Fifth IFSA World Congress (1993), 873-876

A DESIGN METHOD OF LYAPUNOV-STABLE MMG FUZZY CONTROLLER

Fumio Hara
Science University of Tokyo
1-3 Kagurazaka, Shinjuku-ku, Tokyo 162, JAPAN
and
Kazuomi Yamamoto
NTT Data Communication

ABSTRACT

A fuzzy controller designed by mini-max-gravity(MMG) method is essentially nonlinear with respect to the controller's input and output relationship, and stability analysis is thus needed to construct a stable control system. This paper deals with a design method of a position-type MMG fuzzy controller stable in a sense of Lyapunov when considered is a single-input-single-output linear, stable plant. We first introduce a method to construct a Lyapunov function by using an eigenvalue of A matrix of the linear, stable plant dynamics and then we derive an asymptotic stability condition in terms of scale factors for fuzzy state variables and controller gain. The stability condition is found reasonably practical through comparing the theoretical stability region with that obtained from simulations.

1. Introduction

One of the most important and fundamental concepts concerning the dynamic characteristics of fuzzy control system is stability; and because of a strong nonlinearity in the input-output relationship of fuzzy controller, few works have been done in investigating the dynamic stability condition of fuzzy control system. Kitamura[1] investigated the dynamic stability condition by applying the circle criterion to a position-type fuzzy control system; Tanaka and Sugeno[2], Tanaka and Sano[3] showed a stability condition to a special fuzzy control system constructed by weighted sum of linear-stable subsystems with an algebric-product-sum inference scheme controller. Maeda and his colleagues[4] proposed a method to investigate the stability of both position and velocity type fuzzy control systems by introducing two parameters for linear approximation of a fuzzy controller input-output relationship. Recently Hara and Ishibe[5] showed a counterexample to the stability conjecture derived by Maeda and et al[4] and showed the existence condition of limit cycle oscillation in the velocity-type, MMG fuzzy control system.

This paper discusses the determination of scale factors of a position-type, MMG fuzzy controller for ensuring the asymptotic stability of the fuzzy control system in Lyapunov sense. This paper shows how to construct the Lyapunov function of the fuzzy feedback control system and verifies the proposed method with comparing the analytical result with that obtained from numerical simulations.

2. Fuzzy Control System

Plant Dynamics

Let us assume the plant to be considered is linear and stable, and its dynamics is, for simplicity, represented by a 2nd order state vector equation

$$\dot{x} = Ax + Bu \quad , \quad y = C^T x \qquad (1)$$

where **x**=state vector, u=fuzzy control input, y=observed output and matrix **A**, **B** and **C** are assumed as

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \qquad (2)$$

When the sampling interval τ is designated as unit time step for discrete form of eq.(1), eq.(1) is rewritten in the form

$$x(t+1) = \Phi x(t) + \Psi u(t) \qquad (3)$$

where Φ is a transition matrix defined as $\exp(A\tau)$ and $\Psi = (\exp(A\tau - I)A^{-1}$

For a position-type fuzzy controller, the control input u(t) is written as

$$u(t) = u \qquad (4)$$

where u = output of a position-type fuzzy controller.

When we introduce three scale factors Kp, Kv, and Ku to the fuzzy controller f, u is written as

$$\left.\begin{aligned} & u = Ku \cdot f(\mathbf{k}^T \cdot \mathbf{e}) \\ & \mathbf{k} = \left[\frac{1}{Kp}, \frac{1}{Kv} \right]^T \\ & \mathbf{e} = [e_p, e_v]^T = \mathbf{x}_0 - \mathbf{x}, \ \mathbf{x}_0 = \text{target vector}. \end{aligned}\right\} \qquad (5)$$

Fuzzy Control

The basic membership functions for the fuzzy labels to be used in this paper NB(Negative Big), NM(Negative Medium), NS(Negative Small), ZO(Zero), PS(Positive Small), PM(Positive Medium), and PB(Positive Big) are defined on the support [-6, 6] as shown in Fig.1, and are used to specify the range of each of fuzzy variables such as displacement e_p, velocity e_v and control gain after introducing scale factors Kp, Kv and Ku for each of them respectively. The fuzzy control rules to be used are shown in Fig.2 and the control plane generated by the mini-max-gravity(MMG) center method is, for example, shown in Fig.3 for the case of three scale factor Kp=10, Kv=10 and Ku=10. If we obtain the value **e** at time step t, then the output of the position-type fuzzy controller u in eq.(5) is simply determined by using a control

