'94 KACC (1994. 10. 17 ~ 20)

Impact Control of Redundant Manipulators Using Null-Space Dynamics

W. J. Chung°

Dept. of Precision and Mechanical Eng.
Chang-Won National University
#9 Salim-Dong, Chang-Won
641-773 KOREA

S. L. Choi, I. H. Kim, and G. J. Chung

Automatic Control Lab.
Korea Institute of Machinery and Metals
Chang-Won P.O. Box 41
641-600 KOREA

Abstract

This paper presents an impact control algorithm for reducing the potentially damaging effects by interation of redundant manipulators with their environments. In the proposed control algorithm, the redundancy is resolved *at the torque level* by locally minimizing joint torque, subject to the operational space dynamic formulation which maps the joint torque set into the operational forces. For a given pre-impact velocity of the manipulator, the proposed approach is on generating joint space trajectories throughtout the motion near the contact which instantaneously minimize the impulsive force which is a scalar function of manipulator's configurations. This is done by using the *null space dynamics* which does not affect the motion of an end-effector. The comparative evaluation of the proposed algorithm with a local torque optimization algorithm without reducing impact is performed by computer simulation. The simulation results illustrate the effectiveness of the algorithm in reducing both the effects of impact and large torque requirements.

1 Introduction

Increasing attention has been paid to the issue of redundancy in the literature concerning robot manipulators. The research on the use of redundancy has been focused on the optimal coordination of joint motion or torque trajectories based on the optimization of applicable performance criteria. Several performance criteria have been implemented including joint range availability [1], singularity avoidance [2], local kinetic energy minimization [3], and obstacle-avoidance [4]. However, little attention has been paied to the possibility of using kinematic redundancy to address the issue of how best to interact an environment while minimizing potentially damaging collision and impact effects in contact tasks.

Notably, for the case of redundant manipulators, Walker [5] has established a model for the instantaneous (impulsive) effects of impact and derived the impulsive force as a function of manipulator's configuration. A framework for the minimization of the magnitude of impulsive forces has been implemented by solving the (velocity level) inverse kinematics. Specifically, he discussed how to specify the joint velocities that can reduce the undesirable effects of the impact. But he ignored the fact that manipulators are actually controlled by specifying joint torques or especially joint accelerations in the model-based control.

In order to generate the command joint acceleration of redundant manipulator, the dynamic resolution of redundancy which incorporates manipulator dynamics should be taken into account. A dramatic difficulty with using kinematic redundancy in the dynamic control of redundant manipulator is the instability problem illustrated in [6]. This instability means that the command joint acceleration to track a long end-effector trajectory can result in large joint velocities which may require unrealistic torque requirements. Torque stability, specifically keeping joint torques within torque bounds, is not a separate subtask such as singularity or obstacle avoidance, but prerequisite to the trajectory tracking of an end-effector. In a practical sense, any task and/or subtask of a manipulator cannot be accomplished when torque stability is not guaranteed. In this work, a new dynamic control algorithm with torque stability for reducing the potentially damaging effects by interation of redundant manipulators with their environments is proposed.

2 Resolution of Redundancy At The Torque Level

For a n-link manipulator operating in m-dimensional space where $n > m$, the forward kinematics is given by

$$x = f(\theta) \tag{1}$$

where x is an m-dimensional operation space vector with θ being the vector of n-joint coordinates. Differentiation of this relationship leads to the following expression

$$\dot{x} = J(\theta)\,\dot{\theta} \tag{2}$$

where $J \in \Re^{m\times n}$ is the manipulator Jacobian. Differentiating Eq. (2) with respect to time, we have the forward kinematics at the acceleration level as follows:

$$J(\theta)\,\ddot{\theta} = \ddot{x} - \dot{J}(\theta)\,\dot{\theta}\,. \tag{3}$$

The joint space dynamic model of the manipulator can be written in the form

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau \tag{4}$$

where τ is an n-dimensional joint torque vector of the manipulator with $M(\theta)$ and $N(\theta, \dot{\theta})$ being its $n \times n$ symmetric inertia matrix and an n-dimensional vector containing terms such as Coriolis, centripetal, and gravity torques, respectively.

