Decentralized Suboptimal H_2 Filtering

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ABSTRACT

In this paper, the decentralized suboptimal H_2 filtering problem is considered. An additional term is added to the centralized optimal H_2 filter so that the whole filter is decentralized. We derive a sufficient condition for existence of such decentralized filters. By employing the solution procedure for the exact model matching problem, we obtain a set of decentralized H_2 filters, and choose a suboptimal filter from this set of decentralized H_2 filters. Naturally the resulting filter is guaranteed to be stable.

1. INTRODUCTION

The problem of designing filters for reconstructing the system states in a decentralized framework for large-scale interconnected systems has been recognized as an important one and many interesting results have been reported in the literature. When a large-scale system is concerned, the centralized pattern often fails to hold due to either lack of the overall information or lack of the centralized computing capability. Although much of the early work dealing with linear interconnected systems by multilevel approaches have been made, these estimators still require information transfer among subsystems. With the constraint that only local information is observed by the local station, some studies concerning this problem have been proposed, but these are not satisfactory. Hence, we present a new algorithm for the design of decentralized filtering schemes for large-scale interconnected systems, which doesn't require information transfer among subsystem. We insert a auxiliary term into centralized H_2 optimal filter so that the decentralized filtering is possible. The existence of this filter is guaranteed under the certain conditions. Moreover we can compute the H_2 norm of proposed filter so that the degree of suboptimality is easily known. With the help of [7] we can obtain the set of those decentralized filter with ease. Finally, in this design procedure we select a suboptimal filter such that the H_2 norm of those decentralized filter is minimized.

The plan of the paper is as follows: in Section 2, we proposed the suboptimal H_2 filters of which H_2norm can be computed. In section 3, it will be shown that under what conditions, the proposed suboptimal H_2 filters reduced to decentralized suboptimal H_2 filter. In section 4, we present the design procedure for this decentralized suboptimal H_2 filter using the solution prodecure for the exact model matching problem. Section 5 is a brief conclusion. Before going to the detail of the approach, we will introduce some notations.

A transfer matrix T(s) with a state space realization $D + C(sI-A)^{-1}B$ will be denoted by

$$G(s) = \left[\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right]$$

Let P(s) be a partitioned matrix with a state-space realization given by

$$P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Then a linear fractional transformation of the partitioned matrix P and a matrix K is defined as

$$F(P,K) = P_{11} + P_{12}K(I-P_{22}K)^{-1}P_{21}$$

2. Suboptimal H_2 filtering

In this section, we consider suboptimal H_2 filtering of LTI system driven by noise process w with unit variance:

$$\dot{x} = Ax + Bw \tag{2.1}$$

$$y = Cx + Dw \tag{2.2}$$

where

$$\begin{bmatrix} B \\ D \end{bmatrix} D^T = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{2.3}$$

We also assume that (A,B) is stabilizable and (C,A) is detectable.

In state estimation problem, we seek to estimate a linear combination of the state vector defined by

$$z = Lx \tag{2.4}$$

Let $\hat{z}$ be an estimate of z generated from the observation y by a filter $K(s)$, that is,

$$\hat{z} = K(s)y \tag{2.5}$$

The estimation error is

$$e = z - \hat{z} \tag{2.6}$$

Then, we can write

$$\begin{bmatrix} e \\ y \end{bmatrix} = \left[\begin{array}{c:cc} A & B & 0 \\ \hdashline L & 0 & -I \\ C & D & 0 \end{array}\right] \begin{bmatrix} w \\ \hat{z} \end{bmatrix} \tag{2.7}$$

In H_2 filtering problem, we seek to minimize the L_2-norm of the transfer function $T_{ew}(s)$ from the noise w to the estimation error e.

