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층간의 상관관계를 고려한 다중 층응답스펙트럼해석
Floor Response Spectra Analysis Including Correlations of Multiple Support Motions

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요 약

주구조물의 여러층에 지지점을 가지는 부구조물의 응답스펙트럼 해석방법에 대하여 연구하였다. 본 연구에서는 지지점 입력간의 상관관계를 고려할 수 있으며 실무에서 부구조물의 내진설계에 쉽게 적용할 수 있는 다중 응답스펙트럼 해석방법을 제시하였다. 다중 응답스펙트럼과 지지점 입력간의 상관계수는 random vibration 이론을 이용하여 설계지반응답스펙트럼으로부터 직접 유도하였다. 예제해석 결과 본 연구에서 제안한 방법은 지지점 입력간의 상관관계를 고려하지 않는 통상의 다중 응답스펙트럼 해석방법보다 정확한 지진응답을 예측함을 알 수 있었다.

1. Introduction

Secondary systems as well as primary structures have to be designed to withstand earthquake loadings, particularly for the cases of important industrial facilities such as those in nuclear power plants. Most secondary systems are supported at multiple supports extended in the space. Hence, the effects of the relative motion between the supports and their correlations can be very significant.

In engineering practice, the single envelope and the multiple response spectra methods have been used for seismic designs, which do not consider the correlation effects in the analysis. In the last decade, random vibration approaches were studied to take into account the correlation effects by several investigators[1-4]. However, those methods are difficult to be employed in practice, since they require very complicated procedures to calculate the auto- and cross-floor spectra.

In this study an efficient floor response spectra method is proposed for the secondary systems, in which the correlation effects between the multi-support motions

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can be considered. The floor response spectra and their cross-correlation coefficients are evaluated from the design ground response spectrum, based on the random vibration theory.

Example Analyses are carried out for several cases of the primary and secondary systems. The validity of the proposed method is investigated by comparing the results with those obtained by other methods, such as the conventional multiple response spectra method without considering the correlation of support motions, the random vibration method, and the time history analysis method. The results show that the proposed method yields more accurate results than those by the conventional multiple response spectra method.

2. Equation of Motion

For a coupled system of the primary and secondary structures subjected to earthquake loading, the equation of motion can be obtained as

$$\begin{bmatrix} M_S & 0 & 0 \\ 0 & M_F & 0 \\ 0 & 0 & M_P \end{bmatrix} \begin{Bmatrix} \ddot{X}_S^t \\ \ddot{X}_F^t \\ \ddot{X}_P^t \end{Bmatrix} + \begin{bmatrix} C_{SS} & C_{SF} & 0 \\ C_{FS} & C_{FF} & C_{FP} \\ 0 & C_{PF} & C_{PP} \end{bmatrix} \begin{Bmatrix} \dot{X}_S \\ \dot{X}_F \\ \dot{X}_P \end{Bmatrix} + \begin{bmatrix} K_{SS} & K_{SF} & 0 \\ K_{FS} & K_{FF} & K_{FP} \\ 0 & K_{PF} & K_{PP} \end{bmatrix} \begin{Bmatrix} X_S \\ X_F \\ X_P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

where subscripts P, S and F denote the primary and the secondary structures and the supports of the secondary one; M, C, and K are the mass, damping and stiffness matrices; $\{X\}$ and $\{\dot{X}\}$ are the relative displacement and velocity vectors to the ground; and $\{\ddot{X}^t\}$ is the absolute acceleration vector.

Assuming that the effect of the secondary structure on the response of the primary structure is negligible and the damping matrix is proportional to the stiffness matrix, and representing the displacement of the secondary structure as a sum of the pseudo-static and relative dynamic displacements, the equation for the relative dynamic displacement of the secondary structure $\{X_S^d\}$ can be written as [5]

$$[M_S]\{\ddot{X}_S^d\} + [C_{SS}]\{\dot{X}_S^d\} + [K_{SS}]\{X_S^d\} = -[M_S][L]\{\ddot{X}_F^t\} \tag{2}$$

where $[L] = -[K_{SS}]^{-1}[K_{SF}]$

Utilizing the modal solution for the secondary structure as

$$\{X_S^d\} = [\Phi_S]\{q_S\} \tag{3}$$

where $[\Phi_S]$ is the mode shape matrix of the secondary structure; and $\{q_S\}$ is the vector of the modal coordinate, the j-th modal equation of motion can be obtained as

$$\ddot{q}_{sj}(t) + 2\zeta_{sj}\omega_{sj}\dot{q}_{sj}(t) + \omega_{sj}^2 q_{sj}(t) = -\langle \Gamma_S \rangle_j \{\ddot{X}_F^t(t)\} \tag{4}$$

where ω_{sj} and ζ_{sj} are the j-th natural frequency and modal damping ratio of the secondary structure; $<\Gamma_S>_j$ is the j-th row vector of the modal participation factor matrix $[\Gamma_S]=[\mu_s]^{-1}[\Phi_S]^T[M_S][L]$, whose element Γ_{Sjk} is associated with the j-th mode and the k-th support motion; and $[\mu_s]$ is the diagonal matrix of the modal mass.

