'94 KACC (1994. 10. 17 ~ 20)

# Identification of the Process in Closed-loop Control System

Kunihiko OURA †, Kageo AKIZUKI † and Izumi HANAZAKI ‡

† Department of Electrical Engineering, Waseda University. 3-4-1, Ohkubo, Shinjuku-ku, Tokyo 169, Japan
‡ Faculty of Science and Engineering, Department of Systems Engineering, Tokyo Denki University.
Ishizaka, Hatoyama-machi, Hiki-gun, Saitama, 350-03, Japan

**Abstract:** *In this paper, we consider a problem to estimate process parameters using input-output data collected from the process operating in closed-loop control system. When orders and delay-time of the process are known correctly, under some conditions of identifying experiments, it is reported that accurate identification results can be obtained by applying prediction error method. To get accurate estimates, it is necessary to know orders and delay-time of the process. It is difficult to determine them in closed-loop identification, because ill-condition for identification are easily caused by selection of unsuitable order or delay time. Furthermore, the procedures to select orders and delay-time in open-loop identification aren't always available in closed-loop identification. The purpose of this paper is to determine a delay-time under suitable assumption that order of the process are known as the first step.*

## 1 Introduction

*It is often necessary in practice to identify the process operating in closed-loop control system. Such identification is essentially difficult because process input has correlation with output through feedback loop. According to estimation of the process in closed-loop system, it is said that consistent estimators are given by least square method using input-output data of the process if there are at least one time-lag and additive noise in the loop. These conditions requires that the structure of the process and the controller are known. It is important to select model structure (i.e. orders and delay-time). Selection of model orders and delay-time means verification of the estimated models with various orders and delay-time. Model verification is basically performed by comparing model output with actual output for the same input. It is why characteristics of the objective process reflect input and output. There are some criterions for model selection:AIC,condition numbers of the data-matrix, pole-zero placement and so on. But controlled process in closed-loop system doesn't show its property on its output. So these criterions aren't always useful for model verification in closed-loop identification. In this paper, the problem to select delay-time is discussed.*

## 2 Problem Statement

*Consider a following closed-loop control system as shown in fig.1.*

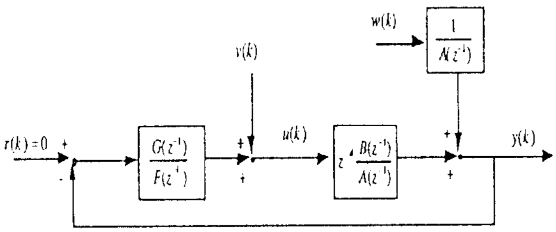


Figure 1: Closed-Loop Control System

$$y(k) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(k) + \frac{1}{A(z^{-1})}w(k) \quad (1)$$

$$u(k) = \frac{G(z^{-1})}{F(z^{-1})}\{r(k) - y(k)\} + v(k) \quad (2)$$

*in these equations,*

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b}$$
$$F(z^{-1}) = 1 + f_1 z^{-1} + \cdots + f_{n_f} z^{-n_f}$$
$$G(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{n_g} z^{-n_g}$$

$$E\{w(k)\} = 0, E\{w(j)w(k)\} = \sigma_w^2 \delta_{jk}$$
$$E\{v(k)\} = 0, E\{v(j)v(k)\} = \sigma_v^2 \delta_{jk}$$
$$E\{w(j)v(k)\} = 0$$

*where $u(k)$ is input to the process and $y(k)$ is output, $w(k)$ is additive noise to the process output and $v(k)$ is additive noise to the process input.*

*The input $u(k)$ to the process is determined from output feedback by a regulator. That is $r(k) = 0$. The output $y(k)$ and the input $u(k)$ are available to estimate process parameters $\{a_1, a_2, \cdots, a_{n_a}, b_0, b_1, \cdots, b_{n_b}\}$. It is assumed that a structure of regulator is given and orders of process $n_a$ and $n_b$, are known a priori, but delay-time isn't known in advance. Least square method using ARX model is adopted as identification method. We consider how to choose suitable model among estimated models of various orders and delay-time.*