The equation of motion of a system that is constrained by a task-dependent operational space trajectory can be ontained by globally minimizing the constrained Lagrangian. Using an $m \times 1$ vector of Lagrange multipliers, λ, the constrained Lagrangian, L, can be written as

$$L = T - U + \lambda^T(f(\theta) - x) \tag{5}$$

where the kinetic energy of the system is given by

$$T = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} \tag{6}$$

with U being the potential energy due to the gravity forces acting on the system.

The objective function, I, of the global minimization of the constrained Lagrangian is given by

$$I = \int_{t_i}^{t_f} L(\theta, \dot{\theta}, \lambda)\, dt \tag{7}$$

where t_i and t_f are the initial and final time of an end-effector movement, respectively. Using the calculus of variations, the necessary conditions for minimizing I result in (see ref. [7])

$$\lambda = (JM^{-1}J^T)^{-1}\left(\ddot{x} - \dot{J}\dot{\theta} + JM^{-1}N\right). \tag{8}$$

Interpreting λ as the operational forces of the redundant manipulator, F, yields

$$F = \overline{J}\tau, \tag{9}$$

where

$$\overline{J} = (JM^{-1}J^T)^{-1}JM^{-1}. \tag{10}$$

Based upon Eq. (9), the local joint torque minimization scheme can be addressed as follows:

$$Minimize\ \ \frac{1}{2}\tau^T\tau\ \ subject\ to\ \ F = \overline{J}\tau.$$

The minimum-norm joint torque solution including the null space component, in terms of the Moore-Penrose generalized inverse (or pseudoinverse), is directly obtained as

$$\tau = \overline{J}^+F + (I - \overline{J}^+\overline{J})\epsilon \tag{11}$$

where ϵ is an n-dimensional arbitrary vector and

$$\overline{J}^+ = MJ^+_{M^2}(JM^{-1}J^T) \tag{12}$$

with the squared-inertia weighted pseudoinverse, $J^+_{M^2}$, given by

$$J^+_{M^2} = M^{-2}J^T(JM^{-2}J^T)^{-1}. \tag{13}$$

It is worth while noticing that the redundancy is resolved *at the torque level*, rather than at the acceleration level.

The null space dynamics can be expressed as

$$\overline{J}\tau_N = O_m \tag{14}$$

where τ_N is the null space joint torque, which does not affect the motion of the end-effector because the resulting operational force F is zero. Thus the second term of Eq. (11) is just the null space joint torque. In the proposed algorithm, the null space dynamics including the arbitrary vector, ϵ, will play an important role in reducing the undesirable effects of the impact in order to guard the redundant manipulator for the contact for a given end-effector motion.

As suggested in [6], the (unweighted) pseudoinverse, J^+, given by

$$J^+ = J^T(JJ^T)^{-1}, \tag{15}$$

can stabilizes further the global behavior of joint torques rather than any weighted pseudoinverse. By using J^+ instead of $J^+_{M^2}$, the resulting joint acceleration is obtained as follows:

$$\begin{aligned}\ddot{\theta} &= J^+(\ddot{x} - \dot{J}\dot{\theta}) - (I - J^+J)M^{-1}N \\ &\quad + (I - J^+J)M^{-1}\epsilon.\end{aligned} \tag{16}$$

This substitution does not affect the null space dynamics, which can be easily shown. In addition, J^+ gains an advantage over $J^+_{M^2}$ with respect to computational efficiency. Therefore the substitution of J^+ for $J^+_{M^2}$ can lead to an effective and efficient algorithm from the viewpoint of both torque stability and modest computational load.