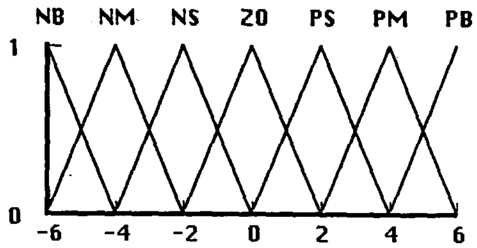
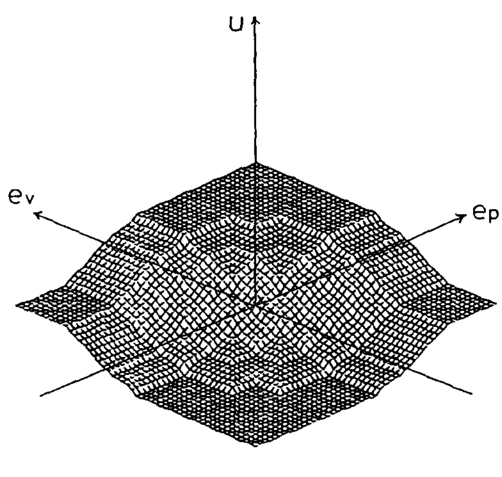


Fig.1 Membership function to each of fuzzy labels, NB, NM,..., PB

Ev \ Ep	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZO
NM	NB	NB	NM	NM	NS	ZO	PS
NS	NB	NM	NM	NS	ZO	PS	PM
ZO	NB	NM	NS	ZO	PS	PM	PB
PS	NM	NS	ZO	PS	PM	PM	PB
PM	NS	ZO	PS	PM	PM	PB	PB
PB	ZO	PS	PM	PB	PB	PB	PB

Fig.2 Control rules specified by fuzzy labels Ep and Ev



Kp=10 Kv=10 Ku=10

Fig.3 Fuzzy control map in terms of e_p and e_v

plane for the case of given scale factors Kp, Kv and Ku.

3. Stability Analysis

Lyapunov Function

Since the position-type fuzzy controller specifies its output u_m by position error e_p and velocity error e_v as shown in Fig.2, eq.(2) must be rewritten in terms of error vector $e(t)=(e_p, e_v)^T$

$$\mathbf{e}(t) = \mathbf{x}_0 - \mathbf{x}(t) \tag{6}$$

i.e.,

$$\mathbf{e}(t+1) = \mathbf{\Phi}\,\mathbf{e}(t) - \mathbf{\Psi}\,\mathrm{v}(t) \tag{7}$$
$$\mathrm{v} = \mathrm{u} - \mathbf{B}^{-1}\mathbf{A}\,\mathrm{x}_0$$

Note here that v is determined by the fuzzy control plane as $v=u_m$. From eqs.(5) and (7), the error vector dynamics is written in the form

$$\mathbf{e}(t+1) = \mathbf{g}(\mathbf{e}(t))\ , \tag{8}$$
$$\mathbf{g} = \text{nonlinear vector function}$$

Designating one eigenvalue of the matrix **A** in eq.(1) as λ, we introduce the following function V for the dynamic system (eq.(8))

$$V = \{ -\lambda\, e_p(t) + e_v(t) \}^2 \tag{9}$$

and investigate its property in a view of the Lyapunov function.

(1) From the property of mini-max-gravity center inference scheme employed in determining the fuzzy controller output u, it is easily found that

$$u = 0 \quad \text{for} \quad e_p(t) = e_v(t) = 0\,. \tag{10}$$

Thus the function V can satisfy the condition for any t

$$V(\mathbf{e}(t)=0) = 0 \qquad \text{when} \quad \mathbf{e}(t) = 0\,. \tag{11}$$

(2) It is easily found from the definition eq.(9)

$$V(\mathbf{e}(t)) > 0 \quad \text{for} \ \mathbf{e}(t) \neq 0 \tag{12}$$
$$V(\mathbf{e}(t)) \rightarrow \infty \quad \text{for} \ \|\mathbf{e}(t)\| \rightarrow \infty \tag{13}$$

