

Large Slewing Control of Low Earth Orbit Satellite

S. W. Rhee

Senior Member of Research Staff, TT&C Section, Satellite Communications R&D Division,
ETRI, Yusong P.O. Box 106, Taejon, 305-600, Korea.
Tel.: 82-42-860-5653, 6469, Fax: 82-42-860-6430

Keywords : Satellite, Low Earth Orbit, Slewing Control, Quaternion Feedback Control,
Angular Rate Limit

Abstract

A new method of quaternion feedback control for the attitude acquisition of spacecraft is suggested to limit the angular rates of rigid body which are not desirable and make a control algorithm complicate. New attitude acquisition control algorithm is evaluated and compared with the existing quaternion feedback control method for the large slewing maneuvers through simulations. The simulation results reveal that a new method is effective on limiting the angular rates of spacecraft.

1. Introduction

Currently, there are so many satellites in Geosynchronous Orbit(GEO), Sun Synchronous Orbits, Molniya Orbit, Low Earth Orbits(LEO), etc. for communication, scientific tests, and remote sensing purposes. In general, the Earth orbits below 1000 km are called LEO and GEO, about 36000 km above the Earth. The key factors in altitude selection are the satellite's radiation environment and mission objectives. Consequently, GEO is mainly selected for the communication purposes and LEO for the scientific tests or remote sensing purposes. Recently LEO are adopted for mobile communication services using multiple satellites, so called constellation structure. Payloads in LEO include those for reconnaissance, detecting nuclear bursts, local weather and hydrology, oceanography, agriculture, etc. Therefore, a lot of interest in LEO satellite has been continuously shown by aerospace industries since 1960's. However, LEO satellites are exposed to more environmental disturbances than GEO satellite is, and the attitude control systems of LEO satellites are generally more complicate than ones of GEO satellites because of fragile environmental disturbances and short circular period of orbit. Four major disturbances on LEO satellites are gravity-gradient effects, magnetic-field torques on the vehicle, impingement by solar-radiation particles, and aerodynamic torques.

For GEO satellite case, large slewing maneuvers are generally required for the apogee kick motor's(AKM) firing attitude to inject into the GEO and for the Sun-Earth

acquisition to get the nominal mission mode attitude. It is expected that LEO satellites require large slewing maneuvers more frequently than GEO ones, that is, the short circular orbit period and the environmental disturbances sometimes require large slewing motion so as to meet antenna pointing accuracy requirements for the efficient data transfer/communication, or to observe a target as fast as possible because of the short ground contact time.

It is known that a direction cosine matrix, Euler angles, or quaternions can specify the orientation of a rigid body and the use of quaternions for large slewing maneuvers is very attractive since they do not have inherent geometrical singularity. The quaternion feedback concept for an attitude regulator was first introduced by Mortensen¹ in 1968. Wie^{2,3}, et al. studied on the large slewing control of spacecraft using quaternion feedback control laws extensively. However, limiting angular rates of spacecraft was not mentioned in his study, which may be of benefit to controller design. Thus, the quaternion feedback control algorithm suggested by Wie³, is modified to limit angular rates of a rigid spacecraft for large slewing control. The rigid dynamic model of a satellite is considered to test the proposed method in this study. It is assumed that three axes information of angular rate from rate gyros is available. The suggested control algorithm is evaluated and compared with Wie's results for the practical point of view through numerical simulation.

2. System Modeling

In this paper, it is considered that the body-fixed torque devices are used for general case of a rigid body slewing maneuvers. Though the dynamics of body-fixed torquing devices(thrusters, reaction wheels, and control moment gyros) should be considered in system modeling for control design, an ideal control torquing device is assumed in modeling for simplicity. A body-fixed frame is considered with axes coincident with a rigid body's principal axes. The mathematical formulation of attitude dynamics may be found in the common text books.^{4,5}

The angular momentum vector \bar{H}_c of a single rigid body referred to an inertia frame may be expressed as

$$\bar{H}_c = I_c \bar{\omega} \quad (1)$$

where $\bar{\omega} = [\omega_x \ \omega_y \ \omega_z]^T$ is the absolute angular rate vector relative to a body-fixed frame, and I_c is the inertia matrix of a single rigid body. The rotational motion of a rigid body about body-fixed axes with the center of mass can be represented by Euler's equations.

$$\dot{\bar{H}}_c + \Omega \bar{H}_c = \bar{T}_c \quad (2)$$

where Ω is a skew-symmetric matrix defined by

$$\Omega = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (3)$$

, and control torque vector, $\bar{T}_c = [T_{cx} \ T_{cy} \ T_{cz}]^T$.

