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A Design Method of Decentralized Control Systems by Sequential Loop Closing

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Abstract : This paper describes a design method of compensators for decentralized control systems. Decentralized control problem is convenient to design multivariable control systems and formulated as a series of independent designs. The proposed design method is composed of some steps, which is sequentially to close loop of the system diagonalized by regarding interactive subsystem as perturbation for current loop. So, on the basis of H_∞ control theory, decentralized controllers are designed considering robust stability for diagonal systems with perturbations. A numerical example shows that the proposed design method is effective for multivariable control systems.

1 Introduction

The goal of any controller design is that the overall system is stable and satisfies some minimum performance requirements. These requirements should be satisfied at least when the controller is applied to the nominal plant, that is, we require nominal stability and nominal performance. In addition, when decentralized controller is used, it is desiable that the system be failure tolerant. This means that the system should remain stable as individual loops are opened and closed.

A design method of multivariable control systems has been proposed by considering this system as decentralized control systems whose structure is block-diagonal. Some reasons for using decentralized controller are that tuning and retuning is easy and they are easy to make failure tolerant and so on.

The design of decentralized control system involves two steps. First step is sequential loop closing, second step is independent design of each loop. However, the method of first step depends strongly on which loop is designed first and how this controller is designed. Therefore, the method of first step has disadvantages which the design proceeds by "trial-and-error" since there are no guidelines on how to design the controller for each loop.

In this paper, we propose a design procedure which guarantees robust performance taking account of robustness for each diagonal subsystem by regarding off-diagonal subsystems of multivariable control systems as perturbations of each diagonal subsystem and setting bounds to these perturbations as an uncertain transfer function within an upper bound. Mainly for mathematical convenience, we choose to define performance using the H_∞-norm.

2 Preliminaries

Consider the following set of subsystems $\Sigma[G,C]$, as shown in Fig.1. C is decentralized compensator and block-diagonal matrix.[1]

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & & \\ G_{21} & G_{22} & & \\ & & \ddots & \\ & & & G_{NN} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \tag{1}$$

where

$$C = \text{block diag}\{C_1, C_2, \cdots, C_N\} \tag{2}$$

$$y_i \in R^{p_i}, \qquad u_i \in R^{q_i}$$

$$G_{ij} \in R^{p_i \times q_j}, \quad C_i \in R^{q_i \times p_i}$$

$$p = \sum_{i=1}^{N} p_i, \qquad q = \sum_{i=1}^{N} q_i$$

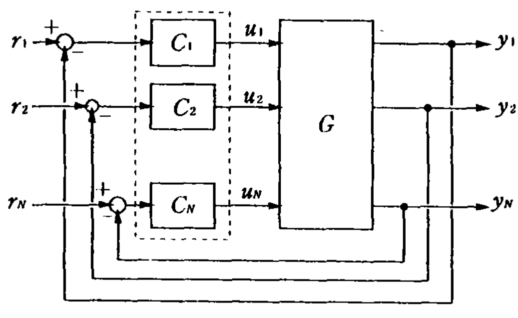

Fig.1 Decentralized control system $\Sigma[G,C]$

Generally G_{ij} is transfer function matrix with q_j inputs and p_i outputs and G cannot be square matrix, however, must be a set of transfer function matrices with $(N \times N)$ dimensions when decentralized control system is discussed. In this paper, so, in the case of $p \neq q$, the control object is rearranged to square matrix by partition of its matrix.

In general, nominal performance is guaranteed when the following condition is satisfied

$$\left\| \begin{matrix} W_1 S W_d \\ W_2 T \end{matrix} \right\|_\infty < 1 \tag{3}$$

$$S = (I + GC)^{-1} \tag{4}$$

$$T = GC(I + GC)^{-1} \tag{5}$$

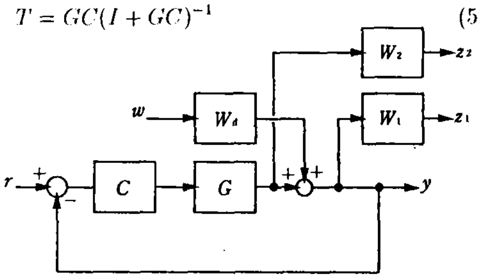

Fig.2 Mixed sensitivity problem

where, W_d is the input weight and often equal to disturbance model. W_1 and W_2 are the output weights and these are used to specify the frequency range over which the sensitivity function and complementary sensitivity function should be small and to weight each output according to its importance.