(3) Replacing the value $-\lambda$ by p in general, and rewriting eq.(7) in the following form

$$\begin{bmatrix} e_p(t+1) \\ e_v(t+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} e_p(t) \\ e_v(t) \end{bmatrix} - \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \tag{14}$$

then

$$\begin{aligned} \Delta V &\equiv V(\mathbf{e}(t+1)) - V(\mathbf{e}(t)) \\ &= \{(pB_1 + B_2)u(t) + (pA_1 + A_3 + p)e_p(t) \\ &\qquad + (pA_2 + A_4 + 1)e_v(t)\} \times \\ &\qquad \{(pB_1 + B_2)u(t) + (pA_1 + A_3 - p)e_p(t) \\ &\qquad + (pA_2 + A_4 - 1)e_v(t)\} \end{aligned} \tag{15}$$

As ΔV is a quadratic form of u(t), ΔV becomes negative if the discriminant D of eq.(15)

$$D = 4(pB_1 + B_2)^2(pe_p(t) + e_v(t))^2 \tag{16}$$

is positive and

$$u_1 < u < u_2 \tag{17}$$

where u_1 and u_2 are the two real roots of the equation

$$\Delta V = 0 . \tag{18}$$

Thus it is easily derived that, as shown in Fig.4,

$$\Delta V < 0 \quad \text{for} \quad u_1 < u < u_2 . \tag{19}$$

Stability Condition in Terms of Scale Factors

When the discriminant D is positive, ΔV becomes negative for the point (e_p, e_v, u) included within the region covered by the two planes (say Lyapunov planes, Fig.5)

$$\left.\begin{aligned} (pB_1 + B_2)u + (pA_1 + A_3 + p)e_p \\ + (pA_2 + A_4 + 1)e_v = 0 \\ (pB_1 + B_2)u + (pA_1 + A_3 - p)e_p \\ + (pA_2 + A_4 - 1)e_v = 0 \end{aligned}\right\} \tag{20}$$

This condition is completely equivalent to eq.(19). The intersection between the two Lyapunov planes (eq.(20)) is easily obtained from eq.(20) and specified by

$$pe_p + e_v = 0 \ , \ \ u=0 \ . \tag{21}$$

since the discriminant eq.(16) becomes zero for the condition $pe_p+e_v=0$, and the degenerate root of $\Delta V=0$ is u=0 due to the property of A_1, A_2, A_3 and A_4.

From the fuzzy control map generated by the fuzzy control rules shown in Fig.2, it is found that

$$u=0 \quad \text{for} \quad Kv\, e_p + Kp\, e_v = 0. \tag{22}$$

When the intersection of the Lyapunov planes coincides with the line specified by eq.(22), the fuzzy control system becomes asymptotically stable if the fuzzy control plane $u = Ku \cdot f(\mathbf{K}^T \cdot \mathbf{e})$ is included within the region covered by the two Lyapunov planes. Then from eqs.(21) and (22),

$$Kv\, e_p + Kp\, e_v = p\, e_p + e_v = 0 \tag{23}$$

$$\therefore \ Kv = pKp = -\lambda Kp$$

The Lyapunov plane with a larger inclination is easily found and is represented by eq.(24) when p is replaced by $-\lambda$:

$$u = -\frac{-\lambda A_1 + A_3 - \lambda}{-\lambda B_1 + B_2} e_p - \frac{-\lambda A_2 + A_4 + 1}{-\lambda B_1 + B_2} e_v \tag{24}$$

Let's consider a tangential plane to the fuzzy control plane $u = Ku \cdot f(\mathbf{K}^T \cdot \mathbf{e})$ and express it by the following equation

$$u = \alpha \frac{Ku}{Kp} e_p + \alpha \frac{Ku}{Kv} e_v \tag{25}$$

where α is the tangential to the nominal fuzzy control plane (Kp=Kv=Ku=1). The Lyapunov plane eq.(24) can cover the fuzzy control plane if the following condition is satisfied:

$$\left.\begin{aligned} \alpha \frac{Ku}{Kp} &< \frac{-\lambda A_1 + A_3 - \lambda}{-\lambda B_1 + B_2} \\ \alpha \frac{Ku}{Kv} &< \frac{-\lambda A_2 + A_4 + 1}{-\lambda B_1 + B_2} \end{aligned}\right\} \tag{26}$$

From the above discussions, the scale factors Kp, Kv and Ku must be selected to satisfy the conditions shown in eqs.(23) and (26) for ensuring the dynamic stability of the position-type MMG fuzzy control system.