Then, the dynamic equation of a rigid body is expressed as

$$I_c \dot{\bar{\omega}} = -\Omega I_c \bar{\omega} + \bar{T}_c \quad (4)$$

It is assumed that rate gyro can measure the angular rate components along the body-fixed axes, which may be used to calculate the body orientation and to feed them back.

The Euler's principal rotation theorem states that a rigid body can be rotated from any initial attitude to any final state by a single rotation of the body through a principal angle ϕ about eigenaxis. The rotation may be expressed by introducing quaternions. The quaternions, also called Euler parameters, consist of the vector part and the scalar part. The vector part represents the direction of the eigenaxis. The four elements of the quaternion are defined as

$$\begin{aligned} q_0 &= \cos(\phi/2) \\ q_1 &= e_x \sin(\phi/2) \\ q_2 &= e_y \sin(\phi/2) \\ q_3 &= e_z \sin(\phi/2) \end{aligned} \quad (5)$$

with $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$

where e_i is the direction cosine of the eigenaxis in the reference frame. The quaternion is used to describe the orientation of a rigid body by

$$\begin{aligned} \dot{\bar{q}} &= \frac{1}{2} \Omega \bar{q} + \frac{1}{2} q_0 \bar{\omega} \\ \dot{q}_0 &= -\frac{1}{2} \bar{\omega}^T \bar{q} \end{aligned} \quad (6)$$

where $\bar{q} = [q_1 \ q_2 \ q_3]^T$ and Ω is the skew-symmetric matrix. Equation (6) is called the quaternion kinematic equation.

3. Quaternion Feedback Control

The linear quaternion feedback law suggested by Mortensen takes the following form:

$$T_{ci} = -k_i q_i - d_i \omega_i, \quad i = 1, 2, 3. \quad (7)$$

Mortensen suggested the design criterion by choosing the control gains k_i to be inversely proportional to the principal moments of inertia. Recently, Wie³ suggested a feedback

controller for eigenaxis rotations and the modified design criterion for the control gains with the following form:

$$T_c = -\Omega I_c \bar{\omega} - K \bar{q} - D \bar{\omega} \quad (8)$$

$$k_i = (\alpha I_{ci} + \beta)^{-1}, \quad i = 1, 2, 3 \quad (9)$$

where Ω is specified in equation (3), I_c is the moment of inertia matrix, $\bar{\omega}$ is the absolute angular rate vector, $K = \text{diag}\{k_1, k_2, k_3\}$, $D = \text{diag}\{d_1, d_2, d_3\}$, I_{ci} is the principal moments of inertia, and α, β are scalars. The $\text{diag}(\ast)$ stands for a diagonal matrix. He showed the control law to be globally stable and to be robust to inertia uncertainties using Lyapunov second theorem. Lyapunov function³ used has the following form:

$$V = \frac{1}{2} \bar{\omega}^T K I_c \bar{\omega} + 2(1 - q_0) \quad (10)$$

The global stability is guaranteed if d_i is a positive scalar and α, β are nonnegative scalars. As seen in equation (8), gyroscopic decoupling torque is included in the control law to cancel out gyroscopic coupling term. However, the coupling term may be neglected in equation (4) if the angular rates of a spacecraft can be kept very small during slewing maneuvers. Thus, Equation (9) is modified to limit the angular rates as following:

$$k_i = (\text{scale_factor} \ast \alpha I_{ci} + \beta)^{-1} \quad (11)$$

The modified quaternion feedback control algorithm can be summarized as

BEGIN

if $\text{abs}(\omega_i) > \omega_{lim}$

$n = n + 1$

if $n > n_{max}$

$n = n_{max}$

$\text{scale_factor} = \omega_{lim} / (n \ast \text{abs}(\omega_i))$

else $\text{scale_factor} = 1$

$k_i = 1 / (\text{scale_factor} \ast \alpha I_{ci} + \beta)$

$T_{ci} = -k_i q_i - d_i \omega_i$

END

As seen in equation (11) and the summarized algorithm, a scale factor and a forgetting factor n are introduced to get a varying weight (scale factor), which may be one of gain scheduling methods of inertia part. Note the scale factor is always greater than zero. Thus, the closed-loop system with the proposed quaternion feedback gains is always stable.

4. Simulation Results and Discussions

For the simplicity, the products of inertia are neglected for the simulation because these terms may be considered as model uncertainty for slewing control problem: $I_c = \text{diag}\{1400, 1700, 1800\}$ Kg-m is selected for the moment of inertia of a rigid body spacecraft. The numerical data, $[q_0 \ q_1 \ q_2 \ q_3] = [0.159 \ 0.57 \ 0.57 \ 0.57]$ is used for the initial quaternion elements at $t=0$ and zeros for the initial angular rates. $[1 \ 0 \ 0 \ 0]$ is selected for the final quaternion elements. Thus, the eigenangle required to be rotating is 161.7 deg.

Initially, $\alpha = 5.e-7$, $\beta = 1.e-4$, and $d = 0.316$ are selected for the simulation. The difference between the suggested modified method and Wie's method is that α is used as a varying weight factor to limit the angular rate of a rigid body in the suggested technique but is a constant value in Wie's one. Figures 1, 2, 3 show the time histories of the control torques for x, y, z directions, respectively. The angular rate profiles of x, y, z axes are shown in figure 4, 5, 6. The maximum angular rates are about (-0.1), (-0.075), (-0.085) rad/sec for each axis with Wie's method but the use of modified method can limit the angular rates less than (-0.04) rad/sec. Figures 7, 8, 9, 10 depict the time histories of quaternion elements, q_1, q_2, q_3, q_0 . The reorientation time is about 40 sec. with Wie's method but the modified method takes about 80 sec. for reorientation, which is two times of the result of Wie's method. Figures 11, 12, 13 show the scale factor variations of the quaternion elements, q_1, q_2 , and q_3 , respectively. As seen in figures, though the angular rates can be limited by the modified method, undesirable chattering phenomena are observed in the control torque and the angular rate profiles. Currently, another modified method is studying to fix this problem, such as using a smooth filter before actuating, as well as torque limiting maneuvers.

5. Conclusions

The quaternion feedback control problem has been studied for an inertially symmetric rigid spacecraft with independent three-axis control torques. A novel technique has been presented for the attitude acquisition of a rigid spacecraft. It has shown that the angular rates of a rigid spacecraft can be limited effectively using the modified quaternion feedback control method. The undesirable chattering phenomena have been observed in control torque and angular rate profiles. The chattering phenomena should be removed for practical applications, which will be the future work.

References

1. Mortensen, R.E., "A Globally Stable Linear Attitude Regulator," *Int. J. of Control*, Vol. 8, no. 3, 1968, pp. 297-302.
2. Wie, B. and Barba, P.M., "Quaternion Feedback for Spacecraft Large Angle Maneuvers," *J. of Guidance, Control, and Dynamics*, Vol. 8, no.3, May-June, 1985, pp. 360-365.
3. Wie, B. and Arapostathis, A., "Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations," *J. of Guidance, Control, and Dynamics*, Vol. 12, no.3, May-June, 1989, pp. 375-380.
4. Wertz, J.R., *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Company, Boston, 1986, pp. 511-523.
5. Fortescue, P.W. and Stark, J.P.W., *Spacecraft Systems Engineering*, John Wiley and Sons, N.Y., 1991, pp. 33-58.