Since

$$y = C(sI-A)^{-1}Bw + Dw, \tag{2.8}$$

we have

$$\begin{aligned} e &= C(sI-A)^{-1}Bw - \hat{z} \\ &= C(sI-A)^{-1}Bw - K(s)[C(sI-A)^{-1}Bw + Dw] \end{aligned} \tag{2.9}$$

We denote by P the solution of Riccati equation

$$AP + PA^T - PC^TCP + BB^T = 0, \tag{2.10}$$

and define

$$F = -PC^T, \quad \hat{A}_0 = A + FC \tag{2.11}$$

We decompose

$$K(s) = K_0(s) + K_1(s) \tag{2.12}$$

where K_0 is chosen as

$$K_0(s) = -L(sI-\hat{A}_0)^{-1}F. \tag{2.13}$$

From the fractional representation

$$(sI-A)^{-1}B = [I + (sI-\hat{A}_0)^{-1}FC]^{-1}(sI-\hat{A}_0)^{-1}B \tag{2.14}$$

We obtain

$$\begin{aligned} e &= L[I+(sI-\hat{A}_0)^{-1}FC]^{-1}(sI-\hat{A}_0)^{-1}Bw \\ &\qquad + L(sI-\hat{A}_0)^{-1}FDw - K_1(s)y \\ &= L(sI-\hat{A}_0)^{-1}Bw + L(sI-\hat{A}_0)^{-1}FDw - K_1(s)y \\ &= L(sI-\hat{A}_0)^{-1}[B + FD]w - K_1(s)y \end{aligned} \tag{2.15}$$

From the fractional representation

$$\begin{aligned} & C(sI-A)^{-1}Bw + Dw \\ &= [I + C(sI-\hat{A}_0)^{-1}F]^{-1}[C(sI-\hat{A}_0)^{-1}(B + FD) + D] \end{aligned} \tag{2.16}$$

We also have

$$\begin{aligned} & K_1(s)y \\ &= K_1(s)[I + C(sI-\hat{A}_0)^{-1}F]^{-1}[C(sI-\hat{A}_0)^{-1}(B+FD) + D]w \end{aligned} \tag{2.17}$$

We let

$$K_1(s) = Q(s)[I + C(sI-\hat{A}_0)^{-1}F] \tag{2.18}$$

that is,

$$Q(s) = K_1(s)[I + C(sI-\hat{A}_0)^{-1}F]^{-1} \tag{2.19}$$

Then, we have

$$\begin{aligned} K_1(s)y &= Q(s)[I + C(sI-\hat{A}_0)^{-1}F]y \\ &= Q(s)[C(sI-\hat{A}_0)^{-1}(B + FD) + D]w \end{aligned} \tag{2.20}$$

$$\hat{z} = K_0(s)y + K_1(s)y$$

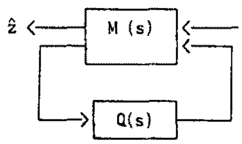
$$= -L(sI-\hat{A}_0)^{-1}Fy + Q(s)[I + C(sI-\hat{A}_0)^{-1}F]y \quad (2.21)$$

Then, the filter $K(s)$ can be represented as a linear fractional representation of the partiontioned matrix $M(s)$ and $Q(s)$ as in Figure 1 where

$$M(s) = \left[\begin{array}{c|cc} \hat{A}_0 & F & 0 \\ \hline -L & 0 & I \\ C & I & 0 \end{array}\right] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (2.22)$$

where

$$M_{11}(s) = -L(sI-\hat{A}_0)^{-1}F, \quad M_{12}(s) = I,$$
$$M_{21}(s) = I + C(sI-\hat{A}_0)^{-1}F, \quad M_{22}(s) = 0 \quad (2.23)$$



<Fig. 1>

With $K(s)$ defined as above, we obtain

$$\begin{aligned} e &= L(sI-\hat{A}_0)^{-1}[B + FD]w \\ &\quad - K_1(s)[I + C(sI-\hat{A}_0)^{-1}F]^{-1}\,[C(sI-\hat{A}_0)^{-1}(B+FD)+D]w \\ &= C(sI-\hat{A}_0)^{-1}[B+FD]w - Q(s)[C(sI-\hat{A}_0)^{-1}(B+FD)+D]w \\ &= G_f w - Q(s)Uw \end{aligned} \quad (2.24)$$

where

$$G_f(s) = \left[\begin{array}{c|c} \hat{A}_0 & B+FD \\ \hline L & 0 \end{array}\right] \quad (2.25)$$

$$U(s) = \left[\begin{array}{c|c} \hat{A}_0 & B+FD \\ \hline C & D \end{array}\right] \quad (2.26)$$

