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Representation of Spatial Relations between Regions in a 2D Segmented Image

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Abstract: We are concerned with developing a robust method for comprehensive scene analysis. In particular, we address the problem of representing spatial relations between regions in a segmented 2D image. Spatial relations are modeled as fuzzy sets. The method has two aspects, symbolic and quantitative, consisting of assigning labels for spatial relations and numeric degrees to which a relation holds respectively. The procedure of deriving a spatial relation is hierarchical taking into account geometric/physical characteristics of the regions in question.

1. Introduction. Image understanding has emerged as an important field of information processing. In particular, it is important to enable machines to recognize the contents of photographs and express it in a way understandable for a user. By contents of a photograph we mean the collection of objects in the photograph as well as the spatial relations between these objects. Even at the level of object recognition spatial relations play an important role. In fact, our current work and interest arose from prior work on object recognition. The top-down model-based object recognition method proposed in [2] was extended in [3] to include spatial relations. We showed that when spatial relations between parts of the object to be recognized were included in the model, the recognition can be both faster and more accurate. While in describing spatial relations linguistic expressions such as *"above"*, *"upper left"*, *"lower right"* etc. are appropriate, the detection of spatial relations is to a large extent a matter of degree. In [3] the degree to which a spatial relation specified in the model held was assessed by a human user. In the current paper we aim at developing an automated method for assessing the degree of spatial relation between two regions/objects, etc.

Furthermore our method proposes that when detecting, or describing spatial relations geometric/physical characteristics of the regions under consideration must also be taken into account. This approach constitutes a sharp departure from previous research in this area such as [4] where spatial relations were defined between points only.

2. Hierarchy of spatial relations between points

2.1. Spatial relations between points. At the level of points spatial relations can be defined in a straightforward manner based on the coordinates of the points considered. Moreover these definitions can be very sensitive to variations in the relative positions of the two points while also making possible to group some positions under the same linguistic label for the spatial relation. To illustrate this let θ be the angle between the line connecting the points Pi, i=1,2, and the positive x-axis. Let denote R a generic relation between P1 and P2. For simplicity, and without loss of generality we assume that P1 is the origin of the system of coordinates. Then the position of P2 with respect to P1, expressed as "P2 is in relation R to P1" will be characterized by $\sin^2\theta$ and $\cos^2\theta$ as follows:

R="*right of*" where

$$\mu_{\text{"right of P1"}}(\theta;P2)=\begin{cases}\cos^2(\theta) & 0\le\theta\le\frac{\pi}{2},\ \frac{3\pi}{2}\le\theta<2\pi\\ 0 & otherwise\end{cases} \quad (1)$$

R="*left of*" where

$$\mu_{\text{"left of P1"}}(\theta;P2)=\begin{cases}\cos^2(\theta) & \frac{\pi}{2}\le\theta\le\frac{3\pi}{2}\\ 0 & otherwise\end{cases} \quad (1')$$

R="*above*" where

$$\mu_{\text{"above P1"}}(\theta;P2)=\begin{cases}\sin^2(\theta) & 0\le\theta\le\pi\\ 0 & otherwise\end{cases} \quad (2)$$

R="*below*" where

$$\mu_{\text{"below P1"}}(\theta;P2)=\begin{cases}\sin^2(\theta) & \pi\le\theta<2\pi\\ 0 & otherwise\end{cases} \quad (2')$$

The relations defined in (1),(1'), (2), and (2'), shown in Fig. 1 are thought of as primitive relations.