3 Robotic Impulsive Model In Contact With Solid Surface

The dynamic equations of the manipulator contacting with an environment is given in the form

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau + J^T(\theta)F_{ce} \tag{17}$$

where F_{ee} denots the force/moment experienced at the end-effector.

An impact occurs when a robotic system contacts its environemt. It is assumed that the impact occurs in a small period of Δt. During the infinitesimally short time interval of impact, all velocities and angular velocities remain finite, and thus there is no change in positions or orientations of any bodies in the system. In the light of this basic assumption, one may integrate both sides of Eq. (17) in an infinitesimally short time and have

$$M(\theta)\{\dot{\theta}(t_0 + \Delta t) - \dot{\theta}(t_0)\} = J^T(\theta)\lim_{\Delta t \to 0}\int_{t_0}^{t_0+\Delta t} F_{ce}\, dt. \tag{18}$$

The imulsive force $\hat{F}_{ce}$ created at the collision point by the impact, denoted by

$$\hat{F}_{ee} = \lim_{\Delta t \to 0}\int_{t_0}^{t_0+\Delta t} F_{ce}\, dt, \tag{19}$$

is a finite quantity. Also, we denote $\dot{\theta}(t_0 + \Delta t) - \dot{\theta}(t_0)$ to be $\Delta\dot{\theta}$. Then from Eqs. (18) and (19) we have, at the instant of collision,

$$\Delta\dot{\theta} = M^{-1}J^T\hat{F}_{ce}. \tag{20}$$

Equation (20) expresses the relationship between the impulsive (contact) force and the instantaneous joint velocity increment corresponding to the induced end-effector velocity. Since the relationship (2) between joint and end-effector velocities still holds at the instant of collision, we may write

$$\Delta\dot{x} = J\,\dot{\theta} = JM^{-1}J^T\,\hat{F}_{ee}\,. \tag{21}$$

Solving for $\hat{F}_{ee}$ in Eq. (21) and substituting into Eq. (20) yields

$$\Delta\dot{\theta} = M^{-1}J^T(JM^{-1}J^T)^{-1}\,\Delta\dot{x} = J_M^+\,\Delta\dot{x}\,, \tag{22}$$

which J_M^+ is defined by

$$J_M^+ = M^{-1}J^T(JM^{-1}J^T)^{-1}\,. \tag{23}$$

For the model of instantaneous collision dynamics, if the velocities of bodies 1 and 2 immediately before collision are v_1 and v_2 respectively, and the change in these velocities immediately following collision are Δv_1 and Δv_2, then we have the following expression:

$$[(v_1 + \Delta v_1) - (v_2 + \Delta v_2)]^T n = -e\,(v_1 - v_2)^T n \tag{24}$$

where n is the normal vector to the plane of contact of the two bodies and $0 \le e \le 1$ is the coefficient of restitution.

We specialize the impact dynamics model to the case of a manipulator contacting a solid object, which does not move (such as a wall or table). In this case, we have

$$\begin{aligned} v_1 &= \dot{x}, \quad \Delta v_1 = \Delta\dot{x}\,, \\ v_2 &= \Delta v_2 = O\,. \end{aligned}$$

Equation (24) becomes

$$(\dot{x} + \Delta\dot{x})^T n = -e\,\dot{x}^T n\,. \tag{25}$$

We also invoke the fact that the impulsive force is directed along the direction of the normal to the common tangent plane to the contact point, that is,

$$\hat{F}_{ee} = \hat{F}_{ee}\,n\,. \tag{26}$$

Using Eqs. (21) and (26), we may solve for the (scalar) magnitude of the impulsive contact force, i.e., $\hat{F}_{ee}$, as (see ref. [5])

$$\hat{F}_{ee} = \frac{-(1+e)\,\dot{x}^T n}{n^T J M^{-1} J^T n}\,. \tag{27}$$

Here we will consider the case of $\dot{x}$ being fixed which is the most likely situation from motion planning. Accordingly, $\hat{F}_{ee}$ becomes a function of the manipulator configuration θ.

