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Minimization of Consumption Energy for a Manipulator with Nonlinear Friction in PPT Motion

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Abstract Robot engineering is developed mainly in the field of intelligibility such as a manipulation. Considering the popularization of robots in the future, however, a robot should be studied from a viewpoint of saving energy because a robot is a kind of machine with a energy conversion. This paper deals with minimizing an energy consumption of a manipulator which is driven in a point-to-point control method. When a manipulator carries a heavy payload toward gravitation or the links are de-accelerated for positioning, the motors at joints generate electric energy. Since this energy can be regenerated to the source by using a chopper, the energy consumption of a manipulator is only heat loss by an electric and a frictional resistance of the motors. The minimization of the sum of these losses is reduced to a two-points boundary-value-problem of an non-linear differential equation. The solutions are obtained by the generalized Newton-Raphson method in this paper. The energy consumption due to the optimum angular velocity patterns of two joints of a two-links manipulator is compared with conventional velocity patterns such as quadratic and trapezoid.

Key Words : Saving energy, Optimal trajectory, PTP controlled manipulator, Generalized Newton-Raphson method

1. Introduction

It is important to save the energy of a robot manipulator since a manipulator is a kind of machine with a energy conversion. The saving energy can contribute to keep good environment of the earth and make an actuator to be small size due to a little heat loss.

There are many reports on velocity pattern of the joints to minimize the time and energy when the end effector of a manipulator traces on the specified path[1] [2] [3] [4] [5]. P.Marinov and P.Kiriazov have studied the optimal trajectory to minimize the time and energy of manipulator point-to-point motion by using a direct method[6]. And T.Teramoto and coworkers study on high-speed motion control of manipulator under a restriction of average heat generation[7]. In these papers, the performance index on energy includes only copper loss by the resistance of DC motors, but not mechanical loss by nonlinear friction. And their minimization is obtained under many restriction on the motion such as maximum velocity and angle of manipulator joints. There are little paper on minimization of energy consumption of a robot manipulator.

This paper deals with minimization of energy consumption of a two-degree-of-freedom manipulator in point-to-point motion. The optimal velocity pattern of two joints are obtained from a two-points boundary-value-problem technique by using the Generalized Newton-Raphson (GNR) method. Since a robot manipulator has a strong nonlinearity, a starting function for GNR is taken into consideration so that the optimal solution does not fall into a local minimum. Last the physical meaning of the optimal velocity pattern is explained.

2. Minimization of energy consumption of a manipulator

2.1 Boundary condition in PTP motion and consumed energy

We consider the motion of a manipulator whose end-effector moves from one specified point to another with a specified velocity at $t=t_0$ and $t=t_f$. The vector $\theta(t)=[\theta_1(t),\cdots,\theta_n(t)]^T$ denotes the angle of n joints. The boundary conditions are represented by the angle and angular velocity vector at $t=t_0$ as

$$\theta(0)=\theta_0, \qquad \dot{\theta}(0)=\omega_0 \tag{1}$$

and at $t=t_f$ as

$$\theta(t_f)=\theta_f, \qquad \dot{\theta}(t_f)=\omega_f \tag{2}$$

On the other hand, the dynamical model of a manipulator is given by using a matrix of inertia $H(\theta(t))$, a centrifugal and Coriolis force $C(\theta(t),\dot{\theta}(t))$, a gravity force $G(\theta(t))$ and Coulomb friction force $F_r=[F_{r1}(\dot{\theta}(t)),\cdots,F_{rn}(\dot{\theta}(t))]$ as

$$H(\theta(t))\ddot{\theta}(t)+C(\theta(t),\dot{\theta}(t))+G(\theta(t))+F_r=K\,i(t) \tag{3}$$

where $K=\mathrm{diag}(K_1,\cdots,K_n)$ is a matrix of the torque constants of DC motors and $i(t)=[i_1(t), \cdots, i_n(t)]^T$ is an armature current vector. Driven by a inertia and gravity force due to link of manipulator, the DC motors can be generator. If the generated power is fedback into the electric source, the energy consumption is written by

$$J=\int_0^{t_f} (i^T(t)Ri(t)+F_r\dot{\theta}(t))dt \qquad (4)$$

where, $R=\mathrm{diag}(R_1,\cdots,R_n)$ is a diagonal matrix of the resistance. The first term of the performance function (4) is due to Joule's heat and the second term is due to frictional heat.

