Fifth IFSA World Congress (1993), 1406-1409

ON MUTUAL AGREEMENT OF SUBJECTIVE RELIABILITY ANALYSIS RESULTS

Takehisa ONISAWA

Institute of Engineering Mechanics
University of Tsukuba
1-1-1, Tennodai, Tsukuba 305, Japan

Abstract – This paper describes a model of the subjective reliability analysis, which uses a fuzzy set, natural language expressions and parameterized operations of fuzzy sets, and reflects analysts' subjectivity. The model has the problem of many different analysis results being obtained since the results depend on their subjectivity. As one of the solutions two kinds of mutual agreements based on the analysis results are considered. One is the intersection and the union of the fuzzy sets obtained by the analysis. The other is the weighted average of the fuzzy sets. This paper gives these interpretations from the viewpoint of system reliability analysis. This paper also shows examples of these considerations.

1. INTRODUCTION

Experts' engineering judgement, i.e., subjectivity, is inherent in a system reliability analysis. For example, numerical values, i.e., a failure probabaility and an error probability, are used in order to analyze system reliability quantitatively. In practice, however, it is not likely that enough amount of immediate data can be collected to estimate these probabilities. The estimation of the failure probability and the error probability cannot help being dependent on experts' engineering judgement. Even if the probability of a system accident is obtained by the analysis, the probability is very very small from the viewpoint of our daily life. Safety evaluation of the analyzed system is closely related to experts' subjectivity. Experts' subjectivity plays an important role in the system reliability analysis. The probability theory is a main tool in the reliability/safety analysis and some related problems for the present. However applications of fuzzy sets theory to this area have increased recently[1][2].

This paper mentions a model of subjective system reliability analysis[3]. The model uses a fuzzy set, natural language expressions and parameterized fuzzy sets operations, and reflects analysts' subjectivity more than our previous model[4]. However for the reason that the analysis results depend on analysts' subjectivity, the subjective reliability analysis has the problem of various analysis results being obtained. In this paper as one of the solutions of the problem two kinds of mutual agreements are considered and discussed from the viewpoint of system reliability analysis.

2. A MODEL OF SUBJECTIVE RELIABILITY ANALYSIS

2.1. Failure Possibility

The model uses a fuzzy set on the unit interval with the membership function

$$F(x) = \frac{1}{1+20\times|x-x_0|^m}, \tag{1}$$

where x_0 and m are parameters.

The fuzzy set is called a failure possibility, which is a subjective unreliability measure. The parameter x_0 gives the maximal grade of the membership and the parameter m is related to fuzziness.

The model considers two kinds of determination methods of parameters x_0 and m[3]. One method deals with the case in which the data of reliability estimate for the system reliability analysis are given by numerical values. The other method deals with the case in which the data of reliability estimate are given by natural language. This paper considers only later case for simplicity. Natural language expressions are more commensurate with the case in which we have not enough amount of data to estimate the probabilities since subjectivity inherent in reliability estimate can be covered well by natural language. In this model the estimation of reliability is expressed in the form of reliability estimate and its fuzziness. The parameter x_0 corresponds to the natural language expressions of reliability estimate shown in Table 1. The failure possibility in the class 1 has the membership function $F(1) = 1(F(x)=0;x\neq 1)$ and that in the class 11 has the membership function $F(0) = 1(F(x)=0;x\neq 0)$. The parameter m corresponds to the expressions of fuzziness of reliability estimate shown in Table 2.

Table 1 EXPRESSIONS OF RELIABILITY ESTIMATE AND PARAMETER x_0

Class	Expressions of Reliability Estimate	Parameter x_0 (Representative Value)
	(System, subsystem, instrument or human operator has)	
1	no reliability	–
2	very low reliability	0.9 – **1.0**(0.95)
3	low reliability	0.7 – **0.9**(0.8)
4	rather low reliability	0.55 – **0.7**(0.625)
5	standard reliability	**0.45** – **0.55**(0.5)
6	rather high reliability	**0.3** – 0.45(0.375)
7	high reliability	**0.2** – 0.3(0.25)
8	quite high reliability	**0.1** – 0.2(0.15)
9	extremely high reliability	**0.05** – 0.1(0.075)
	(Accident, failure or human error is)	
10	next to impossible	**0.0** – 0.05(0.025)
11	impossible	–

Table 2 EXPRESSIONS OF FUZZINESS AND PARAMETER m

Class	Expressions of Fuzziness	Parameter m
1	low	2.0
2	medium	2.5
3	rather high	3.0
4	high	3.5

2.2. Fuzzy Sets Operations

The model applies a fault tree analysis. Basic operations are an **and** and an **or** operations of failure possibilities.