Skogestad and Morari have derived the following conditions for robust stability.[2]

(i) Assume $\tilde{T}$ is stable, and G and $\tilde{G}$ have the same number of RHP poles. Then T is stable (the system is stable when all loops are closed) if

$$\bar{\sigma}(\tilde{T}) \leq \mu_c^{-1}(E_T)\ ,\ \forall\omega\ ,\ E_T = (G - \tilde{G})\tilde{G}^{-1} \tag{6}$$

(ii) Assume $\tilde{S}$ is stable, and G and $\tilde{G}$ have the same number of RHP zeros. Then S is stable if

$$\bar{\sigma}(\tilde{S}) \leq \mu_c^{-1}(E_S)\ ,\ \forall\omega\ ,\ E_S = (G - \tilde{G})G^{-1} \tag{7}$$

where

$$\tilde{G} = \mathrm{diag}\{G_{11}, G_{22}, \cdots, G_{NN}\}$$

$$\tilde{T} = \tilde{G}C(I + \tilde{G}C)^{-1}$$

$$\tilde{S} = (I + \tilde{G}C)^{-1}$$

$\tilde{S}$ and $\tilde{T}$ are sensitivity function matrix and complementary sensitivity function matrix respectively. $\mu(\cdot)$ is the μ-interaction measure and μ is computed with respect to the structure of the decentralized controller C. $\bar{\sigma}(A)$ means the largest singular value of matrix A.

Although condition Eq.(5) and Eq.(6) are, what is called, "fine" condition for robust stability, they give difficulty in stabilization of decentralized control systems, and assume that uncertainties and interactions in G are neglected when each element in C is designed on the basis of the information in $\tilde{G}$ only.

3 Sequential Design Method

3.1 Sequential Loop Closing

In this paper, decentralized controllers are designed by effectively using robust stability condition, applying well-known small gain condition for each subsystem, considering uncertainties effected interactive subsystems, when each diagonal subsystem is sequentially closed.

Genarally the relation between u_k and y_k of k^{th} subsystem is as follows (see Fig.3).[3][4]

$$y_k = P_k u_k + d_k \tag{8}$$

$$P_k = G_{kk} + H_k \tag{9}$$

$$H_k = -G_{kj}C_k^*[I + G_{jj}C_k^*]^{-1}G_{jk} \tag{10}$$

$$d_k = D_k r_j,\ D_k = G_{kj}C_k^*[I + G_{jj}C_k^*]^{-1} \tag{11}$$

where,

$$C_k^* := \text{block diag}\{C_1, C_2, \cdots, C_{k-1}, C_{k+1}, \cdots, C_N\}$$

$$G_{kj} := [G_{k1}, G_{k2}, \cdots, G_{kk-1}, G_{kk+1}, \cdots, G_{kN}]$$

$$G_{jk} := [G_{1k}, G_{2k}, \cdots, G_{k-1k}, G_{k+1k}, \cdots, G_{Nk}]^{\mathrm{T}}$$

$$r_j := [r_1, r_2, \cdots, r_{k-1}, r_{k+1}, \cdots, r_N]^{\mathrm{T}}$$

$$j = 1, 2, \cdots, N,\ j \neq k$$

Hence, Eq.(1) is diagonalized as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} P_1 & 0 & & \\ 0 & P_2 & & \\ & & \ddots & \\ & & & P_N \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix} . \tag{12}$$

H_k implies additive perturbation for current subsystem G_{kk}, which is composed of transfer functions of closed subsystems. d_k implies disturbance to current output y_k, which is composed of reference signal $[r_j]^{\mathrm{T}}$ and transfer functions of closed subsystems.