2.2 Optimal control problem

Assuming $\theta(t)$ and $\dot{\theta}(t)$ to be $x_1(t)$ and $x_2(t)$, respectively, the dynamical equation of (3) is rewritten as

$$\dot{x}_1(t)=x_2(t)$$
$$\dot{x}_2(t)=-H(x_1(t))^{-1}(C(x_1(t),x_2(t)) + G(x_1(t)+F_r(x_2(t)) + H(x_1(t))^{-1}Ki(t) \qquad (5)$$

Under this constraint, the performance index of (4) should be minimized. By using adjoint variables $p_1(t)$ and $p_2(t)$, Hamiltonian is defined as

$$H_a=-i^TRi-F_r^Tx_2+p_1^Tx_2+p_2^T(-H^{-1}(C+G+F_r)+H^{-1}Ki) \qquad (6)$$

and, the optimal current is obtained from $\partial H_a/\partial i=0$ as

$$i=(2R)^{-1}(H^{-1}K)^Tp_2 \qquad (7)$$

Substituting (7) into (6) and arranging,

$$H_a=-F_r^Tx_2+p_1^Tx_2- p_2^TH^{-1}(C+G+F_r)+p_2^TH^{-1}KR^{-1}(H^{-1}K)^Tp_2/4 \qquad (8)$$

is induced. Equation (8) leads Euler canonical equations

$$\dot{x}_1=x_2$$
$$\dot{x}_2=-H^{-1}(C+G+F_r)+H^{-1}KR^{-1}(H^{-1}K)^Tp_2/2$$
$$\dot{p}_1=-\partial H_a/\partial x_1 \qquad (9)$$
$$\dot{p}_2=-\partial H_a/\partial x_2$$

The boundary condition of (1) and (2) is rewritten by

$$x_1(0)=\theta_0, \quad x_2(0)=\omega_0 \qquad (10)$$
$$x_1(t_f)=\theta_f, \quad x_2(t_f)=\omega_f \qquad (11)$$

The optimal current $i(t)$ can be obtained from (7) into which the solution of (9) under these condition is substituted. However, it is difficult to obtain $p_2(t)$ from the two-points boundary-value-ploblem because (9) is a nonlinear differential equation.

3. Generalized Newton-Raphson method

Putting the vector to be $X(t)=[x_1(t)^T\ x_2(t)^T\ p_1(t)^T\ p_2(t)^T]^T$, equation (9) is presented by

$$\dot{X}(t)=F(X,t) \qquad (12)$$

This equation can be solved by the generalized Newton-Raphson(GNR) method. Assuming the m-th approximated solution of (12) to be $X(t)_m$, equation (12) is linearized about $X(t)_m$ by

$$\dot{X}(t)_{m+1}=J(X(t)_m,t)(X(t)_{m+1}-X(t)_m)+F(X(t)_m,t) \qquad (13)$$

where, $J=\partial F(X(t)_m,t)/\partial X(t)_m$ denotes a Jacobian matrix. Putting (m+1)-th solution $X(t)_{m+1}$ to be $X(t)$ again, equation (13) is represented by

$$\dot{X}(t)=A(t)X(t)+B(t) \qquad (14)$$

where, $A(t)=J$, $B(t)=F(X(t)_m,t)-JX(t)_m$. This differential equation is linear but a time variant system. And the initial condition $p(t_0)$ with respect to the adjoint variable $p(t)$ is not given, but only $x(t_0)$ with respect to the state variable $x(t)$ is known. Therefore the value of $p(t_0)$ is assumed and is revised so that $x(t_f)$ can satisfy the boundary conditions. This recurrence technique is called shooting method of GNR.