4. Simulation Study

The validity of the stability conditions (eqs.(23) and (26)) derived in terms of scale factors Kp, Kv and Ku, numerical simulation was carried out for the following conditions:

Dynamic equation : eq.(3)
Initial condition : $\mathbf{x}(0)=[0,0]^T$,

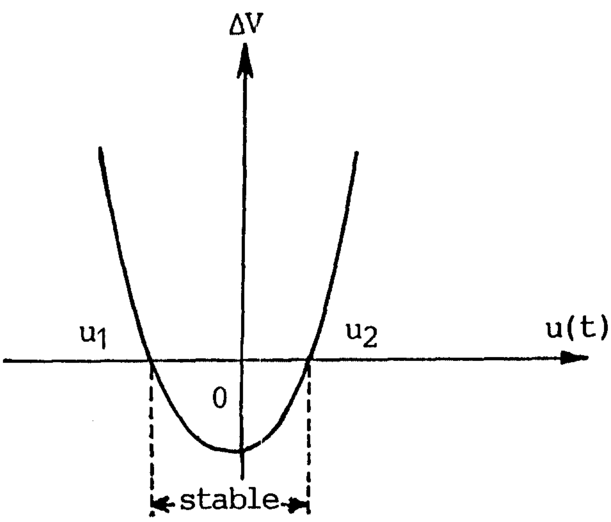


Fig.4 Increment of Lyapunov function ΔV against control u

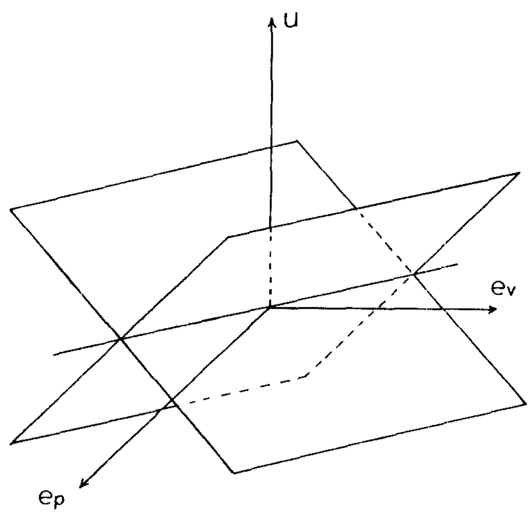


Fig.5 Lyapunov planes in space (e_p, e_v, u)

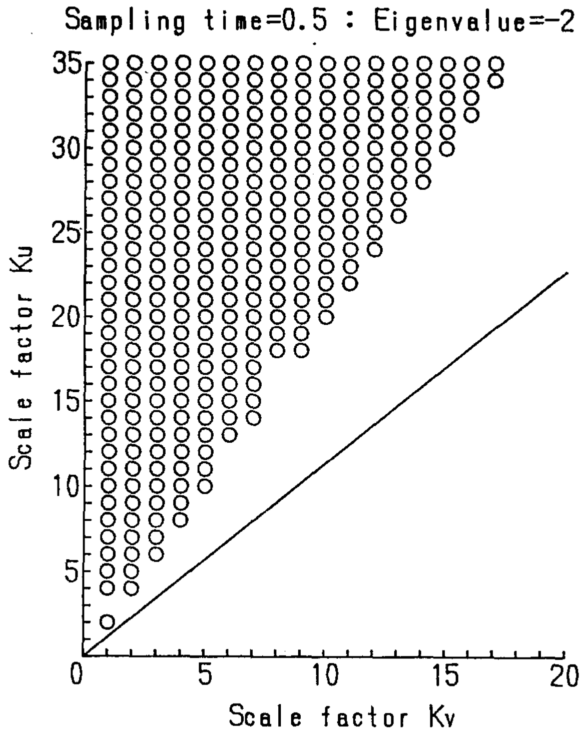


Fig.6 Stability region determined by Lyapunov method in terms of Ku and Kv (solid line) and unstable fuzzy control systems specified by scale factors Kv and Ku for λ=-2

