M-sequence and its Applications to Nonlinear System Identification

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Abstract This paper describes an outline of pseudorandom M-sequence and its applications to measurement and control engineering. At first, generation and properties of M-sequence is briefly described and then its applications to delay time measurement, information transmission by use of M-array, two dimensional positioning, fault detection of logical circuit, fault detection of RAM, linear and nonlinear system identification.

1 Introduction

An M-sequence is one of the pseudo-random sequences which is generated by a linear recurring rule and yet resembles a truly random sequence. It is a deterministic signal with a period, and its properties (autocorrelation function or power spectrum, etc.) over its period are completely determined in a finite time — yet it behaves as though it is a stochastic signal. An M-sequence is easily generated with a shift register with suitable feedback and is widely used in control engineering as a simulation of actual noise, as a transmitted signal for delay time measurement, as a test signal for system identification, as a modulation signal for communication and so on.
M-sequences (maximum-length sequences) are also known as maximum-length linear shift register sequences, shift register sequences, pseudo-random sequences, pseudo-random binary signals(PRBS), linear recurring sequences, chain codes, and pseudo-noise(PN) sequences, amongst other names.
This paper describes the generation and properties of M-sequence and its applications to measurement and control engineering with emphasis on nonlinear system identification.

2 Generation of M-sequences

M-sequences can be generated with an n-stage shift register circuit as shown in Fig.1. Each stage of the shift register contains 0 or 1, and each output is multiplied by a coefficient f_j (equal to 0 or 1) and added mod 2 and fed back. In the circuit, $\oplus$ denotes an exclusive OR circuit. The initial condition of the shift register can be taken arbitrarily except for all zero. When the feedback coefficients f_j are suitably chosen, the generated sequence $\{a_i\}$ has the maximum period $N = 2^n - 1$ and is called an M-sequence. The sequence $\{a_i\}$ is written as

$$a_{i+n} = \sum_{j=0}^{n-1} f_j a_{i+j} \qquad (\text{mod } 2) \qquad (1)$$

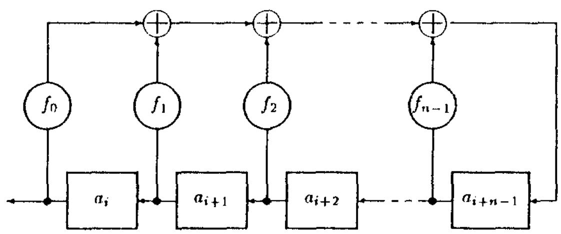

Figure 1: M-sequence generator

Letting $f_n = 1$, we have

$$\sum_{j=0}^{n} f_j a_{i+j} = 0 \qquad (\text{mod } 2) \qquad (2)$$

These expressions are called linear recurrence equations. When we introduce a delay operator x such as $a_{i+j} = x^j a_i$, Eqn.(2) becomes

$$(\sum_{j=0}^{n} f_j x^j) a_i = 0 \qquad (3)$$

Here, the polynomial

$$f(x) = \sum_{j=0}^{n} f_j x^j \qquad (f_0 \neq 0, f_n = 1) \qquad (4)$$

is called the characteristic polynomial. The coefficients f_j or the sequence $\{a_i\}$ are 0 or 1, and the multiplication and addition between them obey mod 2 arithmetic. Therefore, the coefficients f_j or sequence $\{a_i\}$ are considered to belong to Galoir Field GF(2). The necessary and sufficient condition that the sequence $\{a_i\}$ is an M-sequence is that the characteristic polynomial $f(x)$ is a primitive polynomial over GF(2). A primitive polynomial is a polynomial which divides $x^k - 1$ when $k = 2^n - 1$, but dose not divide $x^k - 1$ when $0 < k < 2^n - 1$. Primitive polynomials over GF(2) are found in Peterson (1961) and Zierler and Brillhart (1968).
When the sequence $\{a_i\}$ is represented by a generating function

$$g(x) = \sum_{i=0}^{\infty} a_i x^i \qquad (5)$$

then $g(x)$ is obtained by carrying out the following long division:

$$g(x) = \frac{h(x)}{f^*(x)} \qquad (6)$$

where $f^*(x)$ is the reciprocal polynomial of the characteristic polynomial:

$$f^*(x) = \sum_{j=0}^{n} f_j x^{n-j} \tag{7}$$

The dividend $h(x) = \sum_{j=0}^{n-1} h_j x^j$ is determined from the initial condition of the shift register as follows:

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \cdots \\ h_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ f_{n-1} & 1 & 0 & 0 & 0 \\ f_{n-2} & f_{n-1} & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_1 & f_2 & \cdots & f_{n-1} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \cdots \\ a_{n-1} \end{bmatrix} \tag{8}$$

An M-sequence is also described by a transition matrix expression. Let $U(t)$ be an n-vector representing the state of the shift register at time t:

$$U(t) = [a_1(t), a_2(t), \cdots, a_n(t)]^t \tag{9}$$

Then the state vector $U(t+1)$ at time $t+1$ (after one clock pulse) is given by

$$U(t+1) = TU(t) \tag{10}$$

where

$$T = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 1 \\ f_0 & f_1 & f_2 & \cdots & f_{n-1} \end{bmatrix} \tag{11}$$

This matrix T is called a transition matrix and has the following properties:

$$\begin{aligned} T^N &= I \\ |T - xI| &= f(x) \end{aligned} \tag{12}$$

where I represents the $n \times n$ unit matrix.
If T^k is calculated beforehand and the corresponding circuit is used, it is possible to generate the M-sequence as a series of nonoverlapping k-bit words, each k-bit word being generated in a clock pulse period. This leads to the faster generation of the M-sequence by a factor of k.

3 Properties of M-sequences

Several properties of an nth-order M-sequence with characteristic polynomial $f(x)$ are described below.

(a) In a period $2^n - 1$, there are $2^{n-1} - 1$ zeros and 2^{n-1} ones. That is, the number of ones and zeros in a period is nearly equal.

(b) Every possible n-tuple except all zero appears exactly once in a period.

(c) When a k-shifted version of $\{a_i\}$ is denoted by $\{a_{i+k}\}$, there exists a unique j (mod N) such that

$$\{a_i + a_{i+k}\} = \{a_{i+j}\} \tag{13}$$

This property is called the shift and add property of M-sequences. In general, there exists a unique v such that

$$s_1 a_{i-1} + s_2 a_{i-2} + \cdots + s_n a_{i-n} = a_{i+v} \tag{14}$$

where $s_1, s_2, \cdots, s_n \in \mathrm{GF}(2)$.

(d) When $\{a_i\}(a_i = 0 \text{ or } 1)$ is converted to $\{m_i\}$ ($m_i = 1$ or -1) by $m_i = 1 - 2a_i$, the autocorrelation function $\phi_{mm}(k)$ of the sequence $\{m_i\}$ is given by

$$\begin{aligned} \phi_{mm}(k) &= \frac{1}{N}\sum_{i=0}^{N-1} m_i m_{i+k} \\ &= \begin{cases} 1 & \text{for } k = 0, N, 2N, \cdots \\ -\frac{1}{N} & \text{otherwise} \end{cases} \end{aligned} \tag{15}$$

When N is large, $\phi_{mm}(k)$ is approximately equal to a delta function, so the M-sequence becomes an almost white random signal.

(e) If an M-sequence $\{a_i\}$ is sampled every q digits, the resulting sequence $\{a_{qi}\}$ is again an M-sequence of the same order, if and only if $(q, N) = 1$. The characteristic polynomial of $\{a_{qi}\}$ is the minimal polynomial corresponding to α^q, where α is a root of $f(x)$. if $(q_1, N) = 1$ and $(q_2, N) = 1$, then $\{a_{q_1 i}\} = \{a_{q_2 i}\}$ (except for a phase shift) if and only if q_1 and q_2 belong to the same cyclotomic coset. This leads to the statement that $\{a_{qi}\}$ equals $\{a_i\}$ except for a phase shift when $q = 1, 2, 4, \cdots, 2^{n-1}$. An example of cyclotomic coset is given in the case of $N = 31$:

C_1	:	1	2	4	8	16
C_3	:	3	6	12	24	17
C_5	:	5	10	20	9	18
C_7	:	7	14	28	25	19
C_{11}	:	11	22	13	26	21
C_{15}	:	15	30	29	27	23

(f) In every M-sequence there exists a unique phase such that

$$\{a_{2i}\} = \{a_i\} \tag{16}$$

The M-sequence starting from such a phase is called the characteristic M-sequence. The characteristic M-sequence can be obtained from Eqn.(6) by letting

$$h(x) = \begin{cases} f_e^*(x) & \text{for } n \text{ odd} \\ f_o^*(x) & \text{for } n \text{ even} \end{cases} \tag{17}$$

where $f_e^*(x)$ consists of even power terms of $f^*(x)$ and $f_o^*(x)$ consists of odd power terms (Gold 1966).