4. Saving the energy consumption

4.1 One-link manipulator

When a one-link manipulator is moved in a horizontal plane, the dymanical equation is given by

$$H_1\ddot{\theta}_1(t)+F_{r1}=K_1 i_1(t) \qquad (15)$$

Assuming $\dot{\theta}(t)>0$, (4) is minimized when

$$\theta_q(t)=(t_f(\omega_f+\omega_0)+2(\theta_0-\theta_f))t^3/t_f^3 -(t_f(\omega_f+2\omega_0)+3(\theta_0-\theta_f))t^2/t_f^2+\omega_0 t+\theta_0 \qquad (16)$$

$$\dot{\theta}_q(t)=(3t_f(\omega_f+\omega_0)+6(\theta_0-\theta_f))t^2/t_f^3 -(2t_f(\omega_f+2\omega_0)+6(\theta_0-\theta_f))t/t_f^2+\omega_0 \qquad (17)$$

They are shown in Fig. 1(a) when the boundary condition are

$$\theta_0=-\pi/2,\ \theta_f=\pi/2,\ \omega_0=0,\ \omega_f=0 \qquad (18)$$

This velocity pattern is a well known quadratic function which is independent of H_1 and F_{r1}. This motion is realized when the current of DC motor are

$$i_q(t)=K^{-1}H[6(t_f(\omega_f+\omega_0)+2(\theta_0-\theta_f))t/t_f^3 -2(t_f(\omega_f+2\omega_0)+3(\theta_0-\theta_f))/t_f^2]+K^{-1}F_{r1} \qquad (19)$$

These waveforms are shown in Fig. 1(b), when F_{r1} is 1.2. In these quadratic velocity pattern, the energy consumption J_q is represented by

$$J_{qh}=12H_1^2R_1(\theta_f-\theta_0)^2/(K_1^2t_f^3) +R_1(F_{r1}/K_1)^2t_f+F_{r1}(\theta_f-\theta_0) \qquad (20)$$

On the other hand, when the velocity pattern is a trapezoid and $F_{r1}=0$, the minimum energy consumption is given by

$$J_{th}=(27/2)H_1^2R_1(\theta_f-\theta_0)^2/(K_1^2t_f^3)$$

Therefore the energy of the quadratic velocity

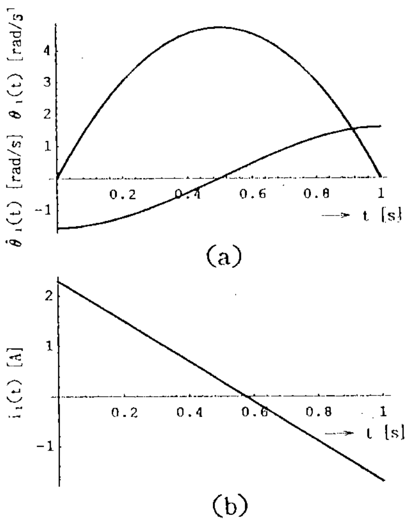

Fig. 1 Optimal motion and current function in a horizontal plane.

pattern can be less than the trapezoidal one by $\eta_h=(J_{th}-J_{qh})/J_{qh}=12.5\%$.

4.2 Two-links manipulator

4.2.1 Without friction

Figure 2 shows a two-links manipulator with ℓ_1 and ℓ_2 in length. Its inertia matrix H is given by

$$H=\begin{bmatrix} h_{11}+m_2\ell_1\ell_2\cos\theta_2 & h_{12}+m_2\ell_1\ell_2\cos\theta_2/2 \\ h_{12}+m_2\ell_1\ell_2\cos\theta_2/2 & h_{22}\end{bmatrix} \quad (21)$$

and a centrifugal and Coriolis's force are written as

$$C=m_2\ell_1\ell_2\sin\theta_2\begin{bmatrix}-(\dot\theta_1\dot\theta_2+\dot\theta_2{}^2/2) \\ \dot\theta_1{}^2/2\end{bmatrix} \quad (22)$$

where $h_{11},\cdots,h_{22}$ and m_2 are constant values. When $\omega_0=\omega_f=0$, the velocity of i-th joint is rewritten from (17) as

$$\dot\theta_{ip}(t)=-6(\theta_{if}-\theta_{i0})t^2/t_f{}^3+6(\theta_{if}-\theta_{i0}))t/t_f{}^2 \quad (23)$$

This velocity shows that the second joint rotates forward direction. If the movable range of the links is not restricted, the following $\dot\theta_{im}(t)=-\dot\theta_{ip}(t)$ can also be adopted.