In order to reduce the damaging effects of the impact the magnitude of $\hat{F}_{ee}$ in Eq. (27) should be minimized. For an environmental 'wall' with the normal to its tangent surface being n and a given desired end-effector velocity $\dot{x}$ at the moment of contact, this means that the denominator of Eq. (27), which is a function of θ, should be maximized. Thus the denominator of Eq. (27) can be used as a performance function to be optimized. Hereinafter it will be noted by $H(\theta)$, that is,

$$H(\theta) = n^T J M^{-1} J^T n\,. \tag{28}$$

4 Proposal of Impact Control Algorithm

The proposed algorithm is based on the formulation of joint acceleration given by Eq. (16). The first term of this equation accounts for the minimization of the norm of joint acceleration, $\|\ddot{\theta}\|$. Hereinafter it is termed the "minimum-norm acceleration," denoted by $\ddot{\theta}_m$. For the accurate tracking of the desired Cartesian trajectory $x_d(\cdot)$, the usual error correcting term $K_v\,\dot{e} + K_p\,e$ is added to $\ddot{x}_d$ in place of $\ddot{x}$. Here $e \stackrel{\Delta}{=} x_d - x$ is the tracking error, and K_v, K_p are m by m constant velocity and position feedback gain matrices, respectively. Then the minimum-norm acceleration is given by

$$\ddot{\theta}_m = J^+(\ddot{x}_d + K_v\,\dot{e} + K_p\,e - \dot{J}\,\dot{\theta})\,. \tag{29}$$

Recalling Eq. (16), the resultant joint acceleration can be decomposed into a combination of the least-squares solution of minimum-norm, i.e., $\ddot{\theta}_m$, plus homogeneous solutions created by the action of a projection operator $(I - J^+J)$ which maps $(-M^{-1}N)$ and $M^{-1}\varepsilon$ into the null space of the Jacobian J. For convenience, these homogeneous solutions are termed "the first homogeneous acceleration" and "the second homogeneous acceleration," denoted by

$$\begin{aligned} \ddot{\theta}_{h1} &= -(I - J^+J)M^{-1}N \qquad (30)\\ \ddot{\theta}_{h2} &= (I - J^+J)M^{-1}\varepsilon\,, \qquad (31) \end{aligned}$$

respectively. In particular, the first homogeneous acceleration corresponds to the local minimization of joint torques since it can be derived from the first term of Eq. (11), i.e., the minimum-norm joint torque solution, by assuming $\varepsilon = O_n$ and substituting J^+ for $J_{M^2}^+$. In addition, the second homogeneous acceleration corresponds to the null space dynamics which is represented by the second term of Eq,. (11).

The role of reducing the undesirable effects of the impact is assigned to the arbitrary vector ε in the second homogeneous acceleration. This is done by using the gradient projection technique [1] as follows:

$$\varepsilon = \kappa\,\nabla H(\theta) \tag{32}$$

where κ is a scalar constant and $\nabla H(\theta)$ is the gradient (n-dimensional column-vector) of the performance function given by Eq. (28). Notice that κ should be a positive constant since $H(\theta)$ is maximized. Especially, the suitable selection of κ in Eq. (32) is based on hardware limits on joint torques as follows:

$$\tau_L^T\tau_L \ge (M\ddot{\theta} + N)^T(M\ddot{\theta} + N)\,, \tag{33}$$

where τ_L is the vector denoting the hardware limit for the joint torques. Equation (33) can be reduced to the second order algebraic equation given by

