Development of a Consistent General Order Nodal Method for Solving the Three-Dimensional, Multigroup Neutron Diffusion Equation

Hyun Dae Kim and Se Kee Oh
Institute for Advanced Engineering

ABSTRACT

A consistent general order nodal method for solving the three-dimensional neutron diffusion equation in (x-y-z) geometry has been derived by using a weighted integral technique and expanding the spatial variable by the Legendre orthogonal series function. The equation set derived can be converted into any order nodal schemes. It forms a compact system for general order of nodal moments. The method utilizes fewer unknown variables in the schemes for iterative-convergence solution than other nodal methods listed in the literatures, and because the method utilizes the analytic solutions of the transverse-integrated one dimensional equations and a consistent approximation for a given spatial variable through all the solution procedures, which renders the use of an approximation for the transverse leakages no longer necessary, we can expect extremely accurate solutions and the solution would converge exactly when the mesh width is decreased or the approximation order is increased.

1. INTRODUCTION

Modern coarse-mesh nodal methods are a very efficient class of numerical methods that have proven to be superior in accuracy, computer storage requirement and computing time to finite difference and finite element methods for the solution of large, multidimensional, neutron diffusion problems that arise in nuclear reactor physics [1]. However, in nodes the transverse currents at their surfaces not varying smoothly, substantial errors subsist and convergence difficulties may arise [2]. These problems *arise mainly from the inconsistent quadratic fit* approximations of the transverse leakage [3]. To overcome them, higher order scheme has been suggested [4]. A higher order nodal scheme for two-dimensional geometry showed greatly accurate results [2]. In the present work, a new general order consistent nodal method without the transverse leakage approximation is derived. We aim at insight synthesis into the nodal method procedure via general order formulation and consistent expansion of spatial variables.

2. GENERALIZED FLUX MOMENT FORMULATION

In a node of widths, Δx_i, Δy_j and Δz_k in x, y and z directions, respectively, the multigroup diffusion equation is written by the dimensionless variables,

$$-4D_g/\Delta x_i^2 \partial^2\Phi_g(u,v,w)/\partial u^2 - 4D_g/\Delta y_j^2\ \partial^2\Phi_g(u,v,w)/\partial v^2$$

$$-4D_g/\Delta z_k^2 \partial^2\Phi_g(u,v,w)/\partial w^2 + \Sigma_{tg}\Phi_g(u,v,w) = S_g(u,v,w), \qquad (1)$$

where

$$S_g(u,v,w) \equiv \sum_{g'=1}^{G}\{\chi_g/k_{\partial}\nu\Sigma_{fg'} + \Sigma_{sg'g}\}\Phi_{g'}(u,v,w) + q_{0g}(u,v,w), \qquad (2)$$

and $-1 \le u,\ v,\ w \le 1$.

We derive the transverse integrated one-dimensional nodal equation set by applying a weighted integration. Each resulting differential equation is for the general order moment of the flux in the transverse spatial direction. For example, for the v and w moments of the u-dependent flux, we multiply Eq. (1) by $P_{n_v}(v)$ and $P_{n_w}(w)$, the Legendre polynomials in v and w variables, respectively, and integrate over $-1 \le v,\ w \le 1$ in the node to arrive at the u-channel differential equation. The result is given by

$$-4D/\Delta x^2 d^2\Phi_{n_v n_w}(u)/du^2 + \Sigma_t\Phi_{n_v n_w}(u) = A_{u n_v n_w}(u), \qquad (3)$$

where

$$\Phi_{n_v n_w}(u) = \int_{-1}^{1}\int_{-1}^{1} dv dw P_{n_v}(v) P_{n_w}(w)\Phi(u,v,w), \qquad (4)$$

$$A_{u n_v n_w}(u) \equiv S_{n_v n_w}(u) - L_v(u,n_v,n_w) - L_w(u,n_v,n_w), \qquad (5)$$

$$L_v(u,n_v,n_w) \equiv 4/\Delta y^2\{J_v^F{}_{n_w}(u) + (-1)^{n_v+1} J_v^N{}_{n_w}(u)$$

$$-\sum_{l=0}^{(n_v-1)/2}(2n_v-4l-1)J_{v(n_v-2l-1)n_w}(u)\}, \qquad (6)$$

and

$$J_v^F{}_{n_w}(u) = \int^{1} dw P_{n_w}(w) J_v(u,1,w) = -D\int_{-1}^{1} dw P_{n_w}(w)\partial\Phi(u,1,w)/\partial v. \qquad (7)$$

The coupling terms, $L_v(u,n_v,n_w)$ and $L_w(u,n_v,n_w)$, in the above equations arise from the intergration of v- and w-channel derivatives, respectively.