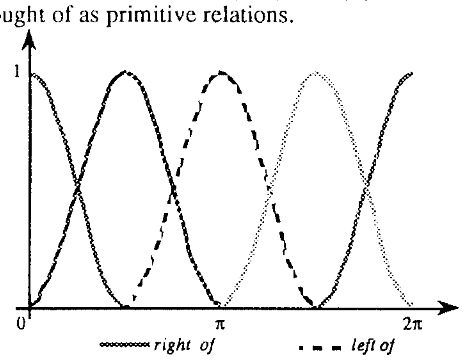


Fig. 1: Primitive spatial relations

Other relations will be defined or expressed in terms of these primitive relations. We can group (1) and (1') under the heading "lateral" or "horizontal position", and (2) and (2') under the heading of "*vertical*" relation by defining:

$$\mu_{\text{"lateral of P1"}}(\theta; P2) = \cos^2(\theta) \qquad (3)$$

$$\mu_{\text{"vertical of P1"}}(\theta; P2) = \sin^2(\theta) \qquad (4)$$

Symbolically the relations *"lateral of"*, and *"vertical of"* are given by the expressions:

"P2 is *lateral of* P1" = "P2 *is left of* P1" **OR** "P2 is *right of* P1"

"P2 is *vertical of* P1" = "P2 *is above* P1" **OR** "P2 is *below* P1"

The relations *"lateral of"*, and *"vertical of"* are shown in Fig. 2:

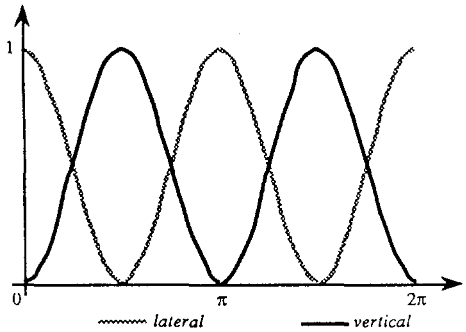


Fig. 2: The spatial relations *"lateral"*, and *"vertical"*.

Remarks:

The spatial relations (1), (1'), (2), (2') are not symmetric. In fact, from the properties of the trigonometric functions we have:

$\mu_{\text{"above P1"}}(\theta; P2) = \mu_{\text{"below P2"}}(\pi+\theta; P1)$ and

$\mu_{\text{"left of P1"}}(\theta; P2) = \mu_{\text{"right of P2"}}(\pi+\theta; P1)$

The spatial relations (3) and (4) are symmetric:

$\mu_{\text{"vertical of P1"}}(\_; P2) = \mu_{\text{"vertical of P2"}}(\_; P1)$

$\mu_{\text{"lateral of P1"}}(\_; P2) = \mu_{\text{"lateral of P2"}}(\_; P1)$

From the basic trigonometric identity: $\sin^2\theta + \cos^2\theta = 1$ and with the standard operator for the complement of a fuzzy set, we have that "*vertical of*" = NOT("*lateral of*"). Actually, "*vertical of*" and "*lateral of*" form a (minimal) fuzzy partition of the universe of discourse for θ, [0, 2π]. More composite spatial relations can be defined. The relation "*oblique*" will be defined as:

$$\mu_{\text{"oblique P1"}}(\theta, P2) = \begin{cases} 1 & \text{if } \mu_{\text{"vertical P1"}}(\theta, P2) = \mu_{\text{"lateral P1"}}(\theta, P2) \\ 0 & \text{if } \mu_{\text{"vertical P1"}}(\theta, P2) = 1 \text{ or } \mu_{\text{"lateral P1"}}(\theta, P2) = 1 \\ \text{piecewise linear interpolation} & \text{otherwise} \end{cases} \qquad (5)$$

This relation can be further refined into "*oblique right*", "*oblique left*" shown in Fig. 3. These relations are obtained from the relation *"oblique"* by replacing *"lateral of"* by its more specific instances of *"left of"*, *"right of"*. Note that it becomes useful to maintain a hierarchy of spatial relations, but this aspect is not considered in the current paper.