## 3 Cross Power Spectrum between Input and Output

*We should select suitable model among candidate models after various considerations. The estimated model is verified based on one-step ahead prediction sequence $\{\hat{y}(k)\}$, which is calculated by estimated parameters and past data $\{y(s), s = 1, \cdots, k-1\}, \{u(s), s = 1, \cdots, k-1\}$. Usually, the model is selected so that prediction errors become as small as possible. Spectra of input and output will give some information about the property of process in frequency domain. But we should pay attention to extract valuable one. In closed-loop system, a pair of input and output sequences doesn't always show the property of the process, because the input to the process consists of the output through feedback loop.*

*In this section, we shall consider whether comparing a nonparametric estimate of the transfer function of the process to models by least square method will be helpful to verification of them. Then power spectrum of the $\{u(k)\}$ and $\{y(k)\}$ is*

$$\Phi_{uu}(\omega) = F\{\phi_{uu}(\tau)\} \quad , \quad \Phi_{yy}(\omega) = F\{\phi_{yy}(\tau)\}$$

*where,*

$$\phi_{uu}(\tau) = E\{u(k)u(k+\tau)\}$$
$$\phi_{yy}(\tau) = E\{y(k)y(k+\tau)\}$$

*From equations (1) and (2), the following equations are given.*

$$y(k) = \frac{z^{-d}B(z^{-1})F(z^{-1})}{A(z^{-1})F(z^{-1}) + z^{-d}B(z^{-1})G(z^{-1})}v(k)$$

$$+ \frac{F(z^{-1})}{A(z^{-1})F(z^{-1}) + z^{-d}B(z^{-1})G(z^{-1})}w(k) \quad (3)$$

$$u(k) = \frac{A(z^{-1})F(z^{-1})}{A(z^{-1})F(z^{-1}) + z^{-d}B(z^{-1})G(z^{-1})}v(k)$$

$$+ \frac{G(z^{-1})}{A(z^{-1})F(z^{-1}) + z^{-d}B(z^{-1})G(z^{-1})}w(k) \quad (4)$$

*Therefore we get*

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)} =$$

$$\frac{|z^{-d}B(z^{-1})F(z^{-1})|^2_{z=e^{j\omega}}\sigma_v^2 + |F(z^{-1})|^2_{z=e^{j\omega}}\sigma_w^2}{|A(z^{-1})F(z^{-1})|^2_{z=e^{j\omega}}\sigma_v^2 + |G(z^{-1})|^2_{z=e^{j\omega}}\sigma_w^2} \quad (5)$$

*If the noise component $v(k)$ doesn't exist in (2), or its level is low compared with one of output noise $w(k)$, equation (5) tends to*

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)} \approx \left|\frac{F(z^{-1})}{G(z^{-1})}\right|^2_{z=e^{j\omega}} \quad (6)$$

*On the other hand, if $v(k)$ stimulates the process enough to identify it, from equations (1) and (2), we have*

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)} \approx \left|z^{-d}\frac{B(z^{-1})}{A(z^{-1})}\right|^2_{z=e^{j\omega}} \quad (7)$$

*From equations (6) and (7), we can notice the followings: If input noise $v(k)$ is small, $\Phi_{yy}/\Phi_{uu}$ is close to the inverse property of the controller. Then comparing $\Phi_{yy}/\Phi_{uu}$ to the frequency characterisitics of models is meaningless. Moreover, since process input $u(k)$ closely correlates to process output $y(k)$, identification with input-output data of the process can be easily ill-conditioned. If input noise $v(k)$ is large, $\Phi_{yy}/\Phi_{uu}$ is close to the property of the process.*

*As mentioned above, the availability of $\Phi_{yy}/\Phi_{uu}$ in verification of the models depends on the levels of $w(k)$ and $v(k)$.*

## 4 Selecting a Delay-time of the Process

*In this section, we consider the procedure to select a suitable delay-time under the assumption that orders of the process and structure of the controller are known a priori. So we can use ARX model for estimation of the process, where the orders of model's denominator and numerator are $n_a$ and $n_b$ respectively and delay-time is changed. As we mentioned above, special care must be taken to the levels of noise $w(k)$ and $v(k)$.*

*If $v(k)$ is large, we can select the model with the smallest prediction error variance among the candidates, because a pair of input and output sequences shows the property which is close to the process property.*