$$A\,\kappa + 2\,B\,\kappa + C \le 0 \tag{34}$$

where

$$\begin{aligned} A &= p^T M^2 p \\ B &= p^T M^2(\ddot{\theta}_m + \ddot{\theta}_{h1}) + p^T M N \\ C &= (\ddot{\theta}_m + \ddot{\theta}_{h1})^T M^2(\ddot{\theta}_m + \ddot{\theta}_{h1}) + 2(\ddot{\theta}_m + \ddot{\theta}_{h1}) \\ &\quad\; M N + \|N\|^2 - \|\tau_L\|^2\,, \end{aligned}$$

wherein

$$p = (I - J^+J)M^{-1}\,\nabla H(\theta)\,.$$

It can be pointed out that the terms A, B, and C are the functions of θ and $\dot{\theta}$. Given the torque limit vector τ_L, the joint position vector $\boldsymbol{\theta}$, and the joint velocity vector $\dot{\boldsymbol{\theta}}$, the values of the parameters can be determined. Thus one candidate for κ will be

$$0 < \kappa \leq \frac{-B + \sqrt{B^2 - AC}}{A}. \tag{35}$$

Thus the second homogeneous acceleration is obtained as

$$\ddot{\boldsymbol{\theta}}_{h2} = \kappa (\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}) M^{-1} \nabla H(\boldsymbol{\theta}). \tag{36}$$

Now, the resulting command acceleration $\ddot{\boldsymbol{\theta}}_d$, incorporating the minimization of the impulsive contact forces (i.e., the maximization of the performance function given by Eq. (28)), becomes

$$\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m + \ddot{\boldsymbol{\theta}}_{h1} + \ddot{\boldsymbol{\theta}}_{h2} \tag{37}$$

where $\ddot{\boldsymbol{\theta}}_m$, $\ddot{\boldsymbol{\theta}}_{h1}$, and $\ddot{\boldsymbol{\theta}}_{h2}$ are given by Eqs. (29), (30), and (36), respectively. However, this form of the command acceleration is not appropriate for direct use in industrial applications due to the inherent instability problem. As a main reason for the instability problem, the previous literature (see refs. [6], [8]) reported that homogeneous accelerations cause homogeneous velocities to increase in performing the subtask such as the local torque minimization, which in turn result in physically unrealizable torques.

In the sense of torque stability, a trade-off among homogeneous accelerations is unavoidable. As a trade-off means, an automatic switching scheme is preferred. As a systematic switching criterion, the stability condition proposed by Maciejewski [8] is adopted in the proposed algorithm, which is given by

$$\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_h \leq 0 \tag{38}$$

where $\dot{\boldsymbol{\theta}}_h$ and $\ddot{\boldsymbol{\theta}}_h$ represent a homogeneous joint velocity and a homogeneous joint acceleration vector, respectively. When Eq. (38) does not hold, the homogeneous acceleration will increase the magnitude of the homogeneous joint velocity and will subsequently increase torque requirements. This, in effect, amounts to a positive feedback system and results in the instability problem. In [8], this condition was proposed on purpose to identify regions of stability and instability for a local torque optimization scheme offline. However, in the proposed algorithm, the condition is incorporated in actively avoiding the instability region of operation online. Especially, considering the homogeneous accelerations, $\ddot{\boldsymbol{\theta}}_{h1}$ and $\ddot{\boldsymbol{\theta}}_{h2}$, the two types of stability condition are used in the following form

$$\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h1} \leq 0 \tag{39}$$

$$\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h2} \leq 0 \tag{40}$$

where $\dot{\boldsymbol{\theta}}_h$ is given by

$$\dot{\boldsymbol{\theta}}_h = (\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\dot{\boldsymbol{\theta}}. \tag{41}$$

The actual value of $\dot{\boldsymbol{\theta}}$ in Eq. (41) can be obtained from a measuring device of a robotic system.

