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On Necessity-Valued Petri Nets

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Abstract : We present here two Petri nets formalisms that can deal with uncertainty by the use of necessity-valued logic. The first and basic model, called necessity-valued Petri nets (NPN), can at the same time deal with uncertainty on markings and on transitions. The second model, called necessity-valued Petri nets (TNPN), is an extension of both NPN and timed Petri nets.

1 - Introduction

One of the recent topics developed in the Petri nets research field has been the treatment of uncertainty. Some models, like the ones proposed in [1] and [2] introduce fuzzy temporal constraints [3] in a Petri net formalism and use imprecise and uncertain markings to monitor flexible manufacturing systems. A similar approach has been used in [4] to model fuzzy programmable logic controllers. In [5], [6], [7], we find studies on how to transform rule-based systems, in which the knowledge bases are pervaded with uncertainty, into Petri nets. In [8] fuzzy Petri net languages are discussed.

In this work, we propose to use elements of propositional possibilistic logic to introduce uncertainty in Petri nets models. In the following section we describe the basic concepts in Petri nets used here. We then briefly expose some fundaments of necessity-based logic, and give a formulation of necessity-based Petri nets and timed necessity-based Petri nets. We conclude with a brief comparison of our models to related works found in the literature.

2 - Basic definitions on Petri nets

A Petri net is a directed graph containing two types of nodes, - places and the transitions - , usually associated to conditions and events respectively (see [9] for a first contact with Petri nets). Places are graphically represented by circles and transitions by bars. A token contained in a place, at a given moment of time, means that a condition associated to the place is satisfied at that moment. The distribution of tokens in the places in a given moment of time is called a *marking* of the net (see Fig. 1.a). A transition is *enabled* when each one of its input places contains the number of required tokens labeled on the arcs (when only one token is required we do not label the arc). At each step of the the execution of the Petri net, one of the enabled transitions is chosen to be fired, generating a new marking. We may also associate external conditions to a transition (e.g. sensors). In this case, the transition only fires if it is enabled and the conditions are true. Formally, a *Petri Net* is defined by a quadruple

$$PN = \langle P, T, I, O\rangle$$

where :

$P = \{p_1, p_2, ..., p_n\}$ is a finite set of places,
$T = \{t_1, t_2, ..., t_k\}$ is a finite set of transitions,
$I : T \rightarrow 2^P$ is the input mapping from transitions to sets of places,
$O : T \rightarrow 2^P$ is the output mapping from transitions to sets of places.

Let p be a place in P, and t be a transition in T. Then $p \in I(t)$ indicates that p belongs to the set of input places of transition t, and accordingly $p \in O(t)$ indicates that p belongs to the set of output places of transition t.

In this work we only consider safe Petri nets, which are those where the number of tokens in a place cannot exceed 1. A *marked safe Petri net* is a pair $N = <R, M_0>$, where R is a Petri net, and $M_0 : P \rightarrow \{0,1\}$ is the initial marking of R. The notation $M_i \rightarrow_t M_{i+1}$ expresses that from marking M_i we obtain marking M_{i+1} through the firing of transition t. M_{i+1} is defined as

$$M_{i+1}(p) = 1, \quad p \in O(t),$$
$$M_{i+1}(p) = 0, \quad p \in I(t) \text{ and } p \notin O(t),$$
$$M_{i+1}(p) = M_i(p), \quad \text{otherwise.}$$

The marking depicted in Fig. 1.a is given by $M_0(p_1) = 0$, $M_0(p_2) = 1$, $M_0(p_3) = 1$, which can be synthetically described by $M_0 = (0\ 1\ 1)$.

A *sequence* of transitions $S = s_1...s_k$, $s_i \in T$, is the concatenation of k transitions fired from an initial marking M_0 ; where s_i denotes the i-th transition fired from M_0. For instance, the sequences of length 2 that we can obtain from the marked net depicted in Fig. 1.a are $S = s_1s_2 = t_2t_1$, and $S' = s_1's_2' = t_4t_1$.