*However, if $v(k)$ does not exist or its level is low compared with $w(k)$, we can't select a suitable delay-time in the way mentioned above. Therefore another method for this case is needed.*

*From equations (1) and (2), we have*

$$\{L(z^{-1})A(z^{-1}) + M(z^{-1})G(z^{-1})\}y(k)$$

$$= \{z^{-d}L(z^{-1})B(z^{-1}) - M(z^{-1})F(z^{-1})\}u(k)$$

$$+ L(z^{-1})w(k) + M(z^{-1})F(z^{-1})v(k) \quad (8)$$

*where $L(z^{-1})$ and $M(z^{-1})$ are arbitrary real-coefficient polynomials. From equation (8), we find that there are various models that represent the relation between obtained input and output sequences. We have to select suitable delay-time among the candidates.*

*We will assume the following model for equation (8) and estimate parameters using input-output data.*

$$\tilde{A}(z^{-1})y(k) = \tilde{B}(z^{-1})u(k) + \tilde{C}(z^{-1})\tilde{w}(k) \quad (9)$$

*Comparing the equation (8) with model (9), the following equations are given about the orders.*

$$order\{L(z^{-1})A(z^{-1}) + M(z^{-1})G(z^{-1})\}$$

$$= order\{\tilde{A}(z^{-1})\}$$

$$order\{z^{-d}L(z^{-1})B(z^{-1}) - M(z^{-1})F(z^{-1})\}$$

$$= order\{\tilde{B}(z^{-1})\}$$

*From these equations, if $\tilde{A}(z^{-1})$ and $\tilde{B}(z^{-1})$ are estimated so that the prediction error variance becomes as small as possible, we can get bounds about suitable delay-time as follows:*

- *If $n_a < n_g$, then $order\{\tilde{A}(z^{-1})\} = ng$, so that suitable delay-time satisfies*

$$\beta - n_b + n_a - n_g \le d \le \beta - n_b \quad (10)$$

- *If $n_a > n_g$, then $order\{\tilde{A}(z^{-1})\} = na$, so that suitable delay-time satisfies*

$$d \le \beta - n_b \quad (11)$$

*here, $\beta$ is the suitable order of $\tilde{B}(z^{-1})$ in the mean that model (9) minimizes its prediction error variance. As the delay-time is large compared with the orders of the process and controller in this kind of control systems, it can be said that $\hat{d} = \beta - n_b$ from equations (10) and (11).*

## 5 Simulation Examples

*The structure of the process and controller are given as equation (12) and (13).*

$$y(k) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(k) + \frac{1}{A(z^{-1})}w(k) \quad (12)$$

$$u(k) = \frac{G(z^{-1})}{F(z^{-1})} + v(k) \quad (13)$$

$$\begin{aligned} A(z^{-1}) &= 1 - 2.3617z^{-1} + 2.0646z^{-2} - 0.6065z^{-3} \\ B(z^{-1}) &= 0.4605 + 0.0904z^{-1} - 0.3197z^{-2} \\ F(z^{-1}) &= 1 - 1.4z^{-1} + 0.4z^{-2} \\ G(z^{-1}) &= 0.018 - 0.015z^{-1} \end{aligned}$$

*where $var\{w(k)\} = 0.01$ and the cases that $var\{v(k)\}$ is $0, 1.0 \times 10^{-6}, 2.5 \times 10^{-5}, 1.0 \times 10^{-4}, 1.0 \times 10^{-2}$ are simulated. Reference input $r(k) = 0$. The number of obtained input and output data is 5000 for each case. It is supposed that the orders $n_a, n_b, n_f, n_g$ and coefficients $f_i(i = 1, \cdots, n_f), g_i(i = 1, \cdots, n_g)$ are known a priori. These simulations show that proposed procedure is available.*

*First, we evaluate prediction errors of the model ($n_a = 3, n_b = 2$) which is estimated as changing its delay-time from 1 to 20. Loss function $J$ which is defined by equation (14) is calculated.*

$$J = \frac{1}{N}\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2 \quad (14)$$

*where $N$ is the number of data and $\hat{y}(k)$ is defined by the following equation.*

$$\hat{y}(k) = -\sum_{i=1}^{n_a} a_i y(k-i) + \sum_{j=0}^{n_b} b_j u(k - \hat{d} - j)$$

*where $\hat{a}$ and $\hat{b}$ are estimated parameters and $\hat{d}$ is estimated delay-time of the model. Loss function for each case is shown in fig.2.*

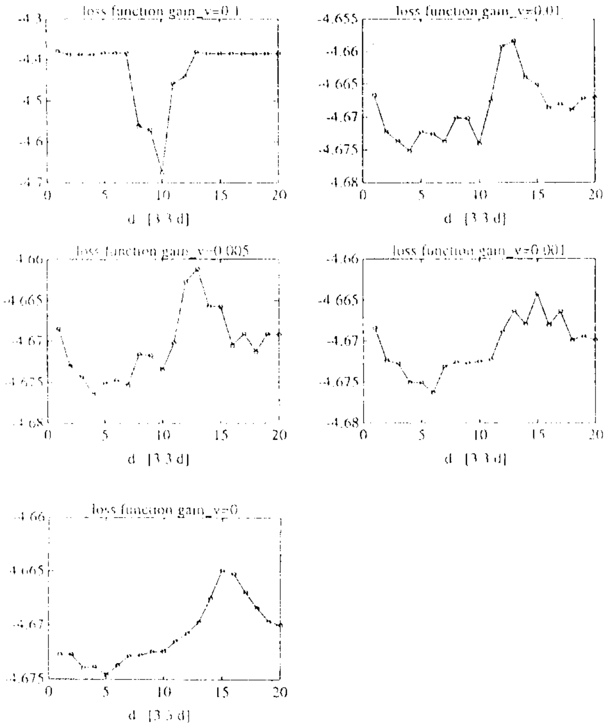


Figure 2: Loss Functions when changes $\hat{d}$

*This figure shows that delay-time giving the minimum of the loss function is true delay-time of the process when variance of $v(k)$ is large* ($1.0 \times 10^{-2}$). *But as the variance becomes smaller, it is more difficult to select delay-time by this procedure.*

*Next, for the case which above selecting method doesn't operate effectively, we shall use model (9). As $n_a > n_g$, the model with order$\{\tilde{A}(z^{-1})\} = n_a$ will be estimated for changed order$\{\tilde{B}(z^{-1})\}$. Loss functions are shown in fig.3.*

*In the case that var$\{v(k)\} = 1.0 \times 10^{-4}$, though the delay-time cannot be selected by first procedure, the second procedure shows that order$\{\tilde{B}(z^{-1})\} = 13$ from fig.3. That is delay-time $d = 10$ by equation (11).*

*Estimated parameters* ($n_a = 3, n_b = 2, \hat{d} = 10$) *and true parameters of the process are summarized in table1.*

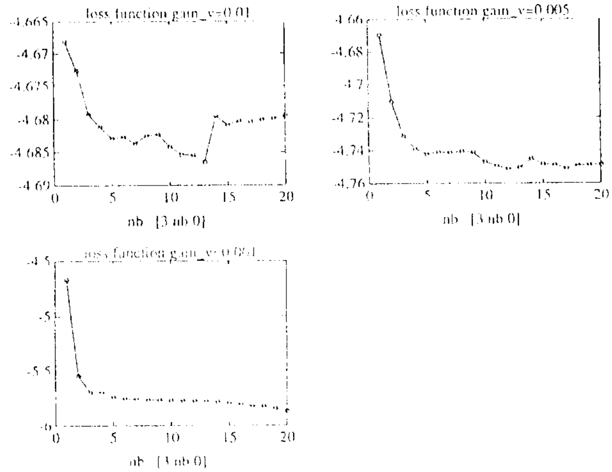


Figure 3: Loss Functions when changes $\hat{n}_b$

table1. Parameters

| | | | | | | |
|---|---|---|---|---|---|---|
| estim. | -2.36 | 2.06 | -0.60 | 0.45 | 0.17 | -0.41 |
| true | -2.36 | 2.06 | -0.61 | 0.46 | 0.09 | -0.32 |

*It is clear that level of the input noise $v(k)$ influences results of identification in closed-loop systems. We can see the level of the input noise $v(k)$ through condition number of the data-matrix (15).*

$$\mathbf{Z}^T\mathbf{Z} = \left(\begin{array}{c|c} \mathbf{Y}^T\mathbf{Y} & \mathbf{Y}^T\mathbf{U} \\ \hline \mathbf{U}^T\mathbf{Y} & \mathbf{U}^T\mathbf{U} \end{array}\right) \tag{15}$$

*where*

$\mathbf{Y}^T\mathbf{Y} =$

$$\begin{pmatrix} \sum y^2_{(k-1)} & \cdots & \sum y_{(k-1)}y_{(k-n_a)} \\ \vdots & \ddots & \vdots \\ \sum y_{(k-1)}y_{(k-n_a)} & \cdots & \sum y_{(k-n_a)}y_{(k-n_a)} \end{pmatrix}$$

$\mathbf{U}^T\mathbf{U} =$

$$\begin{pmatrix} \sum u^2_{(k-\hat{d})} & \cdots & \sum u_{(k-\hat{d})}u_{(k-\hat{d}-n_b)} \\ \vdots & \ddots & \vdots \\ \sum u_{(k-\hat{d})}u_{(k-\hat{d}-n_b)} & \cdots & \sum u^2_{(k-\hat{d}-n_b)} \end{pmatrix}$$

$\mathbf{Y}^T\mathbf{U} = \mathbf{U}^T\mathbf{Y} =$

$$\begin{pmatrix} \sum y_{(k-1)}u_{(k-\hat{d})} & \cdots & \sum y_{(k-1)}u_{(k-\hat{d}-n_b)} \\ \vdots & \ddots & \vdots \\ \sum y_{(k-n_a)}u_{(k-\hat{d})} & \cdots & \sum y_{(k-n_a)}u_{(k-\hat{d}-n_b)} \end{pmatrix}$$

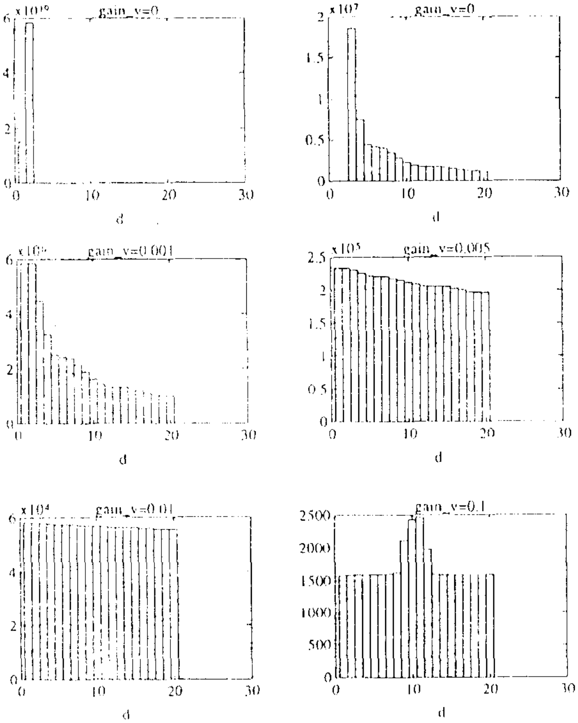


Figure 4: Condition Numbers

*Condition numbers for $\hat{d}$ are shown in fig.4.*

*From fig.4, we can understand that condition numbers are very large when variance of $v(k)$ is small, and especially condition numbers for $\hat{d} < n_a - n_f$ are too large to estimate process parameters. When variance of $v(k)$ is large, condition numbers are uniformly small.*

## 6 Conclusion

*The procedure of selecting a delay-time with input-output data of the process was proposed in this paper. And we verified availability of our procedure through simulations. In identification of the process operating under closed-loop system, it is important to evaluate influence of noises. The main results are summarized as follows:*

- *If the input noise exists and it stimulates the process enough, delay-time is easily selected.*
- *If the input noise which stimulates the process little, selecting a delay-time is difficult. But, it may be selected suitably by proposed procedure.*
- *If there is no input noise added to the process, a delay-time can't be estimated.*
- *The influence of the input noise is evaluated by conditon number of the datamatrix.*

## References

1. *L.Ljung:System Identification - Theory for user,Prentice Hall,1987*
2. *P.E.Wellstead,J.M.Edmunds:*
   *Least Squares Identification of Closed-Loop Systems,International J.of Control, 21-4,689/699,1975*