(g) Since an M-sequence $\{a_i\}$ is a periodic sequence, it can be expanded in a Fourier series. The Fourier coefficients are given by

$$c_k = \frac{1}{N}\sum_{j=0}^{N-1} a_j \exp(2\pi i k j/N) \quad (18)$$

The magnitude of c_k is constant except when k is a multiple of N:

$$c_k = \begin{cases} \dfrac{N+1}{2N} & \text{for } k = N, 2N, \cdots \\ \dfrac{(N+1)^{1/2}}{2N} & \text{otherwise} \end{cases} \quad (19)$$

When $\{m_i\}$ is used instead of $\{a_i\}$, c_k is given by

$$c_k = \begin{cases} \dfrac{1}{N} & \text{for } k = N, 2N, \cdots \\ \dfrac{(N+1)^{1/2}}{N} & \text{otherwise} \end{cases} \quad (20)$$

If a characteristic M-sequence is used as $\{a_i\}$, then $c_{k1} = c_{k2}$ when k_1 and k_2 belong to the same cyclotomic coset.

4 Applications

4.1 Delay time measurement

When we would like to measure a delay time of a process, such as transportation lag in fluid flow system, the M-sequence is well applied. Suppose the delay time to be measured be τ_0, then the output $y(t)$ of the process is,

$$y(t) = u(t - \tau_0) \quad (21)$$

where $u(t)$ is the input to the process. Taking the crosscorrelation between $u(t)$ and $y(t)$, we have,

$$\begin{aligned} \phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \overline{u(t-\tau)u(t-\tau_0)} = \phi_{uu}(\tau - \tau_0) \end{aligned} \quad (22)$$

Since the autocorrelation function becomes maximum when the argument is zero, we can obtain the delay time τ_0 from the maximum point of $\phi_{uy}(\tau)$. In order for $\phi_{uy}(\tau)$ to have a sharp peak, the input is better to have a sharp autocorrelation function; thus the M-sequence is very suited for this purpose.

Continuous-wave radar system uses M-sequence for modulating the transmitted signal to take the crosscorrelation with the received signal, measuring the time delay between the transmitted and the received one(Craig 1965).

The flow rate measuring system by use of M-sequence and a mechanical differential gear is reported in (Isobe *et al* 1966).

4.2 Random number generation

M-sequences are easily used for generating uniformly distributed random signals. Since every possible n-tuple except all zero appears exactly once in a period, the content of the shift register, when considered as a binary number, takes every integer between 1 and $2^n - 1$, and thus becomes a uniformly distributed number. Lewis and Payne (1973) have given a method of generating a uniform random number which is suitable for computer generation. When uniform random numbers are added together, the sum approaches a Gaussian random number as the number of additions increases. In this way, the uniform random number generated from the M-sequence is used for generation of Gaussian random numbers. But, in this case, it should be noted that when the number of addition is increased mutually dependent numbers arising from recurrence relationships increase, causing a skewed amplitude distribution. Therefore, some care should be taken in order to reduce the effect of recurrence relationships. One way of doing this is to choose the characteristic polynomial carefully (Kashiwagi,1978).

4.3 M-array

A two-dimensional pseudo-random array can be constructed by use of an M-sequence (MacWilliams and Sloane 1976). Consider the case where the period of an M-sequence $N = 2^n - 1$ is written as

$$N = N_1 \cdot N_2, \ \ N_1 = 2^k - 1, \ \ N_2 = N/N_1, \ \ N = 2^{k_1 k_2} - 1 \quad (23)$$

For example, $N = 2^4 - 1 = 3 \times 5 (k_1 = k_2 = 2)$. In this case, a pseudo-random array, or M-array, is constructed as follows:

$$\begin{bmatrix} a_0 & a_6 & a_{12} & a_3 & a_9 \\ a_{10} & a_1 & a_7 & a_{13} & a_4 \\ a_5 & a_{11} & a_2 & a_8 & a_{14} \end{bmatrix} \quad (24)$$

This array has almost the same properties (in a two dimensional way) as the M-sequence.