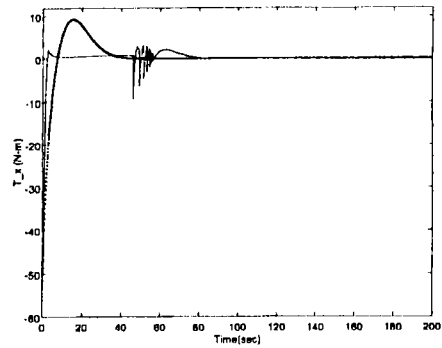


Figure 1. Time History of X-Directional Control Torque (New method: solid, Wie's method: thick dotted line)

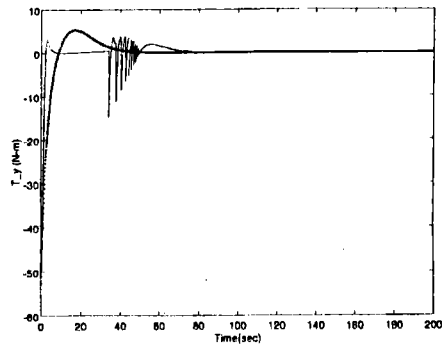


Figure 2. Time History of Y-Directional Control Torque (New method: solid, Wie's method: thick dotted line)

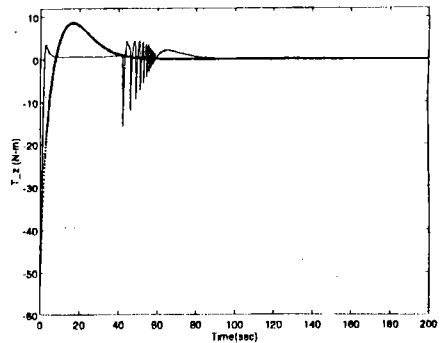


Figure 3. Time History of Z-Directional Control Torque (New method: solid, Wie's method: thick dotted line)

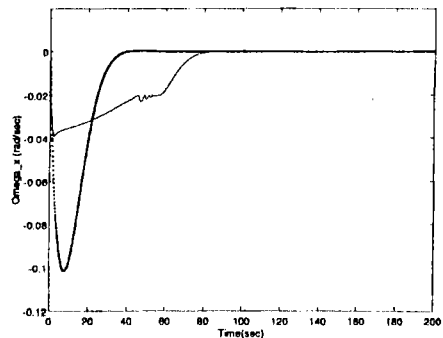


Figure 4. Time History of X-Directional Angular Rate (New method: solid, Wie's method: thick dotted line)

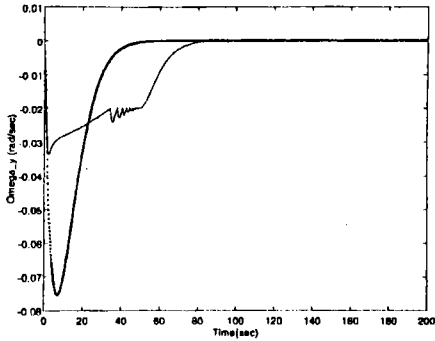


Figure 5. Time History of Y-Directional Angular Rate (New method: solid, Wie's method: thick dotted line)

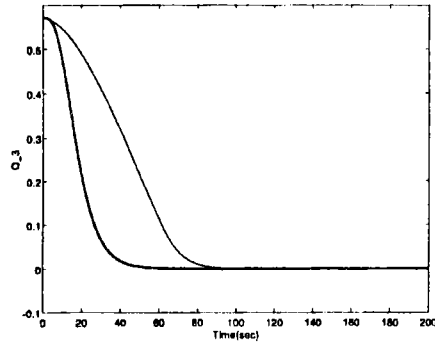


Figure 9. Time History of Quaternion Q3 (New method: solid, Wie's method: thick solid line)