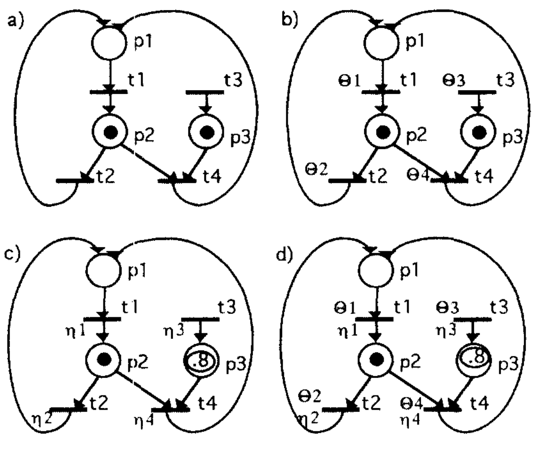

Fig 1- Examples of Petri net formalisms ($\theta_i=\theta(t_i)$, $\eta_i = \eta(t_i)$),
1.a) PN, 1.b) TPN, 1.c) NPN, 1.d) TNPN.

In the kind of Petri nets considered so far there is no concern with time. In fact, the transitions are considered to be instantaneous. One of the Petri net models that deal explicitly with time is the *Timed Petri Net* [12], defined by a quintuple

$$TPN = <P, T, I, O, \theta>$$

where P, T, I, O are defined as in PN, and $\theta : T \rightarrow \mathcal{T}$ associates to each transition the length of time required to accomplish it taken from a time scale $\mathcal{T}$ (see Fig. 1.b). The tokens used in transition t during the time span $\theta(t)$ are not visible anywhere. When a token becomes visible in a place it may be immediately used by any transition having that place as input. For instance, let $M_0 = (0\ 1\ 1)$ be the initial marking at time $\mathcal{T}_0$ on the Petri net depicted in Fig. 1.b. For sequence $S = s_1s_2 = t_2t_1$ a token will be visible at place p_1 at time $\mathcal{T}_1 = \mathcal{T}_0 + \theta(t_2)$, and the net will return to the initial configuration at time $\mathcal{T}_2 = \mathcal{T}_0 + \theta(t_2) + \theta(t_1)$.

3 - Necessity-valued Petri Nets and Timed Necessity-valued Petri Nets

Necessity-valued logic [11], also called PL1, is a specific type of possibilistic logic, in which to each first-order formula φ, representing a statement in a knowledge base, we associate a constraint $N(\varphi) \geq \alpha$, where N is a necessity measure (see [12] for a detailed study in possibility theory, and [11] for a survey in possibilistic logic). Here we are only interested on the case where the φ's are propositional formulae. The constraint $N(\varphi) \geq \alpha$ in PL1 is represented by the pair $(\varphi\ \alpha)$, called a *necessity-valued formula.* The quantity α is called the valuation of formula φ and is denoted by val(φ). Here we call $(\varphi\ \alpha)$, a *necessity-valued proposition* when formula φ consists of a single proposition. In necessity-valued logic we make extensive use of some important properties relative to necessity measures :

$$N(\varphi \wedge \neg\varphi) = 0 \ ; \ N(\varphi \vee \neg\varphi) = 1 \ ;$$

$$N(\varphi \wedge \psi) = \min(N(\varphi), N(\psi)) \ ; \ N(\varphi \vee \psi) \geq \max(N(\varphi), N(\psi))$$

In PL1, the classical modus ponens rule has been extended to the *graded modus ponens* defined as

$$(\varphi\ \alpha), (\varphi \rightarrow \psi\ \beta) \vdash (\psi\ \min(\alpha, \beta))$$

where → denotes the classical logical implication. Expression $(\varphi\ \alpha)$ is here called the *minor necessity-valued premise* and $(\varphi \rightarrow \psi\ \beta)$ the *major necessity-valued premise.*