The two-dimensional autocorrelation function $\rho(i,j)$ is defined as

$$\rho(i,j) = \frac{A - D}{N} \quad (25)$$

where A is the number of places where the elements of the original array and those of the shifted array (shifted i rows below and j columns right) agree, and D is the number of places where they disagree. Then

$$\begin{aligned} \rho(0,0) &= 1 \\ \rho(i,j) &= -\frac{1}{N}, \ 0 \le i < N_1, \ 0 \le j < N_2, \\ & \qquad (i,j) \ne (0,0) \end{aligned} \quad (26)$$

Since a_i obeys a recurrence relationship, the element of M array also satisfies a recurrence vertically and horizontally. If a $k_1 \times k_2$ window is slid over the M-array, each of the $2^{kk} - 1$ nonzero binary $k_1 \times k_2$ arrays is seen exactly once. If the array and its shifted version are added, then the sum is also a shifted version of the original M-array(the shift and add property).

Hadamard matrix is constructed with an M-sequence as follows: Arrange the M-sequence $\{m_i\}$ (=1 or -1) of $2^n - 1$ period in some row; all other rows are made by cyclically shifting the original row. Then add an all 1 row and an all 1 column. The resulting array is a $2^n \times 2^n$ Hadamard matrix.

4.4 Information Transmission

An M-sequence is well applied to spread spectrum communication systems. The signal to be transmitted is modulated with an M-sequence in one of four ways: (a) direct sequence modulated, (b) frequency hopping, (c) pulse-FM or chirp, and (d) time hopping. Then the signal's bandwidth becomes spread and several advantages arise: selective addressing, code-division multiple access, low-density output signals, inherent message security, high-resolution ranging, or interference rejection. Details are given by Dixon (1976). When M-array is used as a masking pattern of a picture to be transmitted, the information transmission can be done in a secret way. Those who know about the masking M-array can only take out the transmitted information. Two dimensional correlation technique is effectively used in these systems.

4.5 Two Dimensional Positioning

A two-dimensional (2D) positioning technique is one of the most important technique in industrial processes such as automatic build-up system, insertion process of capacitors or registers on to a board and so on. 2D positioning system by use of M-array is reported in (Kashiwagi 1988). In this system, the property of M-array is used in that the 2D autocorrelation of M-array has a very sharp peak at its origin. However, since it sometimes occurs the case where the autocorrelation of M-array is too sharp for servo-controlling the two-dimensional positioning, the system was improved by use of autocorrelation of vague M-array which was observed with a TV camera in out-of-focus condition (Kashiwagi 1989a). In addition to the robustness of this system to noise, the system is also robust to misalignment of the TV camera to some extent.

4.6 Fault Detection of Logical Circuit

M-sequence is also applied to detect faults in a logical circuit. A new method of fault diagnosis of logical circuit is reported in (Kashiwagi 1987b), applying a pseudo-random M-sequence to the circuit under test, calculating the crosscorrelation function between the input and the output, and comparing the crosscorrelation functions with the references. This method is called M-SEquence Correlation (MSEC) method, and it has a very small probability that we overlook any faults in the circuit.

4.7 Fault Detection of RAM

M-sequence can be used to generate test patterns for static pattern-sensitive faults in Random Access Memories(RAM). Pattern-sensitive fault in RAM is the fault in which the content of a memory-bit changes due to the influence of surrounding memory values. In order to test whether a RAM has pattern-sensitive faults, it is necessary to generate various patterns in the surrounding memory-bits for all memory-bits. Rao and Kashiwagi(1989) showed that when we use 4th or 5th order M-sequence and write in RAM these sequence in a specified manner, the checking of the pattern-sensitive fault is quite easily and effectively done. This testing schemes use simple hardware to generate test patterns, so this method is easy to implement and suitable not only for on-line testing but also built-in testing of RAM.

4.8 Linear System Identification

Let the impulse response of the system to be identified be $g(t)$. The input and output of the system are denoted as $x(t)$ and $y(t)$, respectively. Then,

$$\phi_{xy}(t) = \int_0^\infty g(\tau)\phi_{xx}(t-\tau)d\tau \tag{27}$$

where $\phi_{xy}(\cdot)$ denotes the crosscorrelation function between $x(t)$ and $y(t)$. If an M-sequence is used as an input $x(t)$, then from Eqn.(15), $\phi_{xx}(t)$ is almost approximated by a delta function. Therefore, Eqn.(27) becomes

$$\phi_{xy}(t) \simeq g(t) \tag{28}$$

Thus, the crosscorrelation function between the input and the output directly determines the impulse response.
Sato(1964) reported a precise method for obtaining the impulse response of a linear system by making use of M-sequence. A method of least-squares estimation of $g(t)$ using an M-sequence as an input is also reported by Clarke and Briggs (1970). Impulse response determination by use of derivatives of correlation function is shown by Kashiwagi (1971). A method for obtaining impulse response of linear system by use of a weighted M-sequence signal is reported by Kashiwagi(1974). A measurement method of frequency response function by use of characteristic M-sequence is reported by Sakata and Kashiwagi(1993).