As shown in Fig.3, Eq.(9),(10) may be denoted as a linear fractional transformation of C_k^*.

$$\begin{aligned} \begin{bmatrix} y_k \\ y_j \end{bmatrix} &= \begin{bmatrix} G_{kk} & G_{kj} \\ G_{jk} & G_{jj} \end{bmatrix} \begin{bmatrix} u_k \\ u_j \end{bmatrix} \\ u_j &= C_k^*(r_j - y_j) \end{aligned}$$

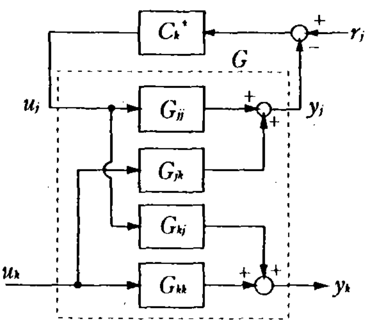

Fig.3 k^{th} subsystem

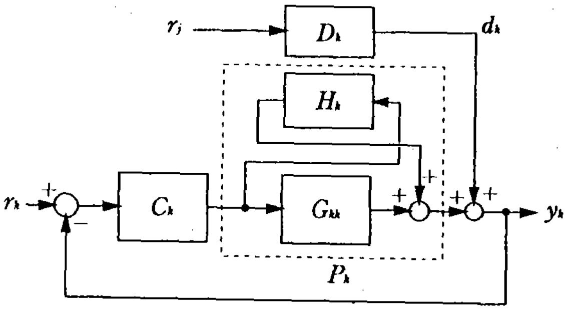

Fig.4 k^{th} subsystem with additive perturbation

3.2 Design Algorithm

When interactive loops are involved in each diagonal subsystems from Eq.(7)~(10), each diagonal subsystems come to have additive perturbations for its nominal systems. In this paper, we consider P_k is nominal transfer function matrix as shown in Fig.4 and propose a sequential design procedure of compensator matrix C_k for P_k introducing the concept of ***diagonal dominance***.[5]

A rational $n \times n$ matrix $Z(s)$ is said to be diagonal dominace if the following conditions are satisfied.

Let D be a large contour in the s-plane consisting of the imaginary axis from $-iR$ to $+iR$, together with a semicircle of radius R in the right half-plane. For each s on

$$D \begin{cases} \text{either } |z_{ii}(s)| - \sum_{j=1, j\neq i}^{n} |z_{ij}(s)| > 0, \quad i = 1, 2, \ldots, n \\ \text{or} \quad |z_{ii}(s)| - \sum_{j=1, j\neq i}^{n} |z_{ji}(s)| > 0, \quad i = 1, 2, \ldots, n \end{cases} \tag{13}$$

The former and latter are said to be diagonal row dominance and diagonal column dominance respectively. For berevity we shall talk simply of row dominance, column dominance and dominance. From the definition it follows that row dominance implies dominance, and column dominance implies dominance.

Let propose a sequential design procedure of C_i as follows. Compensator matrix C_i for subsystem P_i is generally full matrix with $(q_i \times p_i)$ dimensions. The design method considered in this paper uses two compensators $K_i(s)$ and $\tilde{C}_i(s)$ as shown in Fig.5. K_i is used for pseudo-diagonalization of P_i and $\tilde{C}_i$ is diagonal matrix, used for stabilization of each loop.