Let us now see how these concepts can be introduced in a Petri net formalism $PN = <P,T,I,O>$. Let us suppose that it is not well-known if a transition t from a set of places I(t) to a set of places O(t) will be enabled when all the places in I(t) contain tokens. The necessity measure giving the uncertainty that transition t will fire can be represented by the necessity-valued formula $(i_1 \wedge i_2 \wedge ... \wedge i_k \rightarrow o_1 \wedge o_2 \wedge ... \wedge o_n\ \beta)$, where $i_j \in I(t)$ and $o_l \in O(t)$. This formula corresponds in fact to n necessity-valued formulae $(\varphi \rightarrow o_l\ \beta)$, where $\varphi = i_1 \wedge ... \wedge i_k$. The Petri net formal definition can then be extended to incorporate uncertainty by attaching valuation β to its corresponding transition, similarly to what is done with durations in timed Petri nets. Let us now suppose that the exact initial distribution of tokens in the set of places P is not well-known. The necessity measure giving the uncertainty that a token is in a place p can be represented by the necessity-valued proposition ($p\ \alpha$). We incorporate this information in the Petri net formalism by assigning α to $M_0(p)$, in the initial marking of the net.

A *necessity-valued Petri net* (NPN) is then formally defined by

$$NPN = <P, T, I, O, \eta>$$

where P, T, I and O are defined as given before and $\eta : T \rightarrow [0,1]$ associates a valuation to each transition. In this framework, the initial marking for each place p thus consists of the lower bounds on the necessity measures giving the uncertainty that a token is in p. $M(p) = 1$, which stands for $N(p) \geq 1$, means that a token is certainly inside the place, and will be graphicaly represented by a filled circle, as with the usual Petri nets formalisms. $M(p) = 0$, which stands for $N(p) \geq 0$, will be represented by the absence of any symbols inside the place. This does not mean that it is impossible that p contains a token, it only means that the marking on p is not informative. For the intermediate values of M(p), the marking on p is graphically represented by a circle containing the value M(p) (see Fig. 1.c).

In this model a transition t is enabled when $\eta(t) > 0$ and $M(p) > 0$, $\forall\ p \in I(t)$, i.e. when necessity measures related both to the dynamic valuations on the places (represented by the markings), and to the transitions are informative. Let M_0 and M_i respectively be the initial and the i-th marking of an NPN. Let $M_i \rightarrow_t M_{i+1}$. Marking M_{i+1} is defined by

$$M_{i+1}(p) = \min(\inf_{b \in I(t)} M_i(b), \eta(t)), \quad p \in O(t),$$
$$M_{i+1}(p) = 0, \quad p \in I(t), p \notin O(t),$$
$$M_{i+1}(p) = M_i(p), \quad \text{otherwise.}$$

The expression $\inf_{b \in I(t)} M_i(b)$ corresponds to the evaluation of a conjunction of a set of necessity-valued propositions ($p\ \alpha$) at step i. The expression $\min(\inf_{b \in I(t)} M_i(b), \eta(t))$ corresponds to the graded modus ponens ; the first term corresponds to the minor premise and the second to the major premise. Note that the marking definition in NPN's reduce to that of PN's when only certain valuations (=1) are involved in a firing. As an example, let us consider the Petri net in Fig. 1.c with $\eta(t_1) = .7$, $\eta(t_2) = \eta(t_3) = \eta(t_4) = 1$. For the sequence $S = t_2 t_1 t_4$ with $M_0 = (0\ 1\ .8)$, we have $M_1 = (.7\ 0\ .8)$, $M_2 = (0\ .7\ .8)$, and $M_3 = (.7\ 0\ 0)$.

A marking M in any Petri net should not contain tokens in places representing contradictory conditions. For instance, if we have two places p_1 and p_2 representing conditions φ and $\neg\varphi$, we should make sure that we do not have $M(p_1) = 1$ and $M(p_2) = 1$ at the same time. A marking in NPN's should be even more restricted, not allowing $M(p_1) > 0$ and $M(p_2) > 0$ occur at the same time. This should be done to guarantee coherence, since in possibilistic logic when $N(\varphi) > 0$ we have $N(\neg\varphi) = 0$.

