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# 직접구동용 브러쉬없는 직류 전동기를 위한 적분 가변 구조 제어기

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# A INTEGRAL VARIABLE STRUCTURE CONTROLLER FOR BLDDSM WITH PRESCRIBED OPTIMAL OUTPUT DYNAMICS

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**Abstract**

A new integral variable structure system without the reaching phase problems is presented for the prescribed control of the BLDDSM under load variation and parameter uncertainties. The control technique can yields the complete robustness of initially prescribed output dynamics in the sliding surface against the modeling errors. The comparative simulation and experiment studies of the BLDDSM position control are carried out in comparison with two previous algorithms.

## I. Introduction

A direct drive servo motor can provide the very high torque at low to moderate speeds by directly coupling the load to the motor shaft without gears, belts, or any form of the mechanical leverage[13]. Since the load variations and external disturbances directly influence on a servo system, there are some unexpected gains and trade-off, besides the obvious advantages of eliminating the transmission. The inevitable modeling errors resulted from the linearization and parameter uncertainties including load variations are the harmful factors for the control of a brushless direct drive servo motor(BLDDSM).

As a precise and robust algorithm being different from the well known PI type controller, the variable structure system with sliding mode control(SMC) is considered for a brushless servo motor(BLSM)[4-6] or a BLDDSM[7-9]. Unfortunately, there exists an the problems of the reaching phase in the most conventional SMC's for driving of a BLDDSM. The reaching phase decrease the robustness of the algorithm since the system may be sensitive to the parameter variations and disturbances during reaching phase[3]. Moreover, it is difficult to predict the whole output dynamics entirely.

In this paper, a integral variable structure system without the problems of the reaching phase is suggested for the prescribed optimal output control of a BLDDSM subjected the load variation. For the design of this surface, the optimal regulator technique is introduced, which implies the prescription of the output performance in this work. The stability of the position control algorithm is investigated under the modeling errors. The performances of the proposed controller is verified through the simulation and experiment studies on the position control of a BLDDSM in comparison with those of the previous two algorithms.

## II. Hardware Sysem Description

The implemented total hardware configuration for the experiments is shown in Fig. 1, which consists of the BLDDSM, its current controller using VSI, a resolver and reslver-to-digital converter, 486 PC with interface card. The resolver and resolver-to-digital converter is used for detection of the angular position and speed of the rotor with 14 bit resolution per rotation and 12 bit A/D conversion,

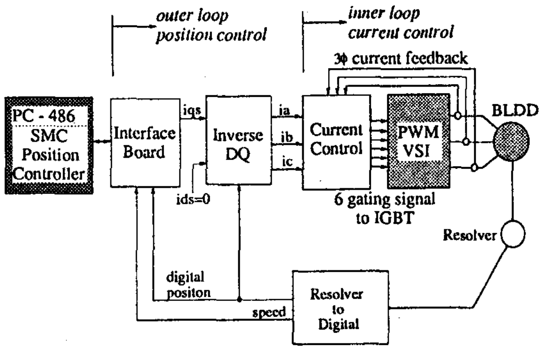


Fig. 1 Total implemented hardware configuration

respectively. The VSS control algorithm embedded in the 486 PC generates the q-axis current command for positioning the BLDDSM to the reference command. Through D/A conversion, this current command is given to the current controller. The current controller of CRPWM VSI using IGBT switching device with 3.4 [kHz] ramp comparison regulates the real motor currents to its command. Eventually, the position of the motor is to be controlled as commanded.

## III. Variable Structure System with Prescribed Optimal Output Dynamics

### A. Modeling of BLDDSM

A vector-controlled linearized BLDDSM can be expressed as follows[7-9]:

$$J \cdot \ddot{\theta}(t) + D \cdot \dot{\theta}(t) + T_L(t,\theta(t)) = \frac{3}{2} p k_t \cdot i_{qs}(t) \qquad (1)$$

| | |
|---|---|
| J | : total moment of inertia |
| D | : coefficient of viscous friction term |
| $p$ | : number of poles |
| $k_t$ | : torque constant |
| $\theta$ | : angle displacement |
| $\dot{\theta}$ | : angular velocity of rotor |
| $i_{qs}$ | : q-axis current |
| $T_L(t,\theta(t))$ | : load variations. |

