =$ (14)

$$\begin{bmatrix} \sum_{j=1}^{n} p_j^1 y(t+i-j) - c_j \hat{y}(t+i-j) + b_j \Delta u(t+i-j) \\ \sum_{j=2}^{n} p_j^1 y(t+i-j+1) - c_j \hat{y}(t+i-j+1) + b_j \Delta u(t+i-j+1) \\ \sum_{j=3}^{n} p_j^1 y(t+i-j+2) - c_j \hat{y}(t+i-j+2) + b_j \Delta u(t+i-j+2) \\ \vdots \\ p_n^1 y(t+i-1) - c_n \hat{y}(t+i-1) + b_n \Delta u(t+i-1) \end{bmatrix}$$

and the matrices A_c, B_c, D_c, and D_c are defined in Equation (3). The optimal solution which minimizes the cost functional (4) is obtained utilizing the dynamic programming(DP) technique[4] and the system equation (13) as the following theorem.

Theorem 4.1 The optimal solution which minimize the cost functional (4) for the system (1) is given as follows:

$$\begin{aligned} \Delta u(t+i) &= -(R + B_c^T F(i) B_c)^{-1} B_c^T [F(i)(A_c X(t+i) \qquad (15) \\ &\quad + D_c e_c(t+i)) + g(t)] \quad i = 1, 2, \cdots, N \end{aligned}$$

where

$$F(i) = A_c^T F(i+1) A_c - A_c^T F(i+1) B_c [R + B_c^T F(i+1) B_c]^{-1}$$
$$\cdot B_c^T F(i+1) A_c + H_c^T Q H_c, \quad i \leq N \qquad (16)$$

$$F(N) = \sum_{j=0}^{N_F-1} A_c^{jT} H_c^T Q_F H_c^T A_c^j \qquad (17)$$

$$g(t+j-1) = \begin{cases} A_c^T [I + F(j+1) B_c R^{-1} B_c^T]^{-1} g(t+j) \\ \quad -H_c^T Q y_r(t+j) \quad for\ j \leq N \\ A_c^T g(t+j) - H_c^T Q_F y_r(t+j) \\ \quad for\ N+1 \leq j \leq N+N_F-1 \end{cases} \qquad (18)$$

with $g(t+N+N_F-1) = H_c^T Q_F y_r(t+N+N_F)$.

Following the same procedure as that of RHPC, we can obtain the periodic solution of GPC when the terminal weighting $Q_F = \infty I$ from Equations (16) -(18) as the following theorem.

Theorem 4.2 Let $Q_F = fI$, then the control law (15)-(18) becomes as follows as $f \to \infty$:

$$\Delta u(t+i) = -R^{-1} B_c^T \mathcal{F}(i)[A_c X(t+i) + s(t+i)] \qquad (19)$$

where $\mathcal{F}(i)$ and $s(t+i)$ are obtained from the following recursions:

$$\begin{aligned} \mathcal{F}(k) &= A_c^{-1} \mathcal{F}(k+1) A_c^{-T} + A_c^{-1} \mathcal{F}(k+1) A_c^{-T} H_c^T Q^{1/2^T} \qquad (20) \\ &\quad (I + Q^{1/2} H_c A_c^{-1} \mathcal{F}(k+1) A_c^{-T} H_c^T Q^{1/2})^{-1} Q^{1/2} H_c A_c^{-1} \\ &\quad \mathcal{F}(k+1) A_c^{-T} + B_c R^{-1} B_c^T \\ \mathcal{F}(N+1) &= B_c R^{-1} B_c^T \\ s(t+j) &= A_c^{-1} [I + \mathcal{F}(k+1) A_c^{-T} H_c^T Q H_c A_c^{-1}]^{-1} s(t+j+1) \qquad (21) \\ &\quad + A_c^{-1} \mathcal{F}(k+1) A_c^{-T} H_c^T Q \\ &\quad \cdot [H_c A_c^{-1} \mathcal{F}(k+1) A_c^{-T} H_c^T Q + I]^{-1} y_r(t+k) \\ s(t+N+1) &= -L_c Y_{rF} \end{aligned}$$

where

$$L_c = [L_{c,0}\ L_{c,1} \cdots L_{c,N_F-1}]$$

with $L_{c,j} \in R^{n \times l}$ such that $H_c A_c^j L_{c,j} = I_l$.

4 Stability Properties of Periodic GPC

In order to consider the stability of the system (1), we consider the stability of the auxiliary system (13) first. The reference sequence $y_r(t+j)$ is assumed to be zero as far as the stability is concerned. Substituting the control law (15) into Equation (13) yields:

$$X(t+i+1) = (A - B_c Z_c(i) A_c) X(t+i) + (D_c - B_c Z_c(i) D_c) e(t+i)$$

where

$$Z_c(i) = (R + B_c^T F(i) B_c)^{-1} B_c^T F(i).$$

Thus, we can see that the stability of the auxiliary system (14) depends on the stability of the matrix $A - B_c Z_c(i) A_c$, $i = 1, 2, \cdots, N$ and the boundedness of the signal $e_c(t)$. The stability properties of the matrix $A - B_c Z_c(i) A_c$, $i = 1, 2, \cdots, N$ is the same as that of periodic receding horizon control of Kwon and Pearson[2] and Nicolao[5]. Furthermore it is well known that the boundedness of the signal $e_c(t)$ is guaranteed if $c(q^{-1})$ of the CARIMA model (1) is a Hurwitz polynomial. Note that the system (1) is stable if $\hat{y}(t)$ and $e_c(t)$ is bounded. From the above arguments we can say that the stability properties of periodic GPC is the same as those of periodic RHC if $c(q^{-1})$ is a Hurwitz polynomial and the periodic GPC enjoy the advantages of periodic RHC over GPC.

5 Conclusions

A periodic GPC which take the intervalwise receding horizon control strategy was derived for I/O models. In order to obtain the periodic GPC, it was required to solve RDE(Riccati difference equation). However, periodic GPC do not require a state estimator and multi-stage output predictors. The stability of the periodic GPC was shown to be determined by that of periodic RHC and that of the one-step ahead predictor. Since the cyclomonotonicity of a RDE can be achieved more easily that the monotonicity of a RDE, it is expected that the stability of periodic GPC can be achieved more easily than the GPC. It must be noted that periodic GPC is different from applying the control inputs which is obtained in the procedure of the standard GPC.

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