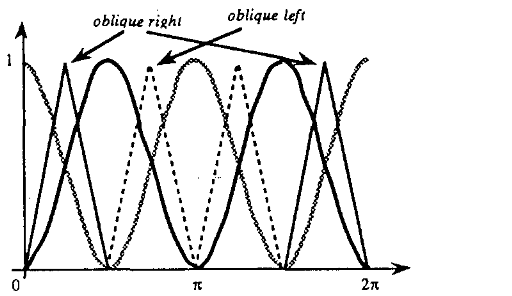


Fig. 3: Composite spatial relations *"oblique right"*, *"oblique left"*

The relations "*oblique right*", and "*oblique left*" can be used to distinguish between mixed spatial relations such as *"upper right"*, and "*to the right and up*":

"upper right" = *"above"* **AND** *"oblique right"*

"to the right and up" = *"right of"* **AND** *"oblique right"*

The membership functions of the newly defined relations (shown in Fig.4) are obtained according to the calculus of fuzzy sets.

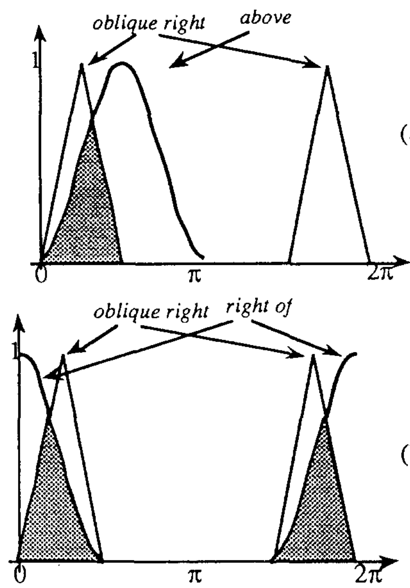


Fig. 4: Composite spatial relations: (a) *"upper right"*, (b) *"to the right and up"*

The impact of distance on spatial relations. So far, in our approach we did not take into consideration any factors which may affect the perception of spatial relations. For instance, the distance between the points P1 and P2 does not affect the spatial relation between them. Yet, from our daily experience it seems that the perception of spatial relations is influenced by other factors, such as distance, in the case of points, and distance, size, shape, etc. in the case of regions. But distances become meaningful only when we take into consideration the total size of the image (window size) in which the spatial relations are analyzed. To illustrate this let us consider the situation shown in Fig. 5:

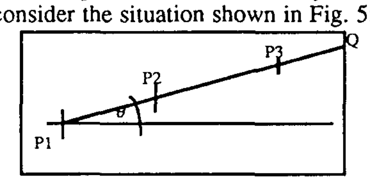


Fig. 5: Perception of spatial relations is affected by distance.

Since P1, P2, and P3 are co-linear, and according to our development so far, the spatial relations of P2 and P3 relative to P1 hold to the same degree. However, our perception is that P3 is both higher and more to the right of P1 than P2. This corresponds to the fact that we must 'travel' further both to the right and above P1 in order to reach P3 than in order to reach P2. To factor in the influence of the distance between points, for a given angle q, the spatial relation of a point P on the line of slope $\tan\theta$ passing through the point P1 we multiply the membership value by the ratio $\frac{d(P1, P)}{d(P1, Q)}$, where Q is (as shown in Fig. 5) the farthest point which can be reached from P1 in the direction of P. Thus we have the following:

$$\mu_{"R\ P1"}(\theta; P,Q)=\mu_{"R\ P1"}(\theta; P)\frac{d(P1, P)}{d(P1, Q)} \qquad (6)$$

3. Linguistic Representation of Spatial Relations

3.1. Spatial relations between regions. A standard approach to analyzing spatial relations between regions consists of reducing these to spatial relations between one or more representative points (such as the center of gravity) of the regions under considerations. However, this approach is unsatisfactory. Indeed, as it can be seen in Fig. 1 the regions A and B have the same centers of gravity in Fig. 1(a) and Fig. 1(b), but region A has different directions. Thus, even though based on the centers of gravity the relations "Region B is above Region A", and "Region A is left of Region B" should hold to the same degree in both cases we feel that the first should hold to a higher degree in the case (b) and the second should hold to a higher degree in the case (a):

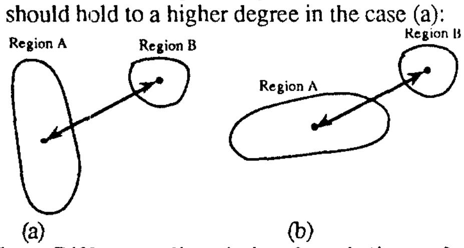
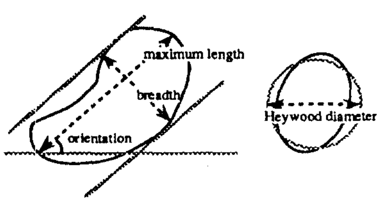


Fig. 6 : Different linguistic description of spatial relations according to different directions (same centers of gravity).