$$\dot\theta_{im}(t)=6(\theta_{if}-\theta_{i0})t^2/t_f{}^3-6(\theta_{if}-\theta_{i0}))t/t_f{}^2 \quad (24)$$

Conventional velocity functions are $\dot\theta_{1p}(t)$ and $\dot\theta_{2p}(t)$ which both the first and second joint are rotated forward. In this case the energy consumption J_{qp} is obtained by a dot dash line in Fig. 3. These values are calculated for several $(\theta_{1f}-\theta_{10})$ under the condition of $\theta_{20}=\theta_{2f}=0$. On the other hand, when only the second link is rotated backward by a function $\dot\theta_{2m}(t)$ of (24), the energy consumption J_{qm} turns into a broken line in Fig. 3. When $(\theta_{1f}-\theta_{10})$ is more than 1[rad], the energy is apparently saved by using $\dot\theta_{2m}(t)$ rather than $\dot\theta_{2p}(t)$. A backward rotation of the second joint makes the inertia small because the elements of (21) become minimum values at $\theta_2=\pi$.

Consequently the trajectory with a small inertia causes the saving energy.

4.2.2 With friction

The energy consumption due to Coulomb friction is given from (4) as

$$J_{ci}=F_{ri}\int_0^{tf}|\dot\theta_i(t)|dt \quad (25)$$

If the sign of $\dot\theta(t)$ does not change during [0 t_f] as (23) and (24), equation (25) is rewritten into

$$J_{ci}=F_{ri}(\theta_{if}-\theta_{i0}) \quad (26)$$

Therefore the increment of energy consumption is

$$\Delta J_i=(F_{ri}/K_i)^2R_it_f+F_{ri}(\theta_{if}-\theta_{i0}) \quad (27)$$

The total energy consumption is represented by

$$\Delta J_m=\sum_{i=1}^{2}(F_{ri}/K_i)^2R_it_f+F_{ri}(\theta_{if}-\theta_{i0})+2\pi F_{r2} \quad (28)$$

for a backward motion, and

$$\Delta J_p=\sum_{i=1}^{2}(F_{ri}/K_i)^2R_it_f+F_{ri}(\theta_{if}-\theta_{i0}) \quad (29)$$

for a forward motion of the second joint.

Consequently the broken line in Fig. 3 is lifted up more than the dot dash line by $2\pi F_{r2}$.

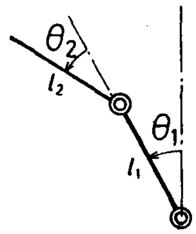

Fig. 2 Two-links manipulator in a horizontal plane.

5. Optimal trajectory

The above mentioned $\dot\theta_{1p}(t)$ and $\dot\theta_{2m}(t)$ not always make the energy consumption minimum. However they satisfies the boundary conditions. Therefore $\dot\theta_{1p}(t)$ and $\dot\theta_{2m}(t)$ can be good starting functions for GNR method. The optimal velocity function is obtained by GNR method by using $\dot\theta_{1p}(t)$ and $\dot\theta_{2m}(t)$ as the starting functions. Then the energy consumption is decreased as shown by the mark ○ in Fig. 3. On the other hand, when $\dot\theta_{1p}(t)$ and $\dot\theta_{2p}(t)$ are used as the starting functions, the energy consumption is also lessened as the marks ● in Fig. 3. It is seen from the figure that the trajectory based on a backward rotation of the second joint is the best except for $(\theta_{1f}-\theta_{10})<0.7$[rad].

Fig. 4(a) shows the angle and angular velocity functions of two joints which can make the energy consumption $J_{mopt}=21.1$[J] at $(\theta_{1f}-\theta_{10})=1.5$[rad]. The bold and thin of solid lines are $\theta_1(t)$ and $\dot\theta_1(t)$, and ones of broken lines show $\theta_2(t)$ and $\dot\theta_2(t)$, respectively. $\dot\theta_2(t)$ is similar to a trapezoidal function not a quadratic one. This slow velocity when $\theta_2(t)=\pi$ can keep the state of small inertia in