After expanding the source term, $A_{u n_v n_w}(u)$ by the Legendre function in u and substituting into Eq. (3), solving this, we obtain the equation for the u-dependent, n_v and n_w moment fluxes, $\phi n_v n_w(u)$,

$$\phi n_v n_w(u)=\{B_u n_v n_w 1+\sum_{n_u=0} S_1 n_u n_v n_w(u)\}\cosh(r_u u)$$
$$+\{B_u n_v n_w 2-\sum_{n_u=0} C_1 n_u n_v n_w(u)\}\sinh(r_u u), \quad (8)$$

where

$$r_u \equiv \Delta x/2\sqrt{\Sigma_t/D}, \quad (9)$$
$$S_1 n_u n_v n_w(u)\equiv\Delta x^2/(4r_u D)(2n_u+1)/2A_u n_v n_w, n_u \int \sinh(r_u u)P n_u(u)du, \quad (10$$
$$C_1 n_u n_v n_w(u)\equiv\Delta x^2/(4r_u D)(2n_u+1)/2A_u n_v n_w, n_u \int \cosh(r_u u)P n_u(u)du, \quad (11$$

and $B_u n_v n_w 1$ and $B_u n_v n_w 2$ are the coefficients which can be expressed by the surface current moments and Eqs. (10) and (11).

If we multiply Eq. (3) by $Pn_u(u)$ and integrate over $-1 \le u \le 1$, we obtain the generalized moment balance equation in (u-v-w) coordinates,

$$L_u(n_u, n_v, n_w)+L_v(n_u, n_v, n_w)+L_w(n_u, n_v, n_w)+\Sigma_t\phi n_u n_v n_w = S n_u n_v n_w. \quad (12)$$

3. GLOBAL COUPLING OF THE LOCAL SOLUTIONS

The solution for the flux moments given by Eq. (12) depends on the availability of the interface net current moments. The spatial coupling parameters which are defined in terms of net current moments across a surface can be generated from the continuity condition for the flux moments at interfaces. When this condition and the equation for flux moments expressed by Eq. (8) are used at a given interface, the spatial dependent flux moments are eliminated and an equation relating the three net current moments at three consecutive interfaces is obtained. For example, for u-direction,

$$E_{u,i-1}J_u^L n_v n_{w,i-1}+E_{u,i}J_u^L n_v n_{w,i}+E_{u,i+1}J_u^L n_v n_{w,i+1}=Q_u n_v n_{w,i}, \quad (13)$$

where

$$E_{u,i-1}\equiv-1/\{r_u D\sinh(2r_u)\}_{i-1}, \quad (14)$$

$$E_{u,i+1}\equiv-1/\{r_u D\sinh(2r_u)\}_i, \quad (15)$$

$$E_{u,i}\equiv-E_{u,i-1}\cosh(2r_{u,i-1})-E_{u,i+1}\cosh(2r_{u,i}), \quad (16)$$

and

$$Q_u n_v n_{w,i}\equiv 1/\{2\cosh(r_{u,i-1})\}\sum_{n_u=0}\{S_1 n_u n_v n_w(u=1)-\ S_1 n_u n_v n_w(u=-1)\}_{i-1}$$

$$+1/\{2\cosh(r_{u,i})\}\sum_{n_u=0}\{S_{1n_un_vn_w}(u=1)-S_{1n_un_vn_w}(u=-1)\}$$
$$+1/\{2\sinh(r_{u,i-1})\}\sum_{n_u=0}\{C_{1n_un_vn_w}(u=1)-C_{1n_un_vn_w}(u=-1)\}_{i-1}$$
$$-1/\{2\sinh(r_{u,i})\}\sum_{n_u=0}\{C_{1n_un_vn_w}(u=1)-C_{1n_un_vn_w}(u=-1)\}_{i}. \quad (17)$$

The process is repeated for all nodes and all moments, giving special treatment only to boundary nodes by incorporating the boundary conditions. The resulting equation set can be solved by the direct method.

4. SUMMARY

A nodal method for solving the steady state multigroup diffusion equation has been derived by using a weighted integral technique and Legendre polynomial expansion. The equation set derived above constitutes a compact system for general order nodal moments. This system can be converted into any orders of nodal moments or the standard finite difference form. When the derived equation set is truncated at finite moment series for a desired order moment, they can be solved by the standard power method for eigenvalue iteration. The finite moment equation set to be solved has more analytic information than the nodal expansion method because of using the analytic solution of the quasi-one-dimensional equation for flux moments in each channel in their derivation, and uses a consistent polynomial expansion for a given spatial variable, therefore, we can expect extremely accurate solutions, and the solution would converge exactly when the mesh width is decreased or the approximation order is increased. Also, the method utilizes fewer unknown variables in iterative-convergence solution procedure than other nodal methods developed [1], and thus requires less computing time and storage requirement.

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