3. Multiple Floor Response Spectra

In this paper, a method for evaluating the maximum modal response for the multi-support excitations as in Eq.(4) is proposed as

$$|q_j|_{\max}=[\sum_{r=1}\sum_{p=1}\Gamma_{Sjr}\Gamma_{Sjp}\cdot\rho_{rp}(\omega_j,\zeta_j)\cdot S_{FrFr}(\omega_j,\zeta_j)S_{FpFp}(\omega_j,\zeta_j)]^{1/2} \tag{5}$$

where $S_{FrFr}(\omega_j,\zeta_j)$ is the floor response spectrum of the r-th floor, and $\rho_{rp}(\omega_j,\zeta_j)$ is the cross-correlation coefficient between two floor response spectra of the r-th and the p-th floors. The floor response spectra and the cross-correlation coefficient are evaluated based on the random vibration approaches as follows :
The expected maximum modal response can be obtained as

$$\begin{aligned}E[|q_j|_{\max}] &= P_j\cdot\sigma_{q_j}\\ &= [\sum_{r=1}\sum_{p=1}\Gamma_{Sjr}\Gamma_{Sjp}\cdot P_j^2\cdot\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FrFp}(\omega)d\omega]^{1/2}\\ &= [\sum_{r=1}\sum_{p=1}\Gamma_{Sjr}\Gamma_{Sjp}\cdot\rho_{rp}(\omega_j,\zeta_j)\cdot P_j(\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FrFr}(\omega)d\omega)^{1/2}\\ &\qquad\cdot P_j(\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FpFp}(\omega)d\omega)^{1/2}]^{1/2}\end{aligned} \tag{6}$$

where σ_{q_j} is the standard deviation of q_j; P_j is the peak factor; $G_{FrFr}(\omega)$ and $G_{FrFp}(\omega)$ are the power- and cross-spectral density functions of the floor excitations; $H_{Sj}(\omega)$ is the frequency response function for q_j from Eq.(4); and $\rho_{rp}(\omega_j,\zeta_j)$ is the cross-correlation coefficient between two floor response spectra defined as

$$\rho_{rp}(\omega_j,\zeta_j)=\frac{\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FrFp}(\omega)d\omega}{(\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FrFr}(\omega)d\omega)^{1/2}\cdot(\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FpFp}(\omega)d\omega)^{1/2}} \tag{7}$$

By approximately replacing the peak factor P_j's in the last expression of Eq.(6) with the peak factors associated with the individual floor response spectra(P_{jr} and P_{jp}), Eq.(6) can be rewritten into the same expression as in Eq.(5) :

$$E[|q_j|_{\max}]=[\sum_{r=1}\sum_{p=1}\Gamma_{Sjr}\Gamma_{Sjp}\cdot\rho_{rp}(\omega_j,\zeta_j)\cdot S_{FrFr}(\omega_j,\zeta_j)S_{FpFp}(\omega_j,\zeta_j)]^{1/2} \tag{8}$$

where the floor response spectrum is defined as

$$S_{FrFr}(\omega_j,\zeta_j)=P_{jr}\cdot[\int_{-\infty}^{\infty}|H_{Sj}(\omega)|^2G_{FrFr}(\omega)d\omega]^{1/2} \tag{9}$$

The peak factors(P_{jr} and P_{jp}) may be evaluated by the conventional methods. In this study, the one proposed by Der Kiureghian[6] is used. The spectral density functions of the floor excitations, $G_{F_rF_r}(\omega)$ and $G_{F_pF_p}(\omega)$, can be computed through the random vibration analysis on the primary structure, once the power spectral density (PSD) function of the ground acceleration is determined. It is customary that the design input ground motion is specified in terms of the ground response spectrum. A compatible PSD function may be obtained using the procedures in References 7 and 8.

Finally, the expected maximum dynamic response of the secondary structure can be evaluated using the conventional methods for the modal response combination. In this study, the square root of the sum of the squares (SRSS) method is employed.