4.9 Nonlinear System Identification

The identification methods for linear system have been developed by many researchers, but the methods for nonlinear system identification are quite few. The reason is that nonlinear systems are complex and difficult to be treated in general.
Barker *et al* (1972) proposed the use of pseudorandom signals, especially antisymmetric M-sequence, for obtaining 2nd-order Volterra kernels with restricted conditions . The authors(Kashiwagi *et al*, 1993a,1993b, 1994) proposed a new method for obtaining not only the linear impulse response, but also Volterra kernels of nonlinear system simultaneously. A pseudorandom M-sequence, specially chosen beforehand, is applied to the nonlinear system, and the crosscorrelation function between the input and the output is calculated. Then the linear impulse response together with several crosssections of the Volterra kernels are obtained. This method is described below.
A nonlinear dynamical system is, in general, described as follows.

$$y(t) = \sum_{i=1}^{\infty} \int_0^\infty \int_0^\infty \cdots \int_0^\infty g_i(\tau_1, \tau_2, \cdots \tau_i)$$

$$\times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2\cdots d\tau_i \qquad (29)$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1, \tau_2, ...)$ is called Volterra kernel of i-th order.
When we take the crosscorrelation function between the input $u(t)$ and the output $y(t)$, we have,

$$\begin{aligned}\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty}\int_0^\infty\int_0^\infty\cdots\int_0^\infty g_i(\tau_1,\tau_2,\cdots\tau_i) \\ &\times \overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)}d\tau_1 d\tau_2\cdots d\tau_i \qquad (30)\end{aligned}$$

where $\phi_{uy}(\tau)$ is the crosscorrelation function of $u(t)$ and $y(t)$ and $\overline{\quad\quad}$ denotes time average.
The difficulty of obtaining $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from $\phi_{uy}(\tau)$ is , in general, due to the difficulty of getting $(i+1)$th moment of the input $u(t)$, because the n-th moment of the signal is very difficult to obtain for actual signals.
When we use an M-sequence as an input to the system, the n-th moment of $u(t)$ can be easily obtained by use of so-called "shift and add property" of the M-sequence. So we can obtain the Volterra kernels $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from simply measuring the crosscorrelation function between the input and output of the nonlinear system.
The $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\begin{aligned}&\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} \\ &= \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)}\end{cases} \qquad (31)\end{aligned}$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 10, $1/N$ is smaller than 10^{-3}. So Eqn.(31) can be approximated as a set of impulses which appear at certain τ's.
Eqn.(31) is due to the so-called shift and add property of the M-sequence; that is, for any integer $k_{i1}^{(j)}, k_{i2}^{(j)}, \cdots$ $k_{i,i-1}^{(j)}$ (suppose $k_{i1}^{(j)} < k_{i2}^{(j)} < \cdots, k_{ii}(j)$), there exists a unique $k_{ii}^{(j)}(modN)$ such that

$$u(t)u(t+k_{i1}^{(j)})u(t+k_{i2}^{(j)})\cdots u(t+k_{i,i-1}^{(j)}) = u(t+k_{ii}^{(j)}) \quad (32)$$

where j is the number of a group $(k_{i1}, k_{i2}, \cdots, k_{i,i-1})$ for which Eqn.(32) holds. We assume that total number of those groups is m_i(that is, $j = 1, 2, \cdots, m_i$). Note that when $k_{ir}^{(j)}(r = 1, 2, ...i)$ satisfy Eqn.(32), then $2^r k_{ir}^{(j)}$ also satisfy Eqn.(32) for any integer p. Therefore Eqn.(31) becomes unity when

$$\tau_1 = \tau - k_{i1}^{(j)}, \tau_2 = \tau - k_{i2}^{(j)}, \cdots, \tau_i = \tau - k_{ii}^{(j)} \qquad (33)$$

Therefore Eqn.(30) becomes

$$\begin{aligned}&\phi_{uy}(\tau) = \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}) \\ &\qquad (34)\end{aligned}$$