The parameterized operations (2) and (3) are used as the **and** and the **or** operations, respectively, where the extension principle[5] is used.

$$H(x,\ y) = \frac{1}{1+(((1-x)/x)^{1/nH}+((1-y)/y)^{1/nH})^{nH}}, \tag{2}$$

where $0<x,y\leq 1$, $H(x,0)=H(0,y)=0$, and nH is a non-negative parameter.

$$G(x,\ y) = \frac{((x/(1-x))^{nG}+(y/(1-y))^{nG})^{1/nG}}{1+((x/(1-x))^{nG}+(y/(1-y))^{nG})^{1/nG}}, \tag{3}$$

where $0\leq x,y<1$, $G(x,1)=G(1,y)=1$, and nG is a non-negative parameter.

The operations H and G are so called Dombi t-norm and Dombi t-conorm, respectively[6]. The operations H and G can cover the range of reliability estimate from the most optimistic estimate through the most pessimistic one. The parameterized operations H and G can reflect analysts' subjectivity. The larger the parameters nH and nG are, the more optimistic the reliability estimate is[3].

2.3. Dependence

2.3.1 Dependence Model

Let's consider a parallel system with dependence as shown in Fig.1. When the failure of the subsystem A has an influence on the failure of the subsystem B as shwon in Fig.1, the dependence should be considered. Let us assume that the failure of the subsystem A has an influence on that of the subsystem B as shown in Fig.1. Let F_A and F_B be failure possibilities of the subsystems A and B, respectively, and let R be a fuzzy set representing the dependence level. The following two cases are considered in the dependence model. The extension principle is used in the following operations (4) through (7).

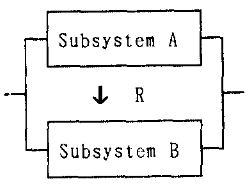


Fig. 1 PARALLEL SYSTEM WITH DEPENDENCE

One is the case in which the failure of the subsystem A has an influence on that of the subsystem B. In this case the failure possibility of the system F_B' is estimated by

$$F_B' = H(F_A,\ R). \tag{4}$$

The other is the case in which the failure of the subsystem A has no influence on that of the subsystem B. As far as the dependence is not complete, the failure of the subsystem A does not always have an influence on that of the subsystem B. In this case the portion of the failure possibility, which has no influence on the failure of the subsystem B, is estimated by

$$F_A = G(F_A',\ F_B'), \tag{5}$$

where F_A' is the portion.

The operation (5) implies that the failure of the subsystem A has an influence on that of the subsystem B or not. The failure possibility of the system is estimated by

$$F' = H(F_A',\ F_B). \tag{6}$$

The operation (6) implies that the failure of the system occurs when the failure of the subsystem A and the failure of the subsystem B occur without influence.

Considering the above two cases the failure possibility of the whole system is estimated by

$$F = G(F_B',\ F'). \tag{7}$$

2.3.2 Natural Language Expressions of Dependence Level

The estimation of the dependence level is expressed by natural language from the same reason as the reliability estimate. In this model it is expressed in the form of the dependence level estimate and its fuzziness. Five kinds of expressions are used as the dependence level estimate and four kinds of expressions are used as its fuzziness. Table 3 shows them. Their meanings are expressed by a fuzzy set with the membership function (8).

$$R(r) = \frac{1}{1+20\times|r-r_0|^{mr}}, \tag{8}$$

where r_0 and mr are parameters.