It is important to note that, although feasible, not all applications require the modeling of uncertainty in both transitions and initial markings. Indeed, we may conceive situations where the initial marking is certain and the transitions are uncertain, - corresponding for instance to ill-known satisfiability of external conditions to the firing - , or where all transitions are certain, but where there is only an imperfect knowledge of the initial localization of tokens in the places.

Until now, transitions (certain or not) in NPN's are considered to be instantaneous. We now propose a necessity-based Petri net formalism that is capable of dealing with transitions requiring an amount of time to be completely accomplished. A *timed necessity-valued Petri net* (TNPN) is then formally defined by

$$TNPN = <P, T, I, O, \theta, \eta>$$

where P, T, I, O, θ, and η are defined as given before (see Fig. 1.d). In TNPN's the markings are defined as in NPN's, and time is treated as in TPN's.

The uncertainty described by TNPN may have diferent meanings depending on the joint interpretation of θ and η. A first interpretation lets the duration θ be exact, and η concerns *firing conditions*. For instance, let us suppose that it is ill-known if some external conditions on a transition t are true. Then it will also be ill-known if t will fire when all the places in I(t) contain tokens. The uncertainty on t would then correspond on the statement the "there is a necessity α that transition t will fire, and in this case it shall take exactly x minutes". Another interpretation situates uncertainty in the *duration* of the firing, corresponding for instance to a statement of the kind "there is a necessity α that the firing of transition t takes x minutes". The uncertainty in the last statement is related to the time spent for the transition to be completed, and not to the satisfiability of the conditions associated to the firing.

Let the Petri net of Fig. 1.d be such that $\eta(t_1) = .7$, $\eta(t_2) = \eta(t_3) = \eta(t_4) = 1$, and $\theta(t_1) = 1h$, $\theta(t_2) = 2h$, $\theta(t_3) = 0h$, $\theta(t_4) = 4h$. Let us suppose that we can yield an initial marking $M_0 = (0\ 1\ .8)$ corresponding to 5.5 hours ago. Let us suppose that we want to know the maximal plausibility that place p_1 contains a token, and that the last marking is kept until the end of the current transition. We have 3 possible sequences with length of time greater than 5.5h. With $S = t_2t_1t_4$, transition t_4 is firing, and the last marking was (0 .7 .8). With $S' = t_2t_1t_2t_1$, t_1 is firing, and the last marking was (.7 0 .8). With $S'' = t_4t_2t_1$, t_2 is firing, and the last marking was (0 .8 0). Then the maximal plausibility (necessity) that there is a token in place p_1 now is .7.

4 - Conclusion

We have presented two Petri nets formalisms based on necessity-valued logic, which is a special kind of possibilistic logic. The first and basic model, called necessity-valued Petri nets (NPN), can at the same time deal with uncertainty on the markings and on the transitions. When uncertainty is present in both the markings and the transitions NPN's can be used to model rule-based systems, requiring only some slight modifications on the markings. Let $M_i \rightarrow_t M_{i+1}$. Then, for all $p \in O(t)$, we will have $M_{i+1}(p) = max(M_i(p), min(inf_{b \in I(t)} M_i(b), \eta(t)))$. This scheme thus represents an alternative approach to those used in [5] and [6].

The second model, called necessity-valued Petri nets (TNPN), is an extension of both NPN and timed Petri-nets. Although dealing with time and possibilistic logic, it is not a direct application of *timed possibilistic logic* [13], in which we deal with dates (e.g. "the lights on the production plant will be on certainly after 8A.M., and before 6A.M."). The model proposed here cannot deal with statements of the type "the transition will take around x minutes to be completed". It has thus less expressive power than the model proposed in [1] and [2], which makes use of fuzzy temporal constraints [3] ; it is however of much easier manipulation. It also represents a more formal alternative approach to that used in [7].

Future research on NPN's and TNPN's include extension to other models of Petri nets, - such as Petri nets with objects for instance - , and a deep study of the relations between TNPN's and timed possibilistic logic.

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