Fifth IFSA World Congress (1993), 849-852

A Chaos Simulator using Analogue Circuit to Model Josephson Junction

Mititada MORISUE, Hiroshi NOGUCHI and Akinori KANASUGI

Department of Electronic Engineering
Saitama University,
Urawa 338, Japan

Abstract This paper proposes a novel chaos generator using the model of Josephson junction. Constructing an equivalent circuit of Josephson element by using an operational amplifier, we have made a chaos generator. The feature of this generator is to generate several kinds of oscillations as well as chaotic oscillations by only changing a DC bias current to the junction. In this paper, it is described in detail how to construct the circuit and what kind of oscillation is realized in the circuit. The experimental results of oscillation modes are compared with simulation results with a satisfactory agreement.