Now we are ready to specify the dynamic control algorithm with torque stability for minimizing the impulsive forces. The proposed approach is to add each homogeneous acceleration term to the minimum-norm acceleration according to the realted stability condition as follows:

if ($\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h1} \leq 0$ and $\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h2} \leq 0$) then
  $\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m + \ddot{\boldsymbol{\theta}}_{h1} + \ddot{\boldsymbol{\theta}}_{h2}$
else if ($\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h1} \leq 0$ and $\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h2} > 0$) then
  $\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m + \ddot{\boldsymbol{\theta}}_{h1}$
else if ($\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h1} > 0$ and $\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h2} \leq 0$) then
  $\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m + \ddot{\boldsymbol{\theta}}_{h2}$
else
  $\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m$.

5 Numerical Simulation

Any dynamic control method of redundant manipulators for minimizing the impact effects has not been presented as far as we know. Thus, in order to explicitly illustrate its effectiveness with respect to minimizing the potentially damaging collision effects, the proposed algorithm is comparatively evaluated with the algorithm given as follows:

if ($\dot{\boldsymbol{\theta}}_h \cdot \ddot{\boldsymbol{\theta}}_{h1} \leq 0$) then
$\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m + \ddot{\boldsymbol{\theta}}_{h1}$
else
$\ddot{\boldsymbol{\theta}}_d = \ddot{\boldsymbol{\theta}}_m$.

The above algorithm proposed by Chung *et al.* [9] is the stability-condition based dynamic control for local joint torque optimization, which does not include the subtask of reducing the effects of impact.

In this example, the simulated manipulator is a 3R planar manipulator without gravity. All links are modeled by a thin uniform rod of a length of 1.0 m and a mass of 10 kg. The desired end-effector trajectory, $\ddot{x}_d(\cdot)$, is a straight-line Cartesian path, starting and ending with zero end-effector velocity, and a constant bang-bang type of acceleration. To demonstrate the issues discussed herein, we utilize a particular example of the collision of the redundant manipulator with a solid surface, or wall. The contact dynamics-based subtask of maximizing $H(\boldsymbol{\theta})$ given by Eq. (28) will guard against damaging collision effects with an environmental wall with the normal to its tangent surface being $\boldsymbol{n} = [-1 \;\; 0]^T$. The wall is located at $x = 1$ parallel to the y-direction, on the way of the planned trajectory.

The command torque $\boldsymbol{\tau}$, which is adopted as a control input, is obtained from Eq. (4) using the command acceleration $\ddot{\boldsymbol{\theta}}_d$. The joint control system is simulated with position and velocity feedback gain matrix $\boldsymbol{K}_p = \mathrm{diag}(256, 256, 256)$ and $\boldsymbol{K}_v = \mathrm{diag}(32, 32, 32)$, respectively. The arm starts from $\boldsymbol{\theta}_0 = [180° \;\; -90° \;\; 0°]^T$ close to a kinematic singularity, with accelerations $\ddot{x}_d = [3 \;\; -2]^T$ m/s^2 and $\ddot{x}_d = [-3 \;\; 2]^T$ m/s^2 for the first and the last half of the path, respectively. Besides, the coefficient of restitution is assumed to be $e = 0.9$ for the collision between the end-effector and the wall.

Figure 1 (a) shows the arm motion generated by the stability-condition based dynamic control *without reducing the effects of impact*. At the instant of collision, the magnitude of impulsive forces calculated using Eq. (27) is obtained as 135.95 N·m·s. As expected, the stable behavior of joint torques is illustrated in Fig. 1 (b) where the peak torque is bounded by about 210 N·m.