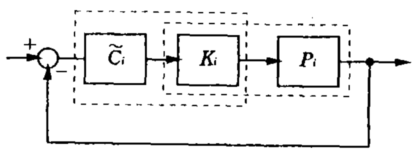

Fig.5 Pseudo-diagonalization problem

step1: For $i = 1$, select C_i^* suitably so that H_i becomes stable.

step2: Design K_i so that P_iK_i becomes pseudo-diagonal dominance. As shown in the following equations,

$$K_i = \begin{cases} K_i' + P_i^{-1}\Delta_i & \text{if } P_iK_i = P_iK_i' + \Delta_i \\ K_i' - P_i^{-1}\bar{\Delta}_i & \text{if } P_iK_i = P_iK_i' - \bar{\Delta}_i \end{cases} \tag{14}$$

$\Delta_i \in R^{p_i \times p_i}$: diagonal matrix
$\bar{\Delta}_i \in R^{p_i \times p_i}$: non-diagonal matrix

A diagonalizable compensator matrix K_i is designed so that P_iK_i satisfies Eq.(12) using a $(q_i \times p_i)$ matrix K_i' and a diagonal matrix Δ_i or a non-diagonal matrix $\bar{\Delta}_i$ anew.

stap3: Design main controller $\tilde{C}_i$ for each diagonal loop by means of frequency shaping method to satisfy

$$\left\| \begin{matrix} W_1 S_i D_i \\ W_2 T_i \end{matrix} \right\|_\infty < 1, \tag{15}$$

$$S_i = (I + P_iK_i\tilde{C}_i)^{-1}, \tag{16}$$

$$T_i = P_iK_i\tilde{C}_i(I + P_iK_i\tilde{C}_i)^{-1}. \tag{17}$$

step4: Let $i := i + 1$, repeat under **step1** until $i = N$.

Here, although the design loop to start may be arbitrary, it must be rounded as $k \to N \to 1, 2 \to (k-1)$.

Feature of sequential design method proposed above is to formulating as a series of independent deisgns throuth regarding interactive loops as a perturbation of current loop and pseudo-diagonalization of this system.

4 Numerical Example

An effectiveness of the proposed design method is shown by a numerical example. Consider the following set of subsystems with $N = 2$.

$$G(s) = \left[\begin{array}{cc|c} g_{11}(s) & g_{12}(s) & g_{13}(s) \\ g_{21}(s) & g_{22}(s) & g_{23}(s) \\ \hline g_{31}(s) & g_{32}(s) & g_{33}(s) \end{array} \right] = \left[\begin{array}{c|c} G_{11}(s) & G_{12}(s) \\ \hline G_{21}(s) & G_{22}(s) \end{array} \right]$$

$$C(s) = \text{block diag}\{C_1(s), C_2(s)\}$$

where,

$$G_{11}(s) = \begin{bmatrix} \dfrac{-(s^2+5s+6)}{(s^2+3s+3)(s+3)} & \dfrac{-(s^2+3s+8)}{(s^2+3s+3)(s+3)} \\ \dfrac{s^2+7s+8}{(s^2+3s+3)(s+3)} & \dfrac{2s^2+12s+13}{(s^2+3s+3)(s+3)} \end{bmatrix}$$

$$G_{12}(s) = \begin{bmatrix} \dfrac{s^2+6s+7}{(s^2+3s+3)(s+3)} \\ \dfrac{s^2+3s+2}{(s^2+3s+3)(s+3)} \end{bmatrix}$$

$$G_{21}(s) = \begin{bmatrix} \dfrac{s^2+2s+4}{(s^2+3s+3)(s+3)} & \dfrac{s^2+2s+5}{(s^2+3s+3)(s+3)} \end{bmatrix}$$

$$G_{22}(s) = \frac{-(s^2+2s+2)}{(s^2+3s+3)(s+3)}$$

step1: For $i = 1$, considering after design, C_i^* is designed

$$C_1^*(s) = -44.0\frac{1+0.45s}{s(1+0.06s)}$$

so as to be robust for a bounded interactive terms of G_{22}.
step2: In order that diagonal element of P_1K_1' has integral characteristics and P_1K_1 becomes diagonal dominance, K_i' and Δ_1 are selected as