The parameters r_0 and mr correspond to the expressions of the dependence level estimate and to the expressions of its fuzziness, respectively. The fuzzy sets in the level 1 and in the level 5 have the same membership functions as the failure possibilities in the class 1 and in the class 11, respectively.

Table 3 EXPRESSIONS OF DEPENDENCE LEVEL ESTIMATE AND ITS FUZZINESS

Level	Expressions of Dependence Level Estimate	
1	complete dependence	
2	high dependence	
3	medium dependence	
4	low dependence	
5	zero dependence	
Class	**Expressions of Fuzziness**	**Parameter mr**
1	low	2.0
2	medium	2.5
3	rather high	3.0
4	high	3.5

2.4. Natural Language Expressions of Analysis Result

In this model the analysis result is expressed by natural language. Let F_R be the failure possibility obtained by the analysis and F_S be the failure possibility with the membership function (1). The distance between F_R and F_S is defined by

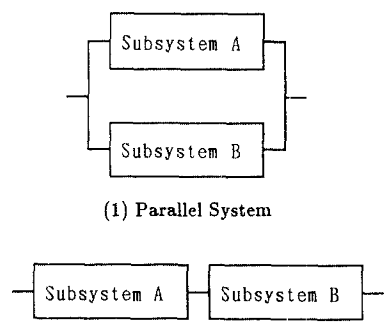
$$d = \int_0^1 \sqrt{(x_{1R}(\alpha)-x_{1S}(\alpha))^2+(x_{2R}(\alpha)-x_{2S}(\alpha))^2}\,d\alpha \tag{9}$$

where α-cuts[5] of F_R and F_S are defined by $(F_R)_\alpha=(x_{1R}(\alpha),\ x_{2R}(\alpha))$ and $(F_S)_\alpha=(x_{1S}(\alpha),\ x_{2S}(\alpha))$, respectively.

The parameters x_0 and m of F_S are selected so as to minimize the distance, and natural language expressions are selected from Tables 1 and 2.

3. DETERMINATIONS OF PARAMETERS

3.1. Parameters nH and nG



(1) Parallel System

(2) Series System

Fig. 2 PARALLEL SYSTEM AND SERIES SYSTEM

Let reliability estimates of the subsystems in Fig. 2 and their fuzziness be *standard reliability* and *medium*, respectively, where the subsystems are assumed to be independent of each other. Parameters nH and nG are determined by analysts' reliability estimates of the systems in Fig.2. For example, when an analyst estimates reliability of the parallel system and that of the series system to be *high*(x_0=0.2) and *standard*(x_0=0.55), respectively, the estimates lead to his determination of the parameters nH=2.5 and nG=2.5.

3.2. Parameters r_0 and mr

The parameter mr is selected from Table 3, i.e., natural language expressions of fuzziness of the dependence level estimate.

The parameter r_0 is determined by the following way. Let reliability estimates of the subsystems in Fig.1 and their fuzziness be *standard reliability* and *medium*, respectively. When the dependence level is estimated to be *complete*, reliability of the system and its fuzziness are estimated to be *standard* and *medium*, respectively. On the other hand when the dependence level is estimated to be *zero*, reliability of the system is estimated by the operation H. Let the parameter x_0 of the failure possibility F_S be x_{0i}, where F_S with x_{0i} is the failure possibility of the system with the dependence level i(i = 1 ; *complete*, i = 2 ; *high*, i = 3 ; *medium*, i = 4 ; *low*, i = 5 ; *zero*). And let the parameter r_0 of the fuzzy set $R(r)$ representing the dependence level i be $r_{0i}(i = 2, 3, 4)$. The parameters $r_{0i}(i = 2, 3, 4)$ are determined so as to satisfy

$$(x_{0i}-x_{05}):(x_{01}-x_{0i}) = \begin{cases} 3:1 & (i = 2) \\ 1:1 & (i = 3). \\ 1:3 & (i = 4) \end{cases} \tag{10}$$

