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Application of Adaptive Predictive Control to an Electric Furnace

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Abstract — This paper shows that the GPC with exponential weighting(GPCEW)[1] can be applied to Electric furnace system which has large time delay. Stability of GPCEW can be guarantee from monotonically non-increasing property of Riccati difference equation. We show that the performance of GPCEW versus GPC and auto-tuning PID control is better than that of GPC or auto-tuning PID.

1. Introduction

Long range predictive control(LRPC) has been attracted since late 1970s, which is based on receding horizon control[1]. Among the receding horizon controls, generalized predictive control(GPC)[2] is well-known and popular control strategy. Especially, it can be applied to real processes which have certain properties such as open-loop unstable, non-minimum phase process, unknown order of process, time delay. Despite its popularity, GPC has some problems, whose stability guarantees on limiting cases (e.g. prediction horizon is large)[3] and it cannot be applied to almost undetectable or unstabilizable system (system have unstable poles and zeros which are very close to each other)[1]. For stability issue, some researchers have studied on guaranteeing stability of GPC or receding horizon predictive control. Consequently, there are predictive control to which is proposed after GPC, that is, Constrained receding horizon control(CRHPC) minimizes the cost function of GPC and future constrained equality[4]. Generalized predictive control with end-point state weighting(GPCW) minimizes the GPC cost function and constrained end-point state vector[5]. The basic idea of guaranteeing the stability of predictive control use the monotonically non-increasing property of the Riccati equation of state space system[6-10]. In comparison with GPC, However, it cannot avoid to be more computational burden although small control tuning knobs is set (prediction horizon, etc. al). GPC with exponential weighting (GPCEW)[1] is applicable to undetectable system. It does not need more computation effort than GPC. Therefore, in this paper, we apply GPCEW which is modified from GPC to an electric furnace and compare with GPC. Section 2 introduces GPCEW control algorithm. In Section 3, stability theorem of GPCEW is proven by the monotonically non-increasing property of Riccati difference equation. Section 4 presents an Electric furnace control system. Section 5 shows results of GPCEW, GPC, and PID.

2. GPC with exponential weighting

The process model is assumed to be ARIMA(Auto - Regressive Integrated Moving - Average) model.

$$A(q^{-1})\Delta y(t) = B(q^{-1})\Delta u(t-1) + C(q^{-1})\xi(t) \quad (1)$$

Consider the following cost function for receding horizon control.

$$J = \left\{ \sum_{j=1}^{N_2} \mu(j)(y(t+j) - w(t+j))^2 + \sum_{j=1}^{N_2} \rho(j)\Delta u(t+j-1)^2 \right\} \quad (2)$$

Cost function (2) is minimized with control increments $\Delta u(t+j)$ are zero after an control horizon N_u.

$$\Delta u(t+j) = 0, \quad \text{for } j \geq N_u$$

Now, consider following exponentially increasing weighting.

$$\mu(i) = \bar{\mu}\beta^i \quad \textit{for tracking error}$$
$$\rho(i) = \bar{\rho}\beta^i \quad \textit{for control increments} \quad (3)$$

In this case, $\bar{\mu}$ is 1 and $\mu(i)$ is to equal to $\mu(N_u)$ for $i > N_u$. For improving the performance, define $\alpha = \beta^{-1/2}$ and (3) can be rewritten as

$$\mu(i) = \alpha^{-2i}, \quad \rho(i) = \bar{\rho}\alpha^{-2i} \quad (4)$$

where $0 \leq \alpha \leq 1$, $0 \leq \bar{\rho} \leq 1$.

The prediction equation of future output can be obtained by diophantine equation[2].

$$\hat{y}(t+j) = G_j u(t) + H_j u(t-1) + F_j y(t) \quad (5)$$

where G_j, H_j, and F_j are related to future control increment, past control increment, and current output, respectively.

The vector form of future output can be rewritten as follows

$$\hat{\mathbf{y}} = G\tilde{\mathbf{u}} + f \quad (6)$$

where $\tilde{\mathbf{u}} = [\ \Delta u(t), \delta u(t+1), \cdots, \Delta u(t+N_u)\ ]$.