Therefore, it follows that

$$T_{\infty}(s) = G_f(s) - Q(s)U(s) \quad (2.27)$$

From the above, we obtained

$$\begin{aligned} UU^{\sim} &= \{C(sI-(\hat{A}_0)^{-1}(B+FD)+D\}\,\{(B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T+D^T\} \\ &= C(sI-\hat{A}_0)^{-1}(B+FD)(B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T \\ &\quad + C(sI-\hat{A}_0)^{-1}(B+FD)D^T + D(B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T + I \\ &= C(sI-\hat{A}_0)^{-1}\,(BB^T+FF^T+P(sI+\hat{A}_0^T)-(sI-\hat{A}_0)P\,) \\ &\qquad (-sI-\hat{A}_0^T)^{-1}C^T + I \\ &= C(sI-\hat{A}_0)^{-1}(AP+PA^T-PC^TCP+BB^T)(-sI-\hat{A}_0^T)^{-1}C^T + I \\ &= I \end{aligned} \quad (2.28)$$

which implies that U is coinner.

Lemma 2.1 $U(s)$ has no transmisson zero in CRHP.

Proof: Suppose that s is a transmission zero of $U(s)$, then

$$\begin{bmatrix} sI-\hat{A}_0 & B+FD \\ -C & D \end{bmatrix} \text{is not full rank.} \quad (2.29)$$

If this matrix is premultiplied by $\begin{bmatrix} I & -F \\ 0 & I \end{bmatrix}$, then

$$\begin{bmatrix} I & -F \\ 0 & I \end{bmatrix}\begin{bmatrix} sI-\hat{A}_0 & B+FD \\ -C & D \end{bmatrix} = \begin{bmatrix} sI-A & B \\ C & D \end{bmatrix}$$

Thus (3.29) implies that there exists nonzero vector $[x_1^* \; x_2^*]$ so that

$$[x_1^* \; x_2^*]\begin{bmatrix} sI-A & B \\ C & D \end{bmatrix} = [0 \; 0] \quad (2.30)$$

From $x_1^*B + x_2^*D = 0$, postmultiplying D^T on each side of this equality, we obtain $x_2^* = 0$. Therefore (3.30) is reduced to

$$x_1^*\,[sI - A \quad B] = 0$$

Since x_1 is nonzero, s is an uncontrollable mode of (A, B). But (A, B) is stabilizable, hence s has negative real part and the proof is completed.

Q.E.D

Remark. $U(s)$ is in RH_{∞}. Thus, by lemma 2.1, $U(s)$ is stable and minimum phase.

Lemma 2.2 There is no unstable pole-zero cancellation in $Q(s)U(s)$

Proof: Note that $DD^T = I$ requires D is a fat matrix and so is $U(s)$. (And let $Q(s)$ is a tall matrix)
Now, the corollary in [1] may be restated that there are pole-zero cancellation in $Q(s)U(s)$ only if some pole of $Q(s)$ is a transmission zero of $U(s)$ or some pole of $U(s)$ is a transmission zero of $Q(s)$.
Since $U(s)$ is stable and minimum phase, all cancelled ploes, if any, have negative real part. Hence there is no unstable pole-zero cancellation in $Q(s)U(s)$.

Q.E.D

Theorem 2.1 $T_{\infty}(s)$ is in RH_2 if and only if $Q(s)$ is in RH_2

proof: (Sufficiency) obvious.
(Necessity) $T_{\infty}(s) \in RH_2$ implies that $Q(s)U(s) \in RH_2$. Therefore all poles of $Q(s)$, except cancelled ones, must have negative real part. Hence lemma 2.2 guarantees that $Q(s) \in RH_2$