For example, let the parameters nH and nG be 2.5 and 2.5, respectively. When the dependence level is estimated to be *complete*, reliability of the system shown in Fig.1 is estimated to be *standard* (x_{01}=0.5). When the dependence level is estimated to be *zero*, reliability of the system is estimated to be *high*(x_{05}=0.20). In this case the parameters $r_{0i}(i = 2, 3, 4)$ are determined such as

$$\left\{\begin{array}{l} r_{02} = 0.99 \\ (x_{02} = 0.43,\ (x_{02} - x_{05}):(x_{01} - x_{02}) = 0.23:0.07 \fallingdotseq 3:1) \\ r_{03} = 0.72 \\ (x_{03} = 0.35,\ (x_{03} - x_{05}):(x_{01} - x_{03}) = 0.15:0.15 = 1:1) \\ r_{04} = 0.63 \\ (x_{04} = 0.28,\ (x_{04} - x_{05}):(x_{01} - x_{04}) = 0.08:0.22 \fallingdotseq 1:3) \end{array}\right. \tag{11}$$

4. EXAMPLE

Fig.3 shows an example of the analyzed fault tree. Let us assume that two analysts analyze this tree and that reliability estimate of each basic event and its fuzziness are *standard reliability* and *medium*, respectively. It is also assumed that the parameters nH and nG, and the dependence estimates are different between two analysts as shown in Table 4.

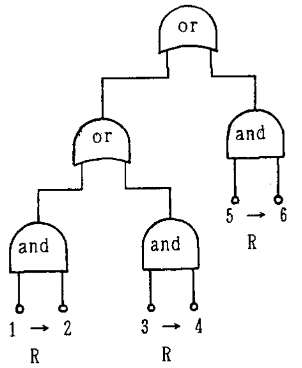


Fig. 3 EXAMPLE OF FAULT TREE

Table 4 Dependence Estimate and Parameters nH and nG

(Dependence Level, Fuzziness)		(nH, nG)
first	(medium, medium)	(2.05, 1.0)
second	(low, medium)	(2.5, 2.5)

Table 4 implies that the first analyst's reliability estimates of the parallel system shown in Fig.2(a) and the series system shown in Fig.2(b) are *high*(x_0=0.25) and *rather low*(x_0=0.63), respectively. On the other hand the second analyst's estimates of them are *high*(x_0=0.2) and *standard*(x_0=0.55), respectively. It is found that the first analyst is more pessimistic than the second analyst, that is, the first analyst's reliability estimate of this system is lower than the second analyst's. Fig.4 shows analysis results of two analysts. The first analyst estimates system reliability to be *rather low* and its fuzziness to be *medium*(Fig.4(1)). On the other hand the second analyst estimates system reliability to be *rather high* and its fuzziness to be *medium*(Fig.4(2)). It is found that the results reflect analysts' subjectivity.

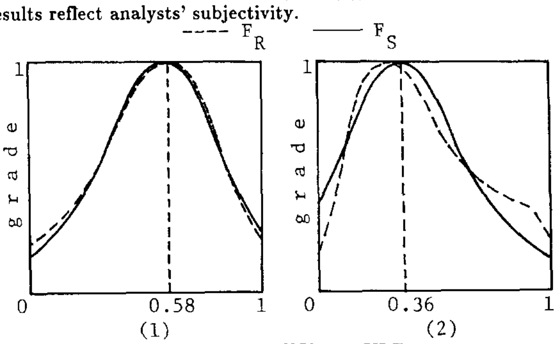


Fig. 4 ANALYSIS RESULTS

5. MUTUAL AGREEMENT

Our previous model[4] have considered only subjectivity of reliability estimate and of dependence level estimate, e.g., subjectivity of reliability estimate of each basic event and subjectivity of dependence level estimate between events in Fig.3. Then the same reliability estimate of each basic event and the same dependence level estimate between events lead to the same analysis result, i.e., the same reliability estimate of the whole system. In practice, however, the same estimates of each basic event reliability and of dependence level do not necessarily lead to the same reliability estimate of the system, since system reliability estimate depends on the analysts' subjectivity toward the analyzed system. The present model deals with this problem by the use of the parameterized operations of fuzzy sets. The parameterized operations can reflect analysts' subjectivity as mentioned before, while there exists a problem that various results, which reflect analysts' subjectivity, are obtained. Even if the analysis results can be classified by natural language expressions of the results, it is necessary to consider further this problem. In this section as one of the solutions of the problem two kinds of mutual agreements are considered. Let F_{Rj} (j=1,2,---,n) be the failure possibility obtained by the j-th analyst, where n is the number of analysts.