Using (4) and (6), cost function (2) can be described by

$$J = \{ (G\tilde{u}+f-w) M (G\tilde{u}+f-w)^T + L\ \tilde{u}^T \tilde{u} \} \quad (7)$$

The optimized control incremental vector is obtained by minimizing (7) with respect to $\tilde{\mathbf{u}}$.

$$\tilde{\mathbf{u}} = (\ G^T M G + \Lambda)^{-1} G^T M (w-f) \quad (8)$$

where $G > 0$, $M \geq 0$, $\Lambda \geq 0$.

$$M = diag[\ \alpha^{-2}, \cdots, \alpha^{-2i}, \alpha^{-2N_u}, \cdots, \alpha^{-2N_u}\ ]$$

$$\Lambda = \bar{\rho} \cdot diag[\ 1, \alpha^{-2}, \cdots, \alpha^{-2i}, \cdots, \alpha^{-2(N_u-1)}\ ]$$

3. Stability property

The stability of GPC is applied on the limiting cases(large prediction horizon)[3]. But most practical applications need to small horizon. For stabilizing GPC with exponential weighting, use the monotonically property of the Riccati difference equation.

Consider a state space system of Eq. (1) without noise term.

$$x(t+1) = A\,x(t) + b\Delta u(t)$$
$$y(t) = c\,x(t) \quad (9)$$

The reference signal is regarded as zero because closed loop stability does not depend on the reference signal. In this paper, the only result is mentioned since the process of stability proof is presented in the paper[1].

From the result[1], control law using the Riccati difference equation can be solved as

$$\Delta u(t) = -K_{N_u-1}\, x(t) \quad (10)$$

where

$$K_k = \frac{1}{\alpha^2\bar{\rho} + b^T P_k b} b^T P_k A \quad (11)$$

$$P_{k+1} = c^T c + a^{-2} (A - b K_k)^T P_k (A - b K_k) + \bar{\rho} K_k^T K_k \quad (12)$$

Theorem 1. The system Eq. (9) with control law Eq. (10), Eq. (11), Eq. (12) is asymptotically stable if $P_i \geq a^2 P_{i+1}$ for $i \leq N_u - 1$.

Proof . proof of this is described in the paper[1].

4. Electric Furnace

4.1 Modelling of Electric Furnace

The mathematical model of the electric furnace can be derived by energy balance law

$$C \frac{dT(t)}{dt} = P(t) - \frac{(T(t) - T_c(t))}{R} \quad (13)$$

where C is thermal capacity, $P(t)$ is power supplied by heater, R is thermal resistor, $T(t)$ is inlet temperature of the furnace, $T_c(t)$ is circumambient temperature. Using the Laplace transformation to Eq. (13), we the rational transfer function as follows

$$Y(s) = \frac{1}{C(s+1/RC)} U(s) \quad (14)$$

where $U(s)$ is $(P(s) + T_c(s)/R)$.

The discrete model of Eq. (14) can be obtained by using z transform. As a result, the discrete formulation of furnace can be written as follows using backward shift operator.

$$G(q^{-1}) = \frac{b_1 q^{-1} + b_1 q^{-2} + \cdots + b_{nb} q^{-nb}}{1 + a_1 q^{-1}} \quad (15)$$

where a_1 implies time constant and b_is are power supplied by heater and circumambient temperature.

4.2 Electric Furnace Control System

A Electric Furnace consists of five component, Furnace, sensor-instrument module, PWM driver module, PIO(Programmable input output) module, and IBM-PC 386. The overall block diagram is presented in Figure 1. It is heated from 0℃ to 150 ℃. The Furnace shows the non-linear response property in relay feedback control. Also, it has large time delay (about 2~4 minute). So, it is difficult to control effectively with conventional linear controller as PID. For instrumenting temperature of furnace, we use PT-100 thermistor. PWM module generates from 0 to 255 per one second. PIO board does A/D conversion from sensor module and puts digital output to PWM generator. The control algorithm is run by 386-PC and complied by Borland C++ ver 3.1.