Here the function $F(\tau)$ is the sum of the odd order Volterra kernels when some of its argument are equal. Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)})$ in Eqn.(34) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$. If the $k_{ii}^{(j)}$ of i-th Volterra kernel g_i are sufficiently apart from each other(say, more than $50\Delta t$, where Δt is the time increment of the measurement time), we can obtain each Volterra kernel $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)})$ from Eqn.(34). Volterra kernels $g_i(\tau_1, \tau_2, \cdots \tau_i)$ are obtained as a set of crosssections along 45 degree lines in $(\tau_1, \tau_2, \cdots, \tau_i)$ space. In order for this to be realized, we have to select M-sequence(As for the selection of M-sequence suitable for obtaining Volterra kernels, see Table 1 of the reference(Kashiwagi,1994a).
An example of obtaining Volterra kernels by this method is shown here.

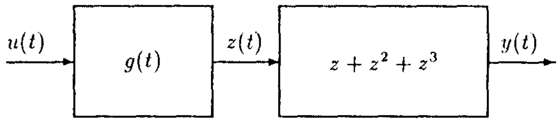

Figure 2: A nonlinear system having up to 3rd Volterra kernels.

The system to be identified is assumed to have up to third Volterra kernel which is actually realized as shown in Fig.2, where $g(t)$ is the impulse response of the linear part of the system. Then the output $y(t)$ can be written as

$$\begin{aligned}y(t) &= z(t) + z^2(t) + z^3(t) \\ &= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 \\ &+ \int_0^\infty\int_0^\infty g(\tau_1)g(\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \int_0^\infty\int_0^\infty\int_0^\infty g(\tau_1)g(\tau_2)g(\tau_3) \\ &\times u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)\,d\tau_1\,d\tau_2 d\tau_3 \qquad (35)\end{aligned}$$

Therefore Volterra kernels are as follows in this case.

$$\begin{aligned}g_1(\tau_1) &= g(\tau_1) \\ g_2(\tau_1,\tau_2) &= g(\tau_1)g(\tau_2) \\ g_3(\tau_1,\tau_2,\tau_3) &= g(\tau_1)g(\tau_2)g(\tau_3) \qquad (36)\end{aligned}$$

When we take the crosscorrelation function between $u(t)$ and $y(t)$, we have

$$\begin{aligned}\phi_{uy}(\tau) &= \int_0^\infty g_1(\tau_1)\overline{u(t-\tau)u(t-\tau_1)}d\tau_1 \\ &+ \int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)}d\tau_1 d\tau_2 \\ &+ \int_0^\infty\int_0^\infty\int_0^\infty g_3(\tau_1,\tau_2,\tau_3) \\ &\times \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)}d\tau_1 d\tau_2 d\tau_3 \quad (37)\end{aligned}$$

When we use the M-sequence having the characteristic polynomial of $f(x) = 424505$(in octal notation, 17 degree), k_{ij}'s in Eqn.(32) are

$$k_{21} = 196, k_{22} = 199, k_{31} = 227, k_{32} = 230, k_{33} = 231$$

Therefore

$$\phi_{uy}(\tau) = \Delta t g(\tau) + F(\tau) + 2(\Delta t)^2 g_2(\tau - 196, \tau - 199) + 6(\Delta t)^3 g_3(\tau - 227, \tau - 230, \tau - 231) + \cdots \quad (38)$$

Computer simulations show that Volterra kernels are well obtained by this method(Kashiwagi 1993a).

This method of Volterra kernel identification is applied to several actual nonlinear systems as shown in Fig.3. The actual output of the system and the estimated output calculated by use of the measured Volterra kernel are compared. The results show a good agreement between them, showing the validity of the method for nonlinear identification.

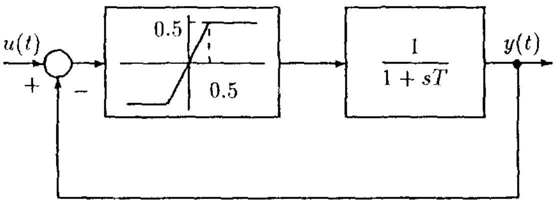

Figure 3: A nonlinear system having feedback loop

5 Conclusion

M-sequence is one of the pseudorandom sequences and widely used in the measurement and control engineering. Generation and properties of M-sequence is briefly explained, and its applications to delay time measurement, random number generation, information transmission, two-dimensional positioning, fault detection of logical circuit/ RAM, linear and nonlinear system identification are described. More information about M-sequence would be obtained from the references listed below.

6 References

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