It is assumed that the system parameters $J$, $D$, and $k_t$ in (1) are bounded as $J \in [J_{min} \;\; J_{max}]$, $D \in [D_{min} \;\; D_{max}]$, and $k_t \in [k_{t_{min}} \;\; k_{t_{max}}]$. Let $X^o \in [X_{min} \;\; X_{max}]$, $X = J, D$, and $k_t$ denotes each estimated nominal parameter. For a SMC, let us define the state vector $X(t) \in \Re^2$ with respect to the desired position command, $\theta_d$ as

$$X(t) \equiv [X_1(t) \quad X_2(t)]^T \qquad (2)$$

where $X_1(t)$ and $X_2(t) \in \Re$ are expressed as

$$X_1(t) \equiv e_1(t) = \theta_d - \theta(t) \qquad (2a)$$

$$X_2(t) \equiv e_2(t) = -\dot\theta(t). \qquad (2b)$$

Finally, the error state equation of a BLDDSM is expressed as

$$\dot X(t)=\begin{bmatrix}0 & 1\\ 0 & -D/J\end{bmatrix}\cdot X(t)+\begin{bmatrix}0\\ 1/J\end{bmatrix}\cdot T_L-\begin{bmatrix}0\\ 3Pk_t/2J\end{bmatrix}\cdot i_{qs}(t) \qquad (3)$$

with an initial condition, $X_1(0)=\theta_d-\theta(0)$ and $X_2(0)=-\dot\theta(0)$. The main objective of the controller design for (3) is to drive the BLDDSM to the command position exactly with prescribed optimal output dynamics even in the presence of the parameter mismatches. To achieve this design goal, the reaching phase must be removed because the output during the reaching phase can not be predetermined by the designer.

**B. Integral-Augmented Sliding Surface and Its Optimal Design**

The conventional switching function is augmented by an integral action with a nonzero initial value in this paper as follows:

$$s(t) \equiv X_2(t)+C_1\cdot X_1(t)+C_0\cdot X_0 \qquad (=0) \qquad (4)$$

$$X_0 \equiv \int_0^t X_1(\tau)d\tau + X_0^0, \quad X_0^0=-(X_2(0)+C_1\cdot X_1(0))/C_0. \qquad (4a)$$

The surface of (4) is obviously defined from any given initial condition in phase plane since (4) always clearly satisfies $s(0)=0$ for any $X^0$ at $t=0$. This point is not considered in [13]. Therefore, the controlled system can slide from the beginning without any reaching phase with (4). From $\dot s(t)=0$, the ideal equivalent control of (4) becomes

$$i_{qs}(t)=\frac{2C_0J}{3Pk_t}\cdot X_1+\left(C_1-\frac{D}{J}\right)\cdot\frac{2J}{3Pk_t}\cdot X_2+\frac{2J}{3Pk_t}T_L(t,\theta(t)). \qquad (5)$$

and the sliding mode dynamics is ideally described as

$$\dot X_1 = X_2 \qquad X_1(0)$$
$$\dot X_2=-C_0\cdot X_1-C_1\cdot X_2, \quad X_2(0) \qquad (6)$$

or in matrix form

$$\dot X = A_c\cdot X, \quad A_c=\begin{bmatrix}0 & 1\\ -C_0 & -C_1\end{bmatrix} \qquad (7)$$

which is the dynamical interpretation of (4) from a given $X(0)$ to origin. And by using the nominal parameters, it can be re-expressed as

$$\dot X(t)=\begin{bmatrix}0 & 1\\ 0 & -D^0/J^0\end{bmatrix}\cdot X(t)-\begin{bmatrix}0\\ 3pk_t^0/2J^0\end{bmatrix}\cdot i_{qs}(t), \quad X(0) \qquad (8)$$

where

$$i_{qs}(t)=K\cdot X \qquad (9a)$$

$$K=[2C_0J^0/3Pk_t^0 \qquad (C_1-D^0/J^0)\cdot 2J^0/3Pk_t^0]. \qquad (9a)$$

Now, the desired coefficient of (4) can be selected straightforwardly using the any pole-assignment to the system (8). The linear quadratic optimal technique is employed in this paper to determine the gain matrix $K$ in (9a). The quadratic performance index $I$ is chosen as

$$I=\int_0^\infty (X^TQX+ri_{qs}^2)dt \qquad (10)$$

where $Q^T=Q>0$ and $r>0$ are the weighting matrix for the state and the scalar for the control, respectively. The weighting matrix $Q$ can be selected as $Q=E^TE$ where $E\in\Re^{1\times2}$ and the pair $(A,E)$ is observable. Then, the optimal gain matrix $K$ to minimize the performance index, $I$ in (10) is given by

$$K_{op}=[K_{op1}K_{op2}]=-1/r\cdot[0 \quad 3Pk_t^0/2J^0]\cdot P \qquad (11)$$

where $P$ is the solution of the matrix Riccati equation:

$$\begin{bmatrix}0 & 1\\ 0 & -\bar D/\bar J\end{bmatrix}\cdot P+P\cdot\begin{bmatrix}0 & -\bar D/\bar J\\ 1 & 0\end{bmatrix}+Q-1/r\cdot P\cdot\begin{bmatrix}0\\ 3\bar p\bar k_t \;/\; 2\bar J\end{bmatrix}\cdot[0 \quad 3p\bar k_t/2\bar J]\cdot P=0. \qquad (12)$$

