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Fuzzy model of comprehensive evaluation and its applications

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Abstract: A fuzzy mathematical model is presented that can be applied to support the inspection of organizations by an internal or external evaluation group. The model offers the opportunity to deal with the situation of common practice in which such evaluations are considered as a time series rather than single events. A hypothetical but realistic example is given to illustrate the computational procedure involved.

A very important subject in management science is the modelling of comprehensive evaluation systems for management in enterprises, research institutes, project engineering and so on. Furthermore in an enterprise a rather detailed evaluation of staff performance is often required as an important part of an overall assessment of the business in a competitive world.

A fuzzy mathematical model of comprehensive evaluation is presented in part 1 of the paper. In part 2 the model is applied to a hypothetical but realistic example.

Part 1. The fuzzy mathematical model of comprehensive evaluation

The fuzzy mathematical model of comprehensive evaluation operates on the following vectors:

Factor vector $F = (f_1, f_2, \cdots, f_n)$.

Evaluation vector $E = (e_1, e_2, \cdots, e_m)$, in which e_1, e_2, $\cdots$, e_m are different qualitative grades. we must emphasize that the elements of the evaluation vector must be ordered from the best to the worst or from the worst to the best. The reason for this condition will be clear from the further description of the model.

Evaluation-time vector $T = (t_1, t_2, \cdots, t_r)$, in which the elements represent the successive points in time at which the evaluations are realized.

Weight-distribution vector with respect to F, $WF = (Wf_1, Wf_2, \cdots, Wf_n)$, such that

$$\sum_{i=1}^{n} Wf_i = 1$$

Weight-distribution vector with respect to T, $WT = (Wt_1, Wt_2, \cdots, Wt_r)$, such that

$$\sum_{j=1}^{r} Wt_j = 1$$

Let $r_{ij}^{(k)}$ be the membership degree of factor f_i with evaluation grade e_j at time t_k. Then we can associate with f_i a corresponding vector of membership degrees at t_k, i. e.

$$R_i^{(k)} = (r_{i1}^{(k)}, r_{i2}^{(k)}, \cdots, r_{im}^{(k)})$$
$$k = 1, \cdots, r \text{ and } i = 1, \cdots n \qquad (1)$$

or in matrix notation,

$$R_i = \begin{bmatrix} r_{i1}^{(1)} & r_{i2}^{(1)} \cdots r_{im}^{(1)} \\ \cdots\cdots\cdots\cdots\cdots \\ r_{i1}^{(k)} & r_{i2}^{(k)} \cdots r_{im}^{(k)} \\ \cdots\cdots\cdots\cdots\cdots \\ r_{i1}^{(r)} & r_{i2}^{(r)} \cdots r_{im}^{(r)} \end{bmatrix} \qquad i = 1, \cdots, n$$

in which the jth column vector

$$R_{ij} = \begin{bmatrix} r_{ij}^{(1)} \\ r_{ij}^{(2)} \\ \vdots \\ r_{ij}^{(r)} \end{bmatrix}$$

contains the membership degrees of factor f_i with grade e_j at the evaluation times t_1, t_2, $\cdots$, t_r in a proper order. With respect to the sequence of evaluation times t_1, t_2, $\cdots$, t_r, the weighted mean membership degree of factor f_i with evaluation grade e_j can be written as

$$r_{ij} = WT * R_{ij} = \sum_{k=1}^{r} Wt_k * r_{ij}^{(k)} \qquad (2)$$

For all the factors f_i and evaluation grade e_j the following evaluation matrix R of weighted mean membership degrees is obtained

$$R = \begin{bmatrix} r_{11} & r_{12} \cdots r_{1m} \\ r_{21} & r_{22} \cdots r_{2m} \\ \cdots & \cdots \\ \cdots & \cdots \\ r_{n1} & r_{n2} \cdots r_{nm} \end{bmatrix} \qquad (3)$$

In general to each of the factors in $F = (f_1, \cdots, f_n)$ some sub-factors can be associated that characterise certain aspects of the factor. In this case the evaluations occur at more than one level and the equations (1) and (2) are identified by extending the above computations in a natural way. This will be illustrated in the example of Part 2 of the paper.

In what follows it is convenient to put the following normalizing condition, which can always be satisfied in a practical problem,

$$\sum_{j=1}^{m} r_{ij}^{(k)} = 1 \quad i = 1, \cdots, n; \quad k = 1, \cdots, r$$

so that,

$$\sum_{j=1}^{m} r_{ij} = \sum_{j=1}^{m} \sum_{k=1}^{r} Wt_k * r_{ij}^{(k)} = \sum_{j=1}^{m} r_{ij}^{(k)} = 1$$

We introduce,

$$S = (s_1, s_2, \cdots, s_m) = WF * R \qquad (4)$$

and it readily follows that $\sum_{j=1}^{m} s_j = 1$. S is called a fuzzy comprehensive evaluation vector. In relation to the vector S the following two approaches for practical applications are obtained.