$$K_1'(s) = \begin{bmatrix} \frac{4}{s} & 0.1 \\ 0.1 & \frac{4}{s} \end{bmatrix},\ \Delta_1(s) = \begin{bmatrix} \frac{20}{s(s+1)} & 0 \\ 0 & \frac{20}{s(s+1)} \end{bmatrix}.$$

step3: For diagonal element, when we select its frequency weighting functions as

$$W_1(s) = \mathrm{diag}\left\{\frac{5}{s^2+0.16s+0.02},\ \frac{5}{s^2+0.16s+0.02}\right\}$$

$$W_2(s) = \mathrm{diag}\{2\times 10^{-4}s^2,\ 2\times 10^{-4}s^2\},$$

a stabilizing compensator matrix satisfing Eq.(14) is obtained as

$$C_1(s) = (K_1' + P_1^{-1}\Delta_1)\begin{bmatrix} \frac{1+1.5s}{1+0.04s} & 0 \\ 0 & \frac{1+s}{1+0.03s} \end{bmatrix}.$$

step4: For $i = 2$, since it is found that the controller assuming at **step1** satisfies

$$\left\| \frac{C_1^*}{1+G_{22}C_1^*} H_2 \right\|_\infty < 1,$$

we set $C_2(s) = C_1^*(s)$ and design procedure are completed. Finaly, actual step responses using these compensators is shown in Fig.5.

5 Conclusions

In this paper, we proposed a design method for decentralized control systems considering robustness by regarding interactive subsystems of multivariable control systems as uncertainty of the diagonal subsystem sequentially. Moreover it was shown that even a simple sequential stabilizing method such as recursion of some steps is effective for multivariable control systems.

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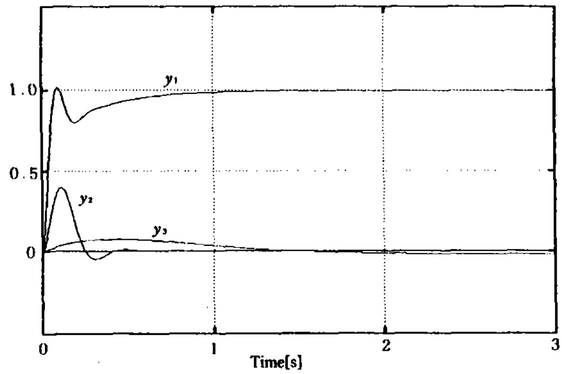
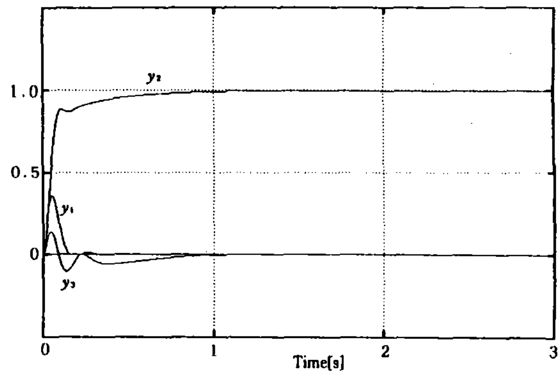
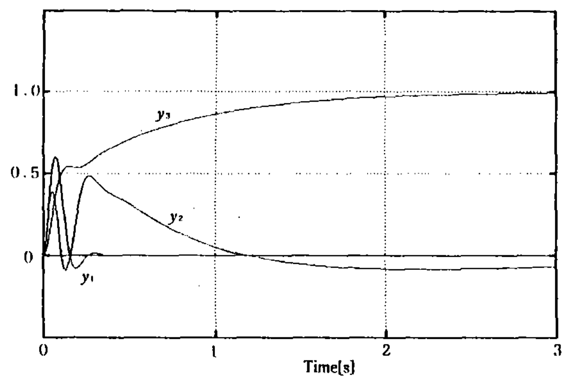


(a) Response for unit step change in r_1



(b) Response for unit step change in r_2



(c) Response for unit step change in r_3

Fig.5 Step responses of the closed-loop system