Q.E.D

From Theorem 2.1, it can be seen that we only need to consider $Q(s)$ in RH_2

From (2.25) and (2.26), we obtain

$$\begin{aligned} G_f(s)U(s)^{\sim} &= L(sI-\hat{A}_0)^{-1}(B+FD)\,\{C(sI-\hat{A}_0)^{-1}(B+FD)+D\}^{\sim} \\ &= L(sI-\hat{A}_0)^{-1}(BB^T(sI+\hat{A}_0^T)^{-1}(-C^T) \\ &\qquad + F + FF^T(sI+\hat{A}_0^T)^{-1}(-C^T)) \\ &= L(sI-\hat{A}_0)^{-1}(BB^T+P(sI+\hat{A}_0^T)+PC^TCP)\,(sI+\hat{A}_0^T)^{-1}(-C^T) \\ &= L(sI-\hat{A}_0)^{-1}(sP+PA^T+BB^T)(sI+\hat{A}_0^T)^{-1}(-C^T) \\ &= L(sI-\hat{A}_0)^{-1}(sI-A+PC^TC)P(sI+\hat{A}_0^T)^{-1}(-C^T) \\ &= -LP(sI+\hat{A}_0^T)^{-1}C^T \end{aligned} \quad (2.31)$$

which implies that $G_fU^{\sim} \in RH_2^{\perp}$

From the above result, we have

$$\begin{aligned} \langle Q(s)U(s), G_f(s)\rangle^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} trace[U^{\sim}(j\omega)Q^{\sim}(j\omega)\,G_f(j\omega)]d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} trace[Q^{\sim}(j\omega)\,G_f(j\omega)U^{\sim}(j\omega)]d\omega \\ &= \langle Q(s), G_f(s)U^{\sim}(s)\rangle \\ &= 0 \end{aligned} \quad (2.32)$$

Similarily, it can be shown that

$$\langle G_f(s), Q(s)U(s)\rangle = 0 \quad (2.33)$$

Thus,

$$\begin{aligned} \|T_{\infty}\|_2^2 &= \langle T_{\infty}, T_{\infty}\rangle \\ &= \langle G_f(s) - Q(s)U(s),\, G_f(s) - Q(s)U(s)\rangle \\ &= \langle G_f(s), G_f(s)\rangle - \langle G_f(s), Q(s)U(s)\rangle \\ &\quad - \langle Q(s)U(s), G_f(s)\rangle + \langle Q(s)U(s), Q(s)U(s)\rangle \\ &= \|G_f\|_2^2 + \|Q\|_2^2 \end{aligned} \quad (2.34)$$

Hence, we have

Lemma 2.3 The family of all filters such that $\|T_{\infty}\|_2^2 \le \|G_f\|_2^2 + \epsilon^2$ is the set of all transfer matrices from y to $\hat{z}$ in Fig 1, where $Q \in RH_2$, $\|Q\|_2^2 \le \epsilon^2$

3. Decentralized Suboptimal H_2 filtering.

In this section, we consider a decentralized filtering problem. We decompose

$$\hat{z} = \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (3.1)$$

We seek to find a decentralized filter $K(s) = block\text{-}diag\,(K_{11}(s), K_{22}(s))$ so that

$$\hat{z}_1 = K_{11}(s)y_1, \quad \hat{z}_2 = K_{22}(s)y_2 \quad (3.2)$$

while satisfying $\|T_{\infty}\|_2^2 \le \|G_f\|_2^2 + \epsilon^2$.

From Fig 1, it follows that

$$K(s) = F(M, Q) = M_{11} + QM_{21} \quad (3.3)$$

where

$$M_{11} = -L(sI-\hat{A}_0)^{-1}F,$$
$$M_{21} = C(sI-\hat{A}_0)^{-1}F + I \quad (3.4)$$

We first determine whether there exists a decentralized filter $K(s)$ so that $T_{\infty} \in RH_2$.

We decompose

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad F = [F_1 \, F_2], \quad Q(s) = \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} \quad (3.5)$$