4. Numerical Analysis and Discussion

Example analyses are carried out for four different cases of the primary and secondary systems(Figure 1 and Table 1) by using the proposed method. For the purpose of comparison, the analyses are also performed by other methods such as the conventional multiple response spectrum method without considering the correlation effects of the support motions, the random vibration method, and the time history analysis method. The natural frequencies of the primary and the secondary structures are shown in Table 2. The modal damping ratios of the structures are assumed to be 3%.

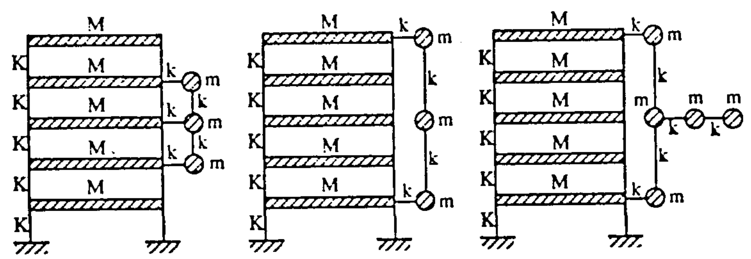

Fig. 1. Example Systems

Table 1. Structural Properties of Each Model

Models	M(kip-s^2/ft)	K(kip/ft)	m(kip-s^2/ft)	k(kip/ft)
Ⅰ	35.5	11,000	0.1	15
Ⅱ	35.5	11,000	0.1	15
Ⅲ	35.5	1,000	0.1	15
Ⅳ	35.5	11,000	0.1	9

For the proposed and the conventional multiple response spectrum methods and the random vibration method, the PSD function of the ground acceleration, which is compatible with the design ground response spectrum (USNRC Reg.Guide 1.60[9]; ζ

=3%, PGA=0.2g), is evaluated by using SIMQKE code[7]. The floor response spectra (FRS) and their cross-correlation coefficients are evaluated based on the PSD function by the random vibration analysis of the primary structures. The time history analysis is carried out on the coupled systems of the primary and secondary structures for twenty different artificial time histories of the ground acceleration generated by using SIMQKE code.

Figure 2 shows the FRS's and their cross-correlation coefficients of Model I. Figure 3 shows the cross-correlation coefficients for other cases. From the results, it can be observed that the values of the cross-correlation coefficients become unity in the low frequency range, namely, in the cases with flexible secondary structures, while they converge to certain constant values in the high frequency range, that is, in the cases with the relatively stiff secondary structures. The maximum relative dynamic responses of the secondary structures, $|\{X_S^d\}|_{max}$, obtained by different methods are compared in Table 3. Compared with the results by the random vibration approach, it has been observed that the proposed method gives much better results than those by the conventional multiple response spectra method. The maximum displacements including the pseudo-static responses are compared in Table 4. In this case, the results by the time history analysis are considered to be most accurate. Compared with them, the proposed method including the correlation effects of the floor motions yields much better results than those by the conventional method.

5. References

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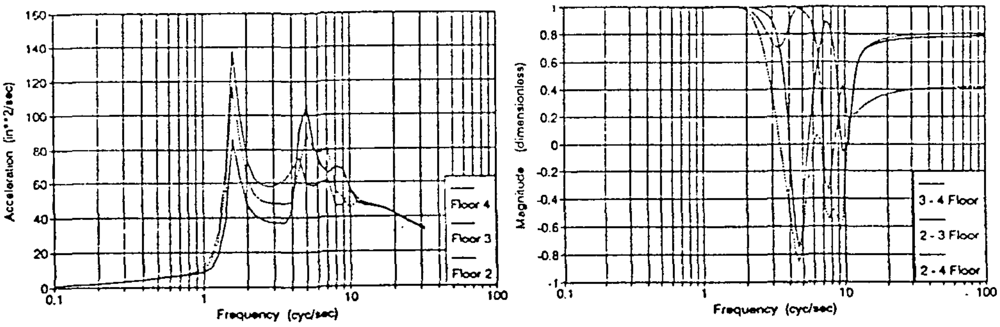
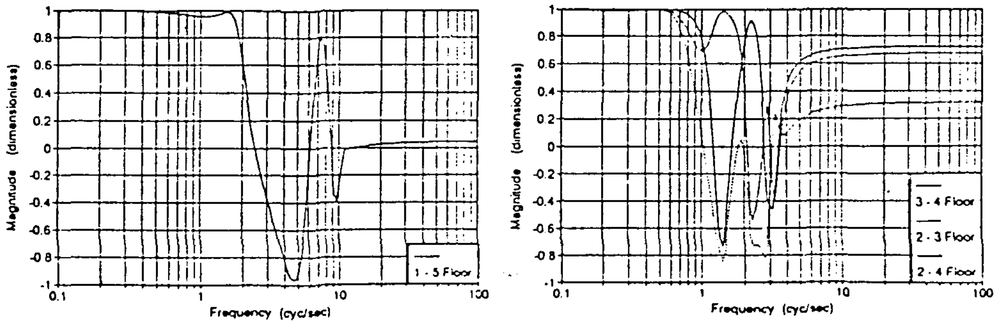