The next simulation was performed for the proposed algorithm along the planned trajectory. The constant κ which

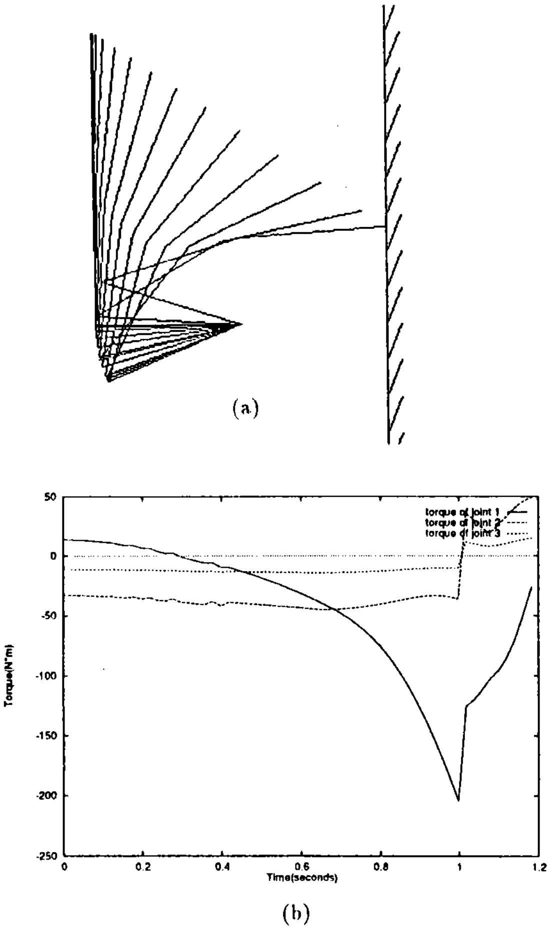

Fig. 1. Simulation results for the stability-condition based method *without reducing the impact effects*: (a) arm motion; (b) torque profiles.

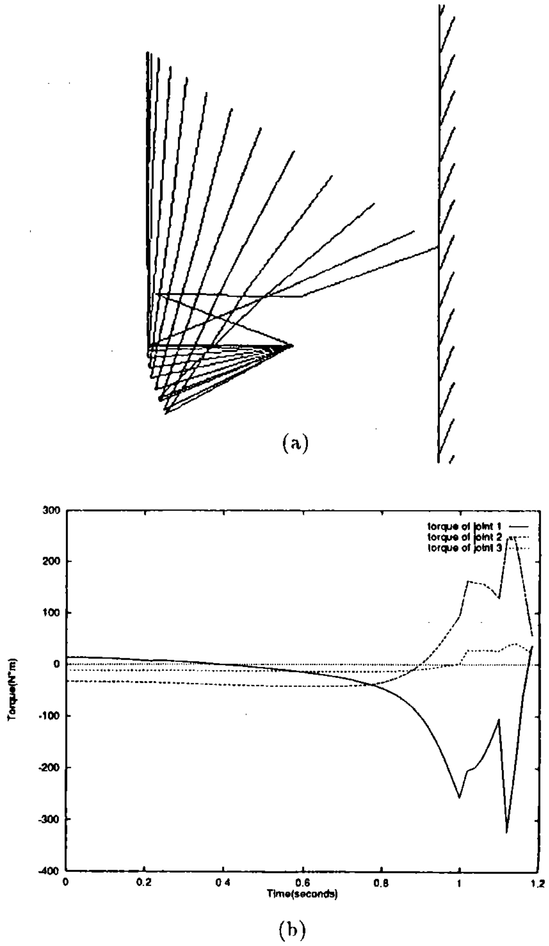

Fig. 2. Simulation results for the proposed algorithm: (a) arm motion; (b) torque profiles.

is required in generating $\ddot{\theta}_{h2}$ was selected as 10^4 considering the bound given by