Then, the optimal coefficient of the sliding surface can be determined as

$$C_0=3pk_t^0\cdot K_{op1}/2J^0, \quad C_1=D^0/J^0+3pk_t^0\cdot K_{op2}/2J^0 \qquad (13)$$

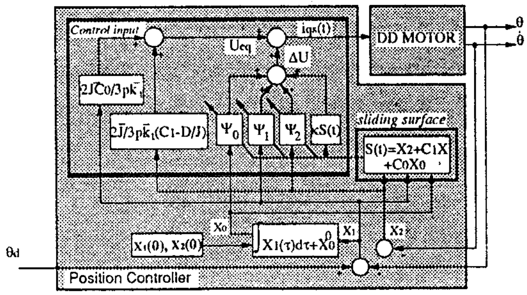


Fig. 2 Proposed algorithm

Since (8) is the nominal system of (3), this design of the sliding surface implies the optimal performance design to the nominal system.

**C. Design of Control Input and Stability Analysis**

As the second design stage of the VSS controller, the following type of the control input for the $q$-axis current of the BLDDSM is considered as

$$i_{qs}(t)=U_{eq}(t)+\Delta U(t) \qquad (14)$$

where $U_{eq}(t)$ is called the available equivalent control of (4) instead of (5) as

$$U_{eq}(t) = 2C_0J^0/3pk_t^0\cdot X_1+(C_1-D^0/J^0)\cdot 2J^0/3pk_t^0\cdot X_2$$
$$= K_{op}\cdot X \qquad (14a)$$

which is directly determined according to the design of the sliding surface. Also, the discontinuous control term to maintain the system on the surface by nullifying the effect of the uncertainties is expressed as

$$\Delta U(t)=\Psi_0\cdot X_0(t)+\Psi_1\cdot X_1+\Psi_2\cdot X_2+\Psi_3+\kappa\cdot s(t) \qquad (14b)$$

where $\kappa>0$ and

$$\Psi_0=\begin{cases}\alpha_0\ge 0 & if \quad s(t)\cdot X_0(t)>0\\ \beta_0\le 0 & if \quad s(t)\cdot X_0(t)<0\end{cases}$$

$$\Psi_1=\begin{cases}\alpha_1\ge \max_t\{-2C_0/3p\cdot k_t^0J/k_tJ^0\} & if \quad s(t)\cdot X_1(t)>0\\ \beta_1\le \min_t\{-2C_0/3p\cdot k_t^0J/k_t\cdot J^0\} & if \quad s(t)\cdot X_1(t)<0\end{cases}$$

$$\Psi_2=\begin{cases}\alpha_2\ge \max_t\left\{\dfrac{4(J\cdot\bar D-\bar J\cdot D)+2(\bar k_t\cdot J-k_t\cdot\bar J)\cdot(C_1-\bar D/\bar J)}{9pk_t\bar k_t}\right\} & if \quad s(t)\cdot X_2(t)>0\\ \beta_2\le \min_t\left\{\dfrac{4(J\cdot\bar D-\bar J\cdot D)+2(\bar k_t\cdot J-k_t\cdot\bar J)\cdot(C_1-\bar D/\bar J)}{9pk_t\bar k_t}\right\} & if \quad s(t)\cdot X_2(t)<0\end{cases}$$

$$\Psi_3=\begin{cases}\xi\ge \max_t\{2T_L(t,\theta(t))/3pk_t\} & if \quad s(t)>0\\ \zeta\le \min_t\{2T_L(t,\theta(t))/3pk_t\} & if \quad s(t)<0.\end{cases} \qquad (14c)$$

By this control input with (4), the BLDDSM can be controlled in the sliding mode so that its output can be completely insensitive to the system uncertainties and load disturbances for $t\ge 0$. Now, the existence of the sliding mode on predetermined sliding surface together with the stability of the closed loop system will be investigated in next theorem

**Theorem 1**: *Given system (3), the position control algorithm (15) with (4) satisfies the sufficient condition of the existence of the sliding mode:*

$$\dot s(t)\cdot s(t)<0 \qquad (15)$$

*and the asymptotic stability.*

*Proof*: Proof is dropped for brevity.