Define

$$X_1 = -L_1(sI-\hat{A}_0)^{-1}F_2, \quad Y_1 = C(sI-\hat{A}_0)^{-1}F_2 + \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (3.6)$$

$$X_2 = -L_2(sI-\hat{A}_0)^{-1}F_1, \quad Y_2 = C(sI-\hat{A}_0)^{-1}F_1 + \begin{bmatrix} I \\ 0 \end{bmatrix} \quad (3.7)$$

then, we have

Lemma 3.1 There exists a decentralized filter $K(s)$ so that $T_{\infty} \in RH_2$ if there exists a $Q(s)$ such that

$$X_1 + Q_1(s)Y_1 = 0 \quad (3.8)$$

and

$$X_2 + Q_2(s)Y_2 = 0 \quad (3.9)$$

Proof: Since

$$K(s) = M_{11} + M_{12}QM_{21}$$

$$= \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (sI_n - \hat{A}_0)^{-1} [F_1 F_2] + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \left[C(sI_n - \hat{A}_0)^{-1} [F_1 F_2] + \begin{bmatrix} I_{p_1} & 0 \\ 0 & I_{p_2} \end{bmatrix} \right]$$

$$= \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix},$$

$$K_{12}(s) = L_1 (sI - \hat{A}_0)^{-1} F_2 + Q_1 \left\{ C(sI - \hat{A}_0)^{-1} F_2 + \begin{bmatrix} 0 \\ I \end{bmatrix} \right\}$$

$$K_{21}(s) = L_2 (sI - \hat{A}_0)^{-1} F_1 + Q_2 \left\{ C(sI - \hat{A}_0)^{-1} F_1 + \begin{bmatrix} I \\ 0 \end{bmatrix} \right\}$$

By the definition of X_1, Q_1, Y_1, X_2, Q_2, Y_2. this complete the proof.

Hence, we have

Lemma 3.2 There exists a decentralized filter $K(s)$ such that $T_\infty \in RH_2$ and $\| T_\infty \|_2^2 \leq \| G_f \|_2^2 + \varepsilon^2$ if the exact model matching problems (3.8) and (3.9) are solvable under the constraint that $Q(s) \in RH_2$, $\| Q(s) \|_2^2 \leq \varepsilon^2$

4. Decentralized Filter Design

To solve (3.8) and (3.9), we consider the stable model matching problem (*SMMP*) such that

$$Y_1^T Q_1^T = -X_1^T \tag{4.1}$$
$$Y_2^T Q_2^T = -X_2^T \tag{4.2}$$

Since $Y_1^T(\infty) = [0 \; I]$ has full row rank, Y_1^T has right inverse, and thus $Q_1^T = -Y_{1r}^T X_1^T$ will be the solution. Since, however, this solution may be unstable or improper, we need the following lemma .

Lemma 4.1 ([7]) Given $P(p \times m,\ p \leq m)$ proper and $T(p \times q)$ proper and stable with $rank(P) = p$, there exist proper and stable solutions M such that

$$PM = T$$

if and only if T has as its zeros all the RHP finite zeros and all the zeros at infinity of P togerher with their associated structure.

Remark : If P is minimum phase and has no zeros at infinity, there exist proper and stable solutions.

Lemma 4.2 The zeros of Y_1 are the unobservable mode of $\{A, C_1\}$

Proof: The zeros of Y_1 are those [1] for which

$$rank \begin{bmatrix} sI - \hat{A}_0, & F_2 \\ -C, & \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} < n + \min(p, p_2) = n + p_2$$

if this matrix is premultiplied by $\begin{bmatrix} I & -F \\ 0 & I \end{bmatrix}$, its rank will not be affected. That is, the matrix

$$\begin{bmatrix} I_n & -F \\ 0 & I_{p_1} \end{bmatrix} \begin{bmatrix} sI_n - \hat{A}_0, & F_2 \\ -C, & \begin{bmatrix} 0 \\ I_{p_1} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} sI_n - \hat{A}_0 + FC, & 0 \\ -C, & \begin{bmatrix} 0 \\ I_{p_1} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} sI_n - A, & 0 \\ \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, & \begin{bmatrix} 0 \\ I_{p_1} \end{bmatrix} \end{bmatrix}$$

will have reduced rank (less than $n + p_2$) only if s is a zero of Y_1. Note, however, that the s which reduce the rank of the last matrix are exactly those which reduce the rank of $\begin{bmatrix} sI_n - A, & 0 \\ C_1, & 0 \end{bmatrix}$ or of $\begin{bmatrix} sI_n - A \\ C_1 \end{bmatrix}$, which are exactly the unobservable poles of $\{A, C_1\}$

Q.E.D

Similarily, it can be shown that the zeros of Y_2 are the unobervable modes of $\{A, C_2\}$

Since $rank \begin{bmatrix} sI_n - A \\ C_1 \end{bmatrix} = rank \begin{bmatrix} sI_n \\ 0 \end{bmatrix} = n$ for $s = \infty$, Y_1 has no eigenvalues at infinity, and so does Y_2. Hence we obtained the sufficient condition for the *SMMP* to be solvale.