A similar argument can be made for considering the size and shapes of the regions under consideration. Thus **orientation, size and shapes** of regions will affect the degree to which spatial relations between regions hold. Moreover, we want these criteria to have a global effect. That is, reducing the regions to a collection of points (rather than to one point) will not be satisfactory.

3.2. Geometric characterization of 2D regions. Information on orientation, size and shape of regions can be calculated in terms of the geometric features which are extracted from regions as shown in Fig. 7:



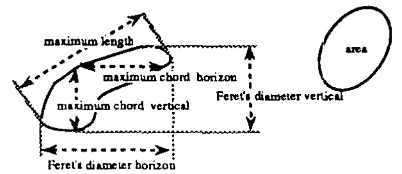


Fig.7 : Examples of geometric features of 2D shapes.

Based on the geometric features extracted the orientation is defined as the angle between the horizontal and the maximum length of the region; the size of the region is defined in terms of the number of pixels. Similarly, the shape can be defined in terms of diameters, vertical and horizontal chords.

3.3. Hierarchical definition of spatial relations between regions. To derive the spatial relation(s) between two regions A1 and A2 we propose the following algorithm:

1.Extract characteristics of each region:

Identify center of gravity: Gi, i=1,2.

Identify direction: Di, i=1,2.

Identify shape: Hi, i=1,2.

Identify size: Si, i=1,2.

2. Find a characterization (the angle θ) of the spatial relations between G1 and G2.

All the relations forming a fuzzy partition of the universe of discourse are evaluated at G1, G2. (Further work is necessary to identify the criteria of choosing among several fuzzy partitions).

3. Use fuzzy reasoning to modify the degrees identified in 2 as follows:

<u>Modification due to directions</u>: Calculate h(D1,D2), the difference between the directions D1 and D2. Modify the degrees by h(D1,D2).

<u>Modification due to size:</u> Calculate f(S1,S2) the difference between the sizes S1 and S2. Modify the degrees by f(S1, S2). In this step we also consider the f(Si, S) where S is the total size of the image (window size, frame size, etc.). The modifications are done such that if f(S1,S2), f(Si,S) are small then the spatial relations between regions are the same as the spatial relations between their centers of gravity.

<u>Modification due to shape</u>: Calculate g(H1,H2), the difference between the shapes H1 and H2. Modify the degrees by g(H1, H2).

Fuzzy reasoning is used in 1-3 to determine the direction and amount of each modification. If sizes, shapes and directions of the two regions are identical, that is the regions are identical but in different locations, the relations determined for their centers of gravity are used for regions as well.

4. Extract the final spatial relation(s) according to the results of step 3.

4. A Simple Example

We illustrate in this section the modifications due to different directions of the region. At the same time it will become obvious why shapes and sizes of regions must be considered as well, and why a fuzzy based reasoning is appropriate to carry out modifications. To indicate the type of results we are looking for we consider three different positions of two regions A and B as indicated in Figure 8. We assume that the viewer position and point of view coincides with the region A. In each case the centers of gravity, shapes and sizes of the two regions are the same, but the directions are different.