5. Experiment Results

The sampling interval is 30 second. The Estimated parameters of furnace are one A parameter and ten B parameters. It is assumed to have 2 sample time delay ($nk = 2$). Figure 2 represents the response of GPC in case of $N_1 = 1$, $N_2 = 20$, $N_u = 8$. Setting N_u=8, GPC has better performance than other one. Figure 3 shows the response of GPC with exponentially weighting. It has same tuning knobs and $a = 0.975$, $\rho = 0$. PI control is shown in Figure 4. The gains of PI control are $K = 23.1$, $T_i = 737.4$. This gain is obtained by Auto-tuning method of Astrom[12].

In Figure 2, GPC control has following results - initial overshoot=83℃, steady state error=1.5℃ in case of reference 80℃ and initial overshoot=120.5 ℃, steady state error=1℃ in case of reference 120 ℃. GPCEW has 82℃ initial overshoot, 0.5℃ steady state error in case of reference 80℃ and 0.5 ℃ steady state error in case of reference 120℃. PI control is poor performance other control in view of initial undershoot and steady state error.

Note that control increment is saturated by -255 ~255. Therefore, it may be include non-linearity and it makes the initial overshoot be not reduce.

6. Conclusion

In this paper, we applied GPCEW to an electric furnace. From experiment results, we know that GPCEW has better performance than GPC, PI control - initial overshoot, steady-state error. Also, theoretically, GPCEW guarantees the stability of closed loop system by tuning the weighting knobs (α, ρ).

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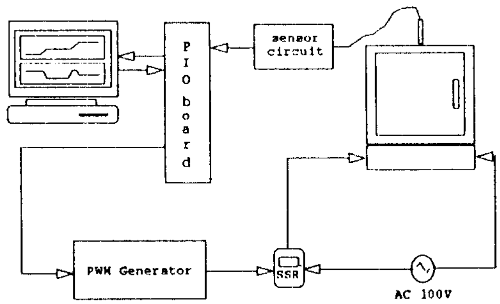

Figure 1. Electric Furnace Control System

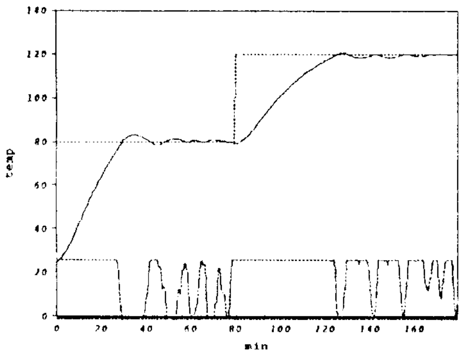

Figure 2. GPC control

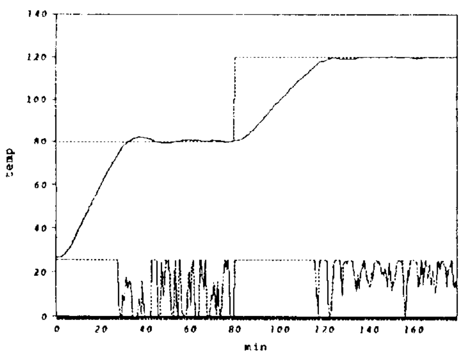

Figure 3. GPCEW control ($\alpha=0.975$, $\rho=0$)

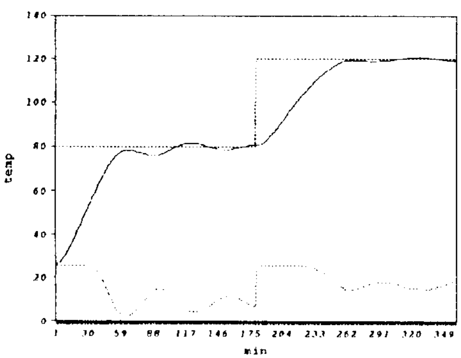

Figure 4. PI control ($K=23.1$, $T_i=737.4$)