5.1. Intersection and Union of F_{Rj}

The intersection and the union of F_{Rj} are defined by $F_I = \cap_j F_{Rj}$ and $F_U = \cup_j F_{Rj}$. A normalized F_I is defined by

$$F_{NI} = F_I / MF_I, \tag{12}$$

where MF_I is the maximal grade of F_I.

Let F_{CU} be the result of the convexity operation on F_U [5]. And let F_{SNI} and F_{SCU} be fuzzy sets which minimize the distance between F_S and F_{NI}, and that between F_S and F_{CU}, respectively, in the sense of Eq. (9). F_{SNI} and F_{SCU} are expressed by natural language in the form of reliability estimate and its fuzziness as mentioned in 2.4.

F_{SNI} is interpreted as the least part of the result which all analysts agree with one another because F_{SNI} includes only the part of the results which all analysts agree with one another. Then the small maximal grade of F_I means the small grade of mutual agreement among the analysts.

On the other hand F_{SCU} is interpreted as the largest part of the result which all analysts agree with one another because F_{SCU} includes the results obtained by all analysts. Then high fuzziness of F_{SCU} means large difference of reliability estimates among the analysts.

The failure possibility in Fig.5(1) is the intersection of F_{Rj} in the example of the previous section. This implies that *system reliability is standard and its fuzziness is low*. In this example MF_I=0.75. The failure possibility in Fig.5(2) is the union of F_{Rj}. This implies that *system reliability is standard and its fuzziness is high.* The results show rather large difference of reliability estimates between two analysts.

The intersection and the union of F_{Rj} have a tendency to show a mean reliability estimate among F_{Rj}.

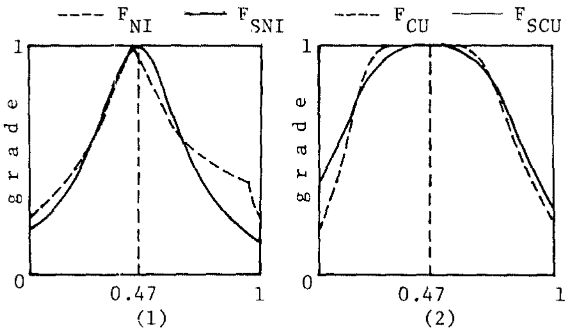


Fig. 5 INTERSECTION AND UNION OF F_{Rj}

5.2. Weighted Average of F_{Rj}

Let W be the weight which means analyst's belief of the result. Five kinds of expressions about the belief are considered as shown in Table 5 and their meanings are expressed by fuzzy sets with the membership function (13).

$$W(w)=\frac{1}{1+20\times|w-w_0|^2}, \tag{13}$$

where w_0 is a parameter and its numerical value is also shown in Table 5.

Table 5 Expressions of Belief and Parameter w_0

Expressions of Belief	Parameter w_0
	(Grade of belief is)
very high	0.9
high	0.75
medium	0.5
low	0.25
very low	0.1

The weighted average of F_{Rj} is defined by

$$F_{WA}=\frac{\Sigma_j\ F_{Rj}\times W_j}{\Sigma_j\ W_j} \tag{14}$$

where W_j is the j-th analyst's weight and the extension principle is used.

Let F_{SWA} be the failure possibility which minimizes the distance between F_S and F_{WA} in the sense of Eq.(9). F_{SWA} is expressed by natural language as mentioned in 2.4.