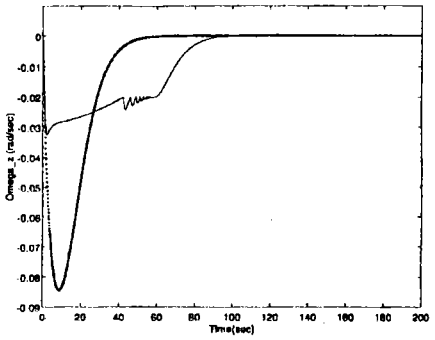


Figure 6. Time History of Z-Directional Angular Rate (New method: solid, Wie's method: thick dotted line)

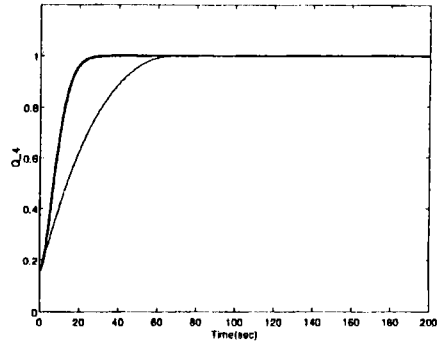


Figure 10. Time History of Quaternion Q0 (New method: solid, Wie's method: thick solid line)

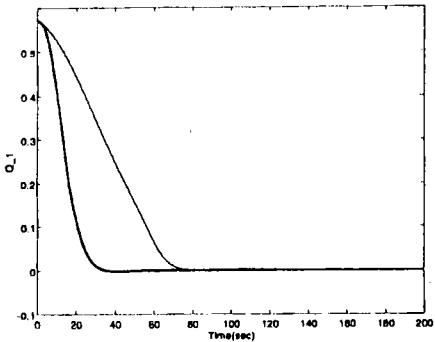


Figure 7. Time History of Quaternion Q1 (New method: solid, Wie's method: thick solid line)

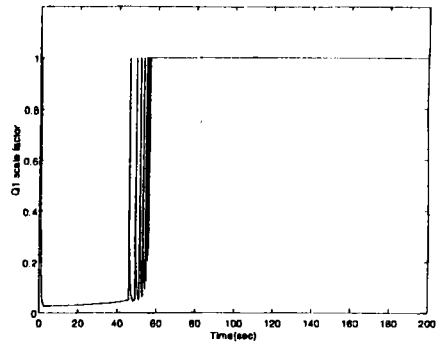


Figure 11. Time History of Scale Factor for Quaternion Q1

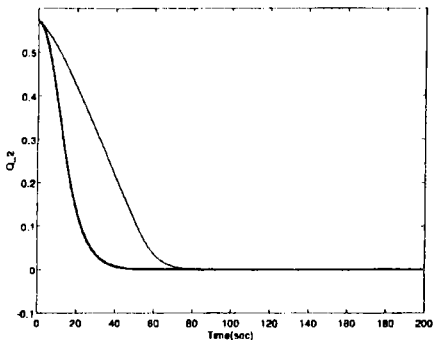


Figure 8. Time History of Quaternion Q2 (New method: solid, Wie's method: thick solid line)

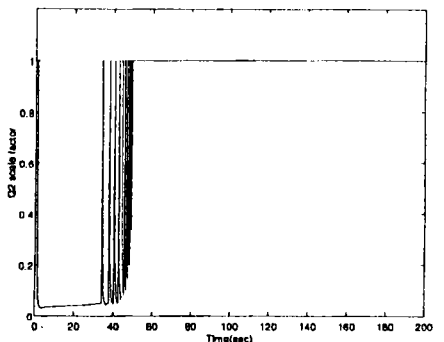


Figure 12. Time History of Scale Factor for Quaternion Q2

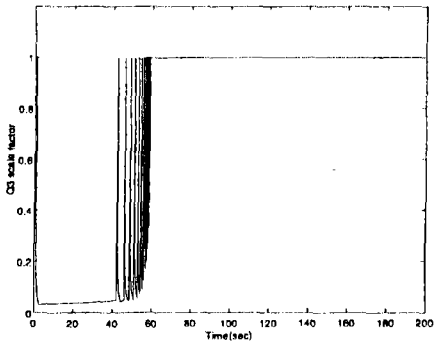


Figure 13. Time History of Scale Factor for Quaternion Q3