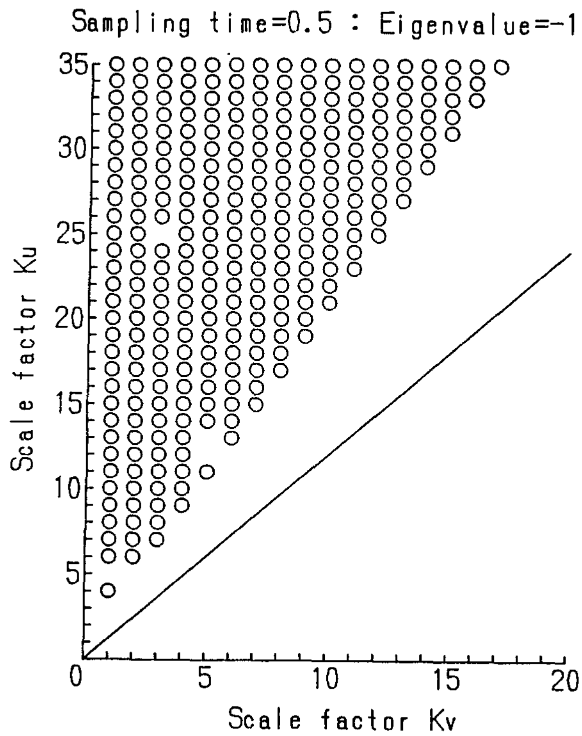


Fig.7 Stability region determined by Lyapunov method in terms of Ku and Kv (solid line) and unstable fuzzy control systems specified by scale factors Kv and Ku for λ=-1

Target state : $\mathbf{x}_0=[40,0]^T$,
Stability criteria in simulation :

$$\left.\begin{array}{l}\lim_{t\to\infty}\mu_{zo}\left(\frac{e_p(t)}{Kp}\right)\geq 0.5\\ \lim_{t\to\infty}\mu_{zo}\left(\frac{e_v(t)}{Kv}\right)\geq 0.5\end{array}\right\} \quad (27)$$

Sampling time τ : 0.5

Simulation results are shown in Fig.6 for $\lambda=-2$, in which open circles mean that the fuzzy control system is unstable in numerical simulation and the solid line corresponds to the condition discribed by eq.(26) under the relationship

$$Kv = -\lambda Kp$$

This figure clearly shows that the stability conditions we have derived in this paper are sufficient for ensuring the fuzzy control system to be stable.

Fig.7 is the distribution of unstable scale factor point (Kv, Ku) obtained by numerical simulation for another eigenvalue $\lambda=-1$. The solid line in this figure is also correspondent to the stability boundary obtained by eq.(26) and the condition Kv=Kp. The unstable scale factor points (Kv, Ku) locate fairly far outside the stable region obtained by eq.(26) in both Figs.6 and 7.

5. Conclusions

This paper has shown the stability condition, in Lyapunov sense, of position-type MMG fuzzy control system and derived the conditions to be satisfied by scale factors Kp, Kv and Ku for ensuring the stability of our fuzzy control system. The conditions were also verified by numerical simulations.

References

[1] S.Kitamura, A Stability Condition for Fuzzy Ruled Control Systems, Trans. SICE, vol.**27**, No.5 (1991) p.532-537

[2] K.Tanaka and M.Sugeno, Stability Analysis and Design of Fuzzy Control Systems, Fuzzy Sets and Systems, vol.**45**, No.2 (1992) pp.135-156

[3] K.Tanaka and M.Sano, Stability Conditions for Design Problem of Fuzzy Controllers, Proc. Korea-Japan Conf. Fuzzy Systems and Engineering, Seoul (1992) pp.21-24

[4] M.Maeda, E.Fukumiya and S.Murakami, Stability Analysis of Fuzzy Control System by 3D Display, Trans. JSME, vol.**58**, No.550 C (1992) pp.1773-1780

[5] F.Hara and M.Ishibe, Simulation Study on the Existence of Limit Cycle Oscillation in a Fuzzy Control System, Proc. Korea-Japan Conf. Fuzzy Systems and Engineering, Seoul (1992) pp.25-28