$$0 < \kappa \leq 18112. \tag{42}$$

This bound was obtained in Eq. (35) by substituting $\boldsymbol{\theta} = [178° \ -89° \ -1°]^T$, $\dot{\boldsymbol{\theta}} = [0 \ 0 \ 0]^T$, and $\boldsymbol{\tau}_L = [10^3 \ 10^3 \ 10^3]^T$ N·m for an approximate initial movement. As shown in Fig. 2(a), the contact configuration of the manipulator generated by the proposed algorithm illustrates that the third link is less perpendicular to the wall when we compare this figure with Fig. 1(a). In this case, the magnitude of impulsive forces is 63.95 N·m·s, about a half of that of the first case, that is, the stability-condition based method without reducing the impact effects. This agrees with our intuition, since in the human arm case, we would expect to obtain less shocks when colliding with obstacles with forearm more folded than in a outstretched configuration. It is seen in Fig. 2(b) that the proposed algorithm leads to a bigger value of the peak torque when compared with the first case. But the peak torque is bounded within about 320 N·m, which is still in a stable region of operation. The above observations leads us to conclude that the proposed algorithm results in a better performance in the sense of reducing the impact effects while providing the stable behavior of joint torques.

6 Conclusion

In this paper, a new dynamic control algorithm with torque stability for redundant manipulators was proposed to reduce the potentially damaging effects of the impact encountered in the interaction of the manipulators with their environments. In the proposed algorithm, the redundancy was resolved at the torque level by locally minimizing joint torques, subject to the operational space dynamic formulation which is the unique functional relationship mapping the joint torque set into the operational forces. The null space dynamics, i.e., the homogeneous component of joint torques, which does not affect the motion of an end-effector, was used to minimize the magnitude of the impulsive forces obtained from the model of instantaneous collision dynamics. The resulting command acceleration is composed of three terms: 1) the minimum-norm acceleration, 2) the first homogeneous acceleration correponding to the local minimization of joint torques, and 3) the second homogeneous acceleration corresponding to the local minimization of the magnitude of the impulsive forces. In the sense of torque stability, the proposed algorithm adopted a switching technique as a trade-off among the three homogeneous accelerations by using the stability condition. The comparative evaluation of the proposed algorithm with the stability-condition based method without reducing the impact effects demonstrated the effectiveness of the proposed algorithm in the sense of the minimization of the impulsive forces. In addition, the proposed algorithm was shown to generate stable joint torques and agree with the human arm case.

References

[1] A. Liégeois, "Automatic supervisory control of configuration and behavior of multibody mechanism," *IEEE Trans. Systems, Man, and Cybernetics*, vol. SMC-7, no. 12, pp. 868–871, 1977.

[2] T. Yoshikawa, "Analysis and control of robot manipulators with redundancy," *Robotics Research: The First International Symposium*, M. Brady and R. Paul, Eds., MIT Press, Cambridge, MA, 1984, pp. 735–747.

[3] O. Khatib, "Dynamic control of manipulators in operational space," in *Proc. of 6th IFToMM World Conf. on Theory of Machines and Mechanisms* (New Delhi, India), 1983, pp. 15-20.

[4] A. A. Maciejewski and C. A. Klein, "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments," *Int. J. Robotics Research*, vol. 4, no. 3, pp. 109–117, 1986.

[5] I. D. Walker "The use of kinematic redundancy in reducing impact and contact effects in manipulation," *Proc. IEEE Int. Conf. on Robotics and Automation* (Cincinnati, OH), May 1990, pp. 434–439.

[6] J. M. Hollerbach and K. C. Suh, "Redundancy resolution of manipulators through torque optimization," *IEEE J. Robotics and Automation*, vol. RA-3, no. 4, pp. 308–316, 1987.

[7] H.-J. Kang and R. A. Freeman, "Null space damping method for local joint torque optimization of redundant manipulators," *J. of Robotic Systems*, vol. 10, no. 2, pp. 249–270, 1993.

[8] A. A. Maciejewski, "Kinetic Limitations on the use of redundancy in robotic manipulators," *IEEE Trans. on Robotics and Automation*, vol. 7, no. 2, pp. 205–210, 1991.

[9] W. J. Chung, W. K. Chung and Y. Youm, "Dynamic control algorithm of redundant manipulators based on stability condition," in *Proc. of the ISCA Int. Conf. on Computer Applications in Industry and Engineering*, (Honolulu, HI), Dec. 1993, pp. 7–11.