Due to Theorem 1, the sliding mode can occur on the every point of $s(t)=0$. Thus output can be obtained as designed in the optimal switching surface for all the bounded load variations and modeling errors by the invariance property of the sliding mode. Fig. 2 shows the proposed algorithm for the position control of a BLDDSM.

The simulation and experiment studies are carried out to show the improved robustness and prescribed output dynamics of the proposed algorithm in comparison with

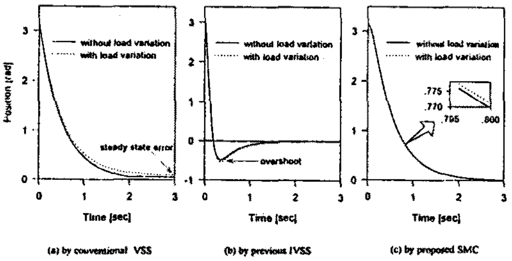


Fig. 3 Output responses by simulations with and without load variations

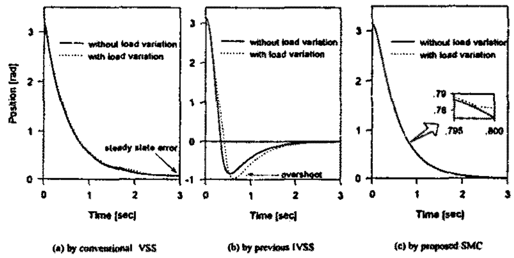


Fig. 4 Output responses by experiment with and without load variations

those of the conventional linear and integral-augmented sliding surfaces[13].

## IV. Simulation and Experiment Studies

### A. Design of Algorithm

Using the parameters . the nominal system becomes

$$\dot{X}(t)=\begin{bmatrix}0 & 1\\ 0 & -54.25\end{bmatrix}\cdot X(t)-\begin{bmatrix}0\\ 12446.\end{bmatrix}\cdot i_{qs}(t). \quad (16)$$

As the first design stage of the suggested position algorithm, $Q$ and $r$ in (10) are selected as

$$Q=\begin{bmatrix}4 & 2\\ 2 & 1\end{bmatrix}, \quad \text{and} \quad r=0.01 \quad (17)$$

which gives rise to the optimal gain, $K_{op}$ as

$$K_{op}=[1.61e^{-3} \quad 3.41e^{-3}] \quad (18)$$

by (11), and consequently, the optimal coefficient of the sliding surface becomes

$$C_0=20.0, \quad C_1=11.8322. \quad (19)$$

In turn, the sliding surface becomes

$$S_P=20\cdot X_0+11.8322\cdot X_1+X_2, \quad X_0=\int_0^t X(\tau)d\tau+X_0^0. \quad (20)$$

Then, the switching gains of (17b) are selected within the range satisfying the inequalities (17c) as

$$\Psi_0=0.1000, \quad \Psi_1=0.0020, \quad \Psi_2=0.0030$$

$$\Psi_3=0.9930, \quad \kappa=0.0001.$$

### B. Results of Simulation and Experiment and Discussions

The position command is given to 3.14 [rad]. Fig. 3 and Fig. 4 show the error output responses of the three algorithms with and without load variation by the computer simulation and experiment, respectively (a) by the conventional VSS with the linear sliding surface, (b) by the previous IVSS[13], and (c) by the proposed SMC. the two case outputs in Fig. 3(a) and 4(a) are different from each other as can be seen because of the sensitivity to the load variation during the reaching phase, and the steady state error exists due to the quasi-sliding mode in digital implementation. In cases of (b), the steady state error is much reduced due to the integral action, but the overshoot as undesirable transient performance appears by the side effect of the integral, because the integral state integrated from zero must re-converge to zero. However, the outputs of two responses in Fig. 3(c) and 4(c) are exactly equal. Because of this, the output behavior can be predictable and prescribed, and no overshoot and negligible steady state error are obtained. The suggested algorithm provides the better features than other two algorithms in view of the overshoot, steady state error, and reaching phase.

From the results of the simulations and the experiments, the suggested position control technique yields better performances than those of the previous two ones.

## V. Conclusions

In this paper, a new integral variable structure system without the reaching phase problems is presented for the improved robust position control of the BLDDSM under load variation. The existence of the sliding mode together with the asymtotic stability of the algorithm is investigated under the parameter uncertainty and load variation as the modeling errors. The simulation and experiment studies verify the usefulness of the algorithm compared with those of the previous two algorithms. The proposed algorithm can provide the improved performances in the respects of the robustness, prediction and prescription of the optimal output.

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