1. According to the maximum membership degree principle, we use the fuzzy comprehensive evaluation vector S to find the summary evaluation grade.

Suppose $s_k = \max_{1 \leqslant j \leqslant m} s_j$

(a) if $\sum_{j=1}^{k-1} s_j < 0.5$ and $\sum_{j=k+1}^{m} s_j < 0.5$, then the summary evaluation grade is e_k;

(b) if $\sum_{j=1}^{k-1} s_j \geqslant 0.5$, then the summary evaluation grade is e_{k-1};

(c) if $\sum_{j=k+1}^{m} s_j \geqslant 0.5$, then the summary evaluation grade is e_{k+1}.

If in the comprehensive evaluation vector there are more maximum elements, the method is still valid. In this situation every maximum number must be evaluated. For example, if $S = (0.1, 0.1, 0.35, 0.35, 0.1)$, then according to (b) the summary evaluation grade is e_3; if $S = (0.25, 0.25, 0.25, 0.25)$, then according to (b) and (c) the summary evaluation grade is e_2 or e_3.

2. Let $L = (l_1, \cdots, l_m)$ be the vector of scores obtained from some scoring system and such that L is compatible with $E = (e_1, e_2, \cdots, e_m)$. Then the total evaluation score is

$$Z = S * L^T = \sum_{j=1}^{m} s_j l_j$$

Part 2.

In what follows we give an example to illustrate how the approach outlined above can be implemented in practical problems.

NOSOGE (North Sea Oil and Gas Exploration) is an Association of twenty one companies with interests in the exploration and production of salt, and hydrocarbons from oil and natural gas at the Dutch continental flat of the North Sea.

An important department of NOSOGE is the Marine Information Service Center (MISC) for the gathering, processing and distribution of a wide range of information. For the on-line monitoring of the relevant conditions of the North Sea system, for example, data from an extensive measurement network are collected for further processing and analyses. Other types of information obtainable from MISC deal with companies and products, scientific publications, industrial management and data processing methods. MISC accommodates several data banks which can be searched separately or in combination to assist operational managers, researchers, planners and policy makers but also environmentalist groups.

We will briefly describe how the fuzzy model for comprehensive evaluations can be suitably used for the assessment of the performance of MISC as a central part in the NOSOGE Association. The following factors and their characterizing sub-factors can be identified.

Factor f_1 is a criterion for data collection, f_2 is a criterion for subject oriented search and f_3 is a criterion for service to clients. Note that these factors are associated with the input, processing and output of the MISC-unit respectively. Furthermore, the sub-factors f_{1j}, f_{2j} $(j = 1,2,3)$ and f_{3j} $(j = 1,2,3,4)$ are defined in the following way.

f_{11} is for completeness of the data banks;

f_{21}, f_{31} are in relation to clients' responses on the work provided by MISC with respect to information search and service respectively;

f_{12}, f_{22}, f_{32} are with respect to the timeliness of the data banks' contents;

f_{13}, f_{23}, f_{33} are with respect to the reliability and accuracy of the data and information;

f_{34} is with respect to the accessibility of the interfaces between MISC and clients.

Therefore

$$F = (f_1, f_2, f_3)$$

in which

$f_1 = (f_{11}, f_{12}, f_{13})$;

$f_2 = (f_{21}, f_{22}, f_{23})$;

$f_3 = (f_{31}, f_{32}, f_{33}, f_{34})$.

Let us assume that the weights allocated to the factors are known such that

$WF = (Wf_1, Wf_2, Wf_3) = (0.3, 0.5, 0.2)$,

$Wf_1 = (Wf_{11}, Wf_{12}, Wf_{13}) = (0.3, 0.4, 0.3)$,

$Wf_2 = (Wf_{21}, Wf_{22}, Wf_{23}) = (0.4, 0.3, 0.3)$,

$Wf_3 = (Wf_{31}, Wf_{32}, Wf_{33}, Wf_{34})$

$= (0.3, 0.3, 0.2, 0.2)$.

Furthermore, we consider two opportunities for the evaluation of MISC. The inspections of the physical system of MISC are performed at t_1 while audits of management and organization are supposed to occur at t_2, such that both opportunities are weighted by

$WT = (Wt_1, Wt_2) = (0.3, 0.7)$.