Fig.6(1) shows the weighted average of F_{Rj} in the example of the previous section, where the grade of both analysts' belief is *very high.* In this case system reliability is expressed by natural language such as *system reliability is standard and its fuzziness is medium.* Fig.6(2) shows the weighted average, where the grade of the first analyst's belief is *medium* and the grade of the second analyst's belief is *very high.* In this case system reliability is expressed by natural language such as *system reliability is rather high and its fuzziness is medium.* If analysts have various weights, the weights reflect the weighted average of F_{Rj}. However the same weights lead to a mean reliability estimate among F_{Rj} as the intersection and the union of F_{Rj}.

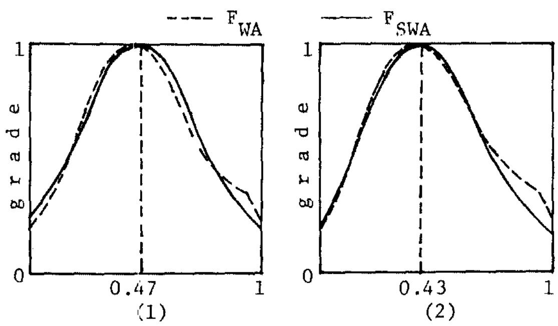


Fig. 6 INTERSECTION AND UNION OF F_{Rj}

6. CONCLUDING REMARKS

This paper mentions a model of subjective reliability analysis. This model uses a fuzzy set and natural language expressions, and considers subjectivity in fuzzy sets operations as well as in reliability estimate and dependence level estimate. The parameterized fuzzy sets operations can reflect analysts' subjectivity toward the analyzed system. So this model can obtain many different analysis results which reflect the analysts' consideration whether the analyzed system is dangerous or not. The model classifies the many different results into a few classes by the natural language expressions. However there exists the problem that it becomes difficult to discuss system reliability based on the obtained different results. This paper also considers two kinds of mutual agreements based on the results of the subjective analysis. One is the intersection and the union of the failure possibilities obtained by the analysis. The intersection and the union are interpreted as the least part of the result and the largest one, respectively, which all analysts agree with one another. The intersection and the union of various obtained failure possibilities have a tendency to show their mean estimate. The other is the weighted average of the failure possibilities. The weighted means the grade of analysts' belief toward the result. Various weights reflect the weighted average of reliability estimates. However the same weights lead to the mean estimate of system reliability as the intersection and the union.

This paper discusses the mutual agreement based on the results of subjective reliability analysis. The mutual agreement of a fault tree construction may be also considered. At any rate, it is necessry to discuss safety of an analyzed system because subjectivity is inherent in estimate of system safety and the difference of subjectivity leads to the various system safety estimates. The probabilistic method of a system reliability analysis eliminates subjectivity from the analysis. In this sense subjective reliability analysis may be against the consideration in the probabilistic method. However subjectivity is inherent in the analysis. It is necessary to consider the subjectivity openly in the analysis rather than to hide the subjectivity behind the numerical value, e.g., probability. This paper considers system reliability from this point of view.

REFERENCES

[1] R. Kenarangui, "Event-Tree Analysis by Fuzzy Probability," *IEEE Trans. on Reliability,* Vol.R-40, No.1, pp.120-124, 1991.

[2] M. Motoka and T. Ohashi, "Risk Analysis of Die-life Shortening with Fuzzy-Language Computations," *Proc. of Korean-Japan Joint Conference on Fuzzy Systems and Engineering, Seoul,* pp.284-287, August 1992.

[3] T. Onisawa, "A Model of Subjective Reliability Analysis," *Proc. of Second IEEE International Conference on Fuzzy Systems, San Francisco,* Vol.2, pp.756-761, March 1993.

[4] T. Onisawa, "Fuzzy Reliability Analysis Using Natural Language Information and Numerical Information," *Proc. of the 4th IFSA Congress, Brussels,* Vol.Eng., pp.163-166, July 1991.

[5] D. Dubois and H. Prade, *Fuzzy Sets and Systems:Theory and Applications,* Academic Press, New York, pp.19-20, pp.36-37, pp.25-26, 1980.

[6] M. Mizumoto, "Pictorial Representations of Fuzzy Connectives, Part I:Cases of t-norms and t-conorms, and Averaging Operators," *Fuzzy Sets and Systems,* Vol. 31, No. 2, pp. 217-242, 1989.