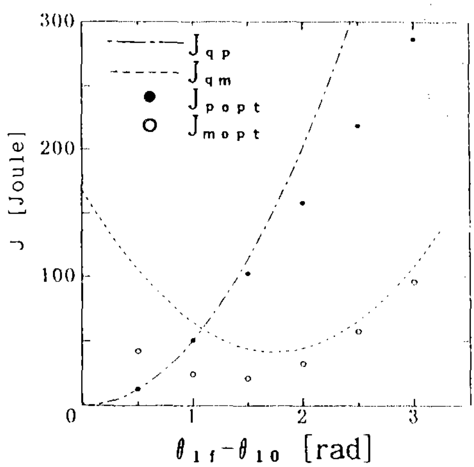

Fig. 3 Energy consumption for various velocity functions.

a long time. Fig. 4(b) shows the trajectory described by $\theta_1(t)$ and $\theta_2(t)$ of Fig. 4(a).

When the energy consumption is $J_{popt}=102.32$[J] at $(\theta_{1f}-\theta_{10})=1.5$[rad], the angle and angular velocity functions are shown in Fig. 5(a). It is remarkable that the velocity of the second joint is zero when $\cos\theta_2(t)$ is lower. Fig. 5(b) shows the trajectory described by $\theta_1(t)$ and $\theta_2(t)$ of Fig. 5(a).

When Coulomb friction force is $Fr=[2.4\ 2.4]^T$, the angle and the angular velocity function are turned into Fig. 6(a) from Fig. 4(a). The velocity function is changed slightly from symmetric into asymmetric about t=0.5. And it is seen from Fig. 6(b) each current is increased by F_{r1}/R_1. The trajectory is also shown in Fig. 6(c). The increments of energy consumption are calculated by (27) as $\Delta J_1=9.36$[J] and $\Delta J_2=20.84$[J]. They are nearly equal to the result of the simulation.

6. Conclusion

The energy consumption of a two-degree-of-freedom manipulator in point-to-point motion has been taken into consideration. The energy can be saved extremely without the restriction of the second link. The optimal velocity pattern of two joints are obtained from a two-points boundary-value-problem technique by using the Generalized Newton-Raphson method. It is seen from the optimal velocity pattern that the velocity of the second joint should be lowered so that the state of the small inertia can be kept as long as possible.

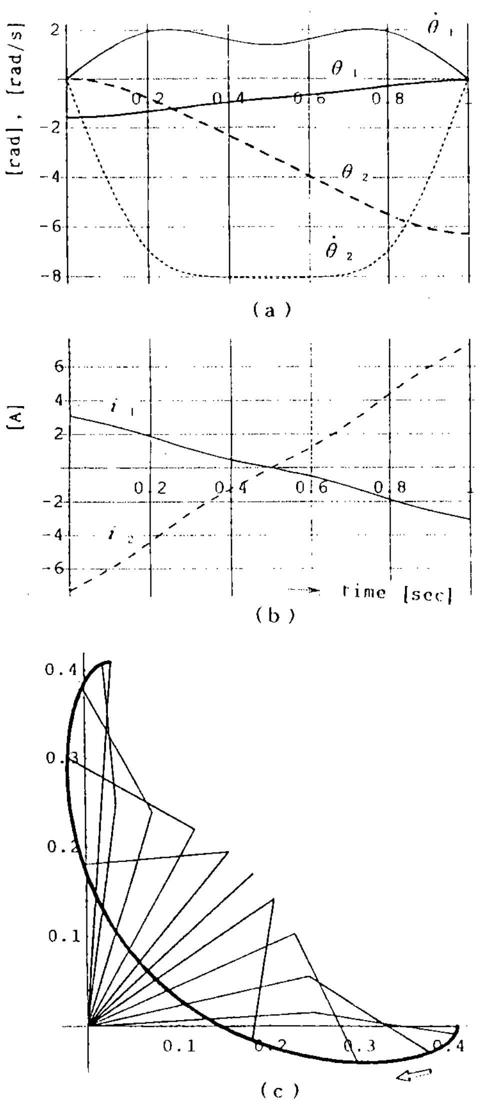

Fig. 4 Minimun energy J_{mopt} at $(\theta_{1f}-\theta_{10})=1.5$
(a) velocity and angle of two joints
(b) current of two joints
(c) trajectory