The expert's grading of inspections and audits as well are expressed on the same evaluation scale

$E = (e_1, e_2, e_3, e_4) =$ (excellent, good, acceptable, unacceptable), or in terms of the corresponding scores

$L = (100, 85, 70, 50)$.

The evaluation matrix R, see (3), We have to specify is

$$R = (r_{ij})_{3\times4} \quad (5)$$

in which the elements r_{ij} according to (2) become

$$r_{ij} = \sum_{k=1}^{2} wt_k r_{ij}^{(k)} \qquad i = 1,2,3, \text{ and } j = 1,\cdots,4 \quad (6)$$

With respect to $f_1 = (f_{11}, f_{12}, f_{13})$ the membership degrees $r_{1j}^{(k)}$, i.e. the entries of the vector $R_j^{(k)}$ in (1) are now,

$$r_{1j}^{(k)} = (wf_1)_{1\times3} * (r_{ij}^{(k)}(f_1))_{3\times4}$$

$$j = 1,\cdots,4 \text{ and } k = 1,2 \quad (7)$$

where $r_{ij}^{(k)}(f_1)$ is the membership degree of the sub-factor f_{1i} of f_1 with evaluation grade e_j at time t_k. Similarly,

$$r_{2j}^{(k)} = (wf_2)_{1\times3} * (r_{ij}^{(k)}(f_2))_{3\times4}$$

$$j = 1,\cdots,4 \text{ and } k = 1,2 \quad (8)$$

$$r_{3j}^{(k)} = (wf_3)_{1\times4} * (r_{ij}^{(k)}(f_3))_{4\times4}$$

$$j = 1,\cdots,4 \text{ and } k = 1,2 \quad (9)$$

Recall that a membership degree of a factor associated with an evaluation grade e_j is defined as the proportion in a group of experts that assigns e_j to the factor under consideration. Assume that the evaluation grades are known for each identified factor at time t_k, $k = 1,2$, see Table 1.

Table 1

F		E							
		k = 1				k = 2			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
f_1	f_{11}	0.5	0.2	0.2	0.1	0.4	0.4	0.2	0
	f_{12}	0.2	0.2	0.6	0	0.3	0.7	0	0
	f_{13}	0.7	0.2	0.1	0	0.8	0.1	0.1	0
f_2	f_{21}	0.6	0.2	0.2	0	0.4	0.4	0.2	0
	f_{22}	0.5	0.4	0.1	0	0.3	0.4	0.2	0.1
	f_{23}	0.4	0.4	0.1	0.1	0.4	0.4	0.1	0.1
f_3	f_{31}	0.7	0.2	0.1	0	0.8	0.2	0	0
	f_{32}	0.6	0.3	0.1	0	0.7	0.2	0.1	0
	f_{33}	0.2	0.2	0.3	0.3	0	0.3	0.4	0.3
	f_{34}	0.3	0.4	0.3	0	0.5	0.4	0.1	0

Then it follows from (7), that the vector $R_i^{(k)}$ in (1), for i = 1, k = 1, 2 becomes,

$$R_1^{(1)} = (0.44, 0.20, 0.33, 0.03)$$

$$R_1^{(2)} = (0.48, 0.43, 0.09, 0.00)$$

so that in combination with equation (6) the elements at the first row of the evaluation matrix $R = (r_{ij})$ can be computed, i. e.

$$r_{11} = 0.3 * 0.44 + 0.7 * 0.48 = 0.468$$

$$r_{12} = 0.3 * 0.20 + 0.7 * 0.43 = 0.361$$

and so on.

Similarly, from (6) in connection with (8) and (9) we can get

$$R_2^{(1)} = (0.51, 0.32, 0.14, 0.03)$$

$$R_2^{(2)} = (0.37, 0.40, 0.17, 0.06)$$

$$R_3^{(1)} = (0.49, 0.27, 0.18, 0.06)$$

$$R_3^{(2)} = (0.55, 0.26, 0.13, 0.06)$$

And we can compute the second and third row in $R = (r_{ij})$ respectively.

$$R = \begin{bmatrix} 0.468 & 0.361 & 0.162 & 0.009 \\ 0.412 & 0.376 & 0.161 & 0.051 \\ 0.532 & 0.263 & 0.145 & 0.060 \end{bmatrix}$$

Plugging WF, R in (4) we can obtain the comprehensive evaluation vector

$$S = (0.4528, 0.3489, 0.1581, 0.0402)$$

According to method 1 outlined earlier we find the maximum element $s_1 = 0.4528$ of S, while $s_2 + s_3 + s_4 > 0.5$, so that the comprehensively evaluated MISC has an overall grade e_2 (good). In applying method 2 the evaluation score is

$$Z = S * L^T = 88$$

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