(a) Floor Response Spectra (b) Correlation Coefficients

Fig. 2. Floor Response Spectra and Cross-Correlation Coefficients of Model I



(a) Model Ⅱ & Ⅲ (b) Model Ⅳ

Fig. 3. Cross-Correlation Coefficients of Models Ⅱ, Ⅲ and Ⅳ

Table 2. Natural Frequencies of Each Model (Hz)

Models	Modes of Primary Structure					Modes of Secondary Structure				
	1	2	3	4	5	1	2	3	4	5
I	1.59	4.65	7.33	9.42	10.7	3.90	5.51	7.80	-	-
Ⅱ	1.59	4.65	7.33	9.42	10.7	2.98	5.51	7.20	-	-
Ⅲ	0.48	1.40	2.21	2.84	3.24	3.90	5.51	7.80	-	-
Ⅳ	1.59	4.65	7.33	9.42	10.7	4.12	9.55	13.5	15.0	19.9

Table 3. Maximum Relative Dynamic Displacements of Secondary Structures (unit : inch)

Models	Nodes	Response Spectrum Analysis		Random Vibration Analysis
		w/o correlation effect	w/ correlation effect (Present Study)	
I	1	1.98 (14.8)*	2.31 (0.76)*	2.32
	2	1.87 (0.85)	1.86 (0.26)	1.85
	3	1.98 (25.4)	1.60 (1.20)	1.58
II	1	2.66 (11.0)	2.32 (3.3)	2.39
	2	3.64 (31.9)	3.05 (10.7)	2.76
	3	2.66 (20.6)	2.32 (5.1)	2.20
III	1	0.528 (10.2)	0.642 (9.2)	0.587
	2	0.512 (10.7)	0.634 (10.3)	0.571
	3	0.528 (7.5)	0.642 (12.6)	0.567
IV	1	4.28(182.3)	1.55 (2.1)	1.52
	2	3.48(178.5)	1.26 (0.6)	1.25
	3	1.14(122.3)	0.433 (15.4)	0.512
	4	2.04(159.5)	0.748 (5.0)	0.787
	5	1.14(134.9)	0.433 (10.6)	0.484

* errors(%) compared with the random vibration analysis results

Table 4. Maximum Displacements of Secondary Structures (unit : inch)

Models	Nodes	Response Spectrum Analysis		Random Vibration Analysis	Time History Analysis
		w/o correlation effect	w/ correlation effect (Present Study)		
I	1	5.59 (6.8)*	5.94 (13.5)*	5.94 (13.6)*	5.24
	2	5.24 (13.8)	5.20 (12.8)	5.20 (12.9)	4.61
	3	5.00 (23.3)	4.65 (14.6)	4.61 (14.5)	4.06
II	1	6.42 (17.7)	6.06 (11.4)	6.10 (12.8)	5.43
	2	6.69 (42.4)	6.10 (29.9)	5.83 (23.7)	4.69
	3	5.04 (41.9)	4.69 (32.3)	4.57 (29.1)	3.54
III	1	16.6 (9.1)	16.7 (8.4)	16.7 (8.8)	18.3
	2	15.4 (6.6)	15.5 (5.9)	15.4 (6.3)	16.5
	3	14.0 (2.6)	14.1 (1.8)	14.0 (2.3)	14.3
IV	1	7.32 (84.1)	4.61 (15.8)	4.57 (14.8)	3.98
	2	6.54 (69.4)	4.29 (11.2)	4.29 (11.2)	3.86
	3	4.88 (11.7)	4.17 (4.5)	4.25 (2.7)	4.37
	4	5.08 (40.2)	3.78 (4.3)	3.82 (5.4)	3.62
	5	3.50 (36.9)	2.80 (9.2)	2.83 (10.8)	2.56

* errors(%) compared with the time history analysis results