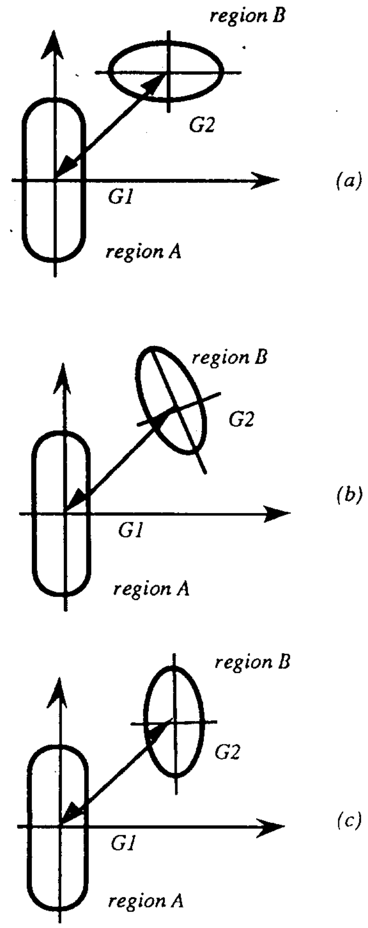


Fig.8: Different positions of two regions with unchanged ged sizes, shapes, and centers of gravity, but different directions.

The slope of the line connecting G1 and G2 corresponds to the angle $\theta=45$. Thus,

$\mu_{\text{"vertical G1"}}(\theta=45; G2)=0.5$	$\mu_{\text{"lateral G1"}}(\theta=45; G2)=0.5$
$\mu_{\text{"above G1"}}(\theta=45; G2)=0.5$	$\mu_{\text{"right of G1"}}(\theta=45; G2)=0.5$
$\mu_{\text{"below" G1"}}(\theta=45; G2)=0$	$\mu_{\text{"left of G1"}}(\theta=45; G2)=0$

It also follows that $\mu_{\text{"oblique right of G1"}}(\theta=45; G2)=1$. Let τ denote the angle made by the main axis of the region B and the horizontal axis of A. Then the angle between the main axis of A and that of B is $\alpha=(90-\tau)$. Thus in (a) $\alpha=90$, in (b) $\alpha=(90-\tau)$ and (c) $\alpha=0$. The perception of the spatial relations between the regions A and B in these three different cases is certainly different. Let $\mu_{\text{"above A"}}(B; y)$, and $\mu_{\text{"right of A"}}(B,y)$ denote the degree to which B is above and right of A respectively. The argument y can take on the value a, b, c according to which case we consider. Then, we have that

$$\mu_{\text{"aboveA"}}(B;a)\geq\mu_{\text{"aboveA"}}(B;b)\geq\mu_{\text{"aboveA"}}(B;c) \qquad (7)$$

$$\mu_{\text{"right ofA"}}(B;c)\geq\mu_{\text{"right ofA"}}(B;b)\geq\mu_{\text{"rightofA"}}(B;a) \qquad (7')$$

The above results can be obtained by changing θ into $\theta'=\theta+\alpha$, and applying (1) and (2) to θ'. It is easy to see that this procedure will not yield satisfactory results in all situations: for example if $\theta=0$, that is the relation "above" between G1 and G2 holds with degree 0 then changing the orientation of B should not always result in a degree >0 for the relation "above". In fact this relation should change according to the relative size of B with respect to A, the larger B is the more the relation should change upon rotations of B (which change its direction). To cope with changes of q as dictated by various criteria involving size, shape, etc. a fuzzy reasoning scheme should be used. In addition to finding the value of q satisfying all criteria the fuzzy reasoning can also take into account the subjective nature of (7) and (7'). Users may specify rules (heuristics) relating spatial relations to the angle between the directions of the regions under consideration. Currently we are experimenting with different ways of deriving such heuristics which will be implemented as fuzzy if-then rules.

Conclusion. According to [1] the necessity for treating spatial relations raises problems which are typical and of and special to vision. Here we propose to use fuzzy logic for describing and analyzing spatial relations between regions in a segmented 2D image. Spatial relations are represented as fuzzy sets. Spatial relations between regions are obtained by modifying spatial relations between representative points of the regions in question by taking into account directions, sizes, shapes, and distances. Fuzzy logic is also used to reason about spatial relations, and to derive composite spatial relations. In addition to a detailed treatment of the ideas proposed in this abstract the extended paper will present several experimental results.

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