Theorem 4.1 If $\{A, C_1\}$ and $\{A, C_2\}$ are each detectable , then there exists a decentralized filter $K(s)$ such that

$$T_\infty \in RH_2$$

The algorithm introduced below to derive such proper and stable $Q_i(s)$ utilizes results from the literature on exact model matching problem [7]. In this algorithm, the poles of the proper right inverse Y_{1ri} will consist of *1) a set of k poles equal to the zeros of Y_1 and 2) a set of n-k poles arbitrarily assignable via linear state feedback F2.*

Assume that Y_I is order n and it has k zeros. Given Y_1^T with rank $rank[Y_1^T] = p$, a right inverse Y_{1r}^T is first determined. In step 7 a solution Q_1^T of SMMP is determined as $Q_1^T(s) = -Y_{1r}^T X_1^T$.

The SMMP Algorithm:

Step 1: Find an irreducible state-space realization of Y_1^T as $\{A, B, C, E\}$, where A, B, C, and E are $n \times n$, $n \times m$, $p \times n$ and $p \times m$ real matrices, respectively.

Step 2: Find an $m \times m$ nonsigular matrix M such that $EM = [I_p \,{:}\, 0]$

Step 3: Calculate $[\underline{B}_1, \underline{B}_2] = BM$ and $A - \underline{B}_1 C$

Step 4: Find an lsvf matrix $\underline{F}_2$ which assigns the n-k controllable poles of $(A - \underline{B}_1 C, \underline{B}_2)$ in th LHP.

Step 5: The desired proper right inverse is

$$\left(A + BM \begin{bmatrix} -C \\ \underline{F}_2 \end{bmatrix}, BM \begin{bmatrix} I_p \\ \underline{G}_2 \end{bmatrix}, M \begin{bmatrix} -C \\ \underline{F}_2 \end{bmatrix}, \begin{bmatrix} I_p \\ \underline{G}_2 \end{bmatrix} \right)$$

where $\underline{F}_2$ was determined above and $\underline{G}_2$ is any $(m - p) \times p$ real matrix (which can be taken as 0 for convenience).

Step 6: Calculate the transfer matrix

$$Y_{1r}^T = C_I (sI - A_I)^{-1} B_I + E_I$$

Step 7: Calculate $Q_1^T(s) = -Y_{1r}^T X_1^T 1$ which is a solution of the *SMMP*.

We can obtain Q_2^T by similar approach.

If we obtain Q1 and Q2 from the above, We can construct a decentralized filter $K(s) = block\text{-}diag\,(K_{11}(s), K_{22}(s))$ so that $\hat{z}_1 = K_{11}(s) y_1$, $\hat{z}_2 = K_{22}(s) y_2$, where

$$K_{11}(s) = L_1 (sI - \hat{A}_0)^{-1} F_1 + Q_1 \left\{ C(sI - \hat{A}_0)^{-1} F_1 + \begin{bmatrix} I \\ 0 \end{bmatrix} \right\}$$

$$K_{22}(s) = L_2 (sI - \hat{A}_0)^{-1} F_2 + Q_2 \left\{ C(sI - \hat{A}_0)^{-1} F_2 + \begin{bmatrix} 0 \\ I \end{bmatrix} \right\}$$

5. Conclusion

Decentralized suboptimal H_2 filtering based on the H_2 output estimation problem[5] has been examined. The stability of this filter is guaranteed. Moreover the degree of suboptimality can be computed with respect to the H_2 optimal filter taken as a reference. A practical procedure for decentralzed filter design based on the stable model matching problem(SMMP) has been proposed.

However, in decentralized filter design the subsystem should be detectable to guarantee the existence of decentralized suboptimal H_2 filter. Futhermore Development of design procedures which incorporate the suboptimality requiremants and dimension reduction of Q(s) seems to be necessary.

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