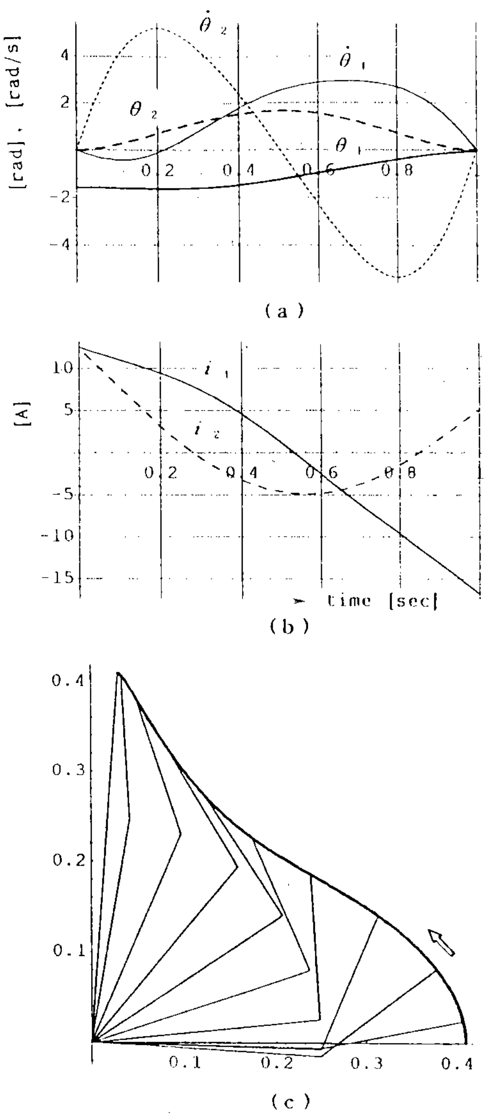

Fig. 5 Minimun energy J_{popt} at $(\theta_{1f}-\theta_{10})=1.5$
(a) velocity and angle of two joints
(b) current of two joints
(c) trajectory

7. References

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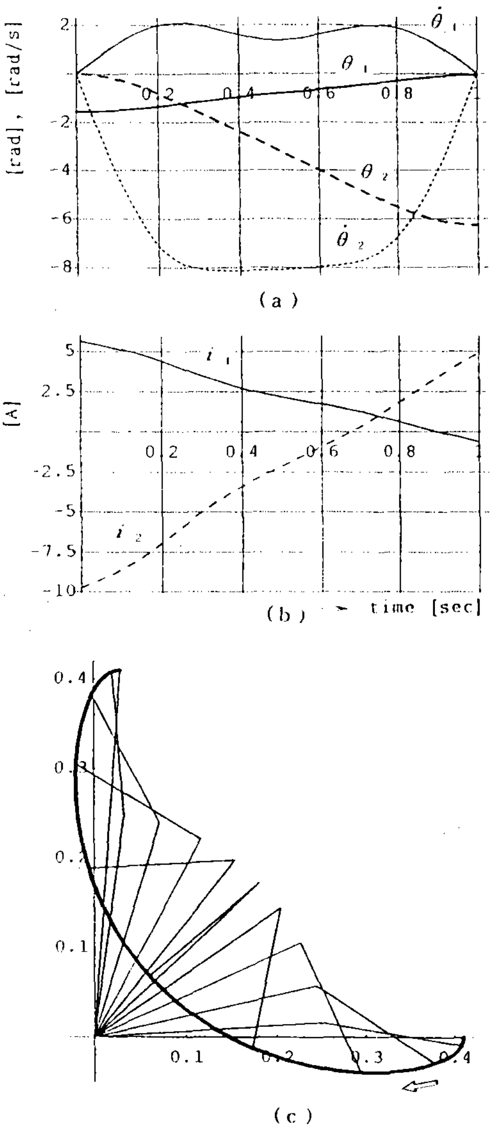

Fig. 6 Minimun energy J_{mopt} in $F_r=[2.4\ 2.4]^T$ at $(\theta_{1f}-\theta_{10})=1.5$
(a) velocity and angle of two joints
(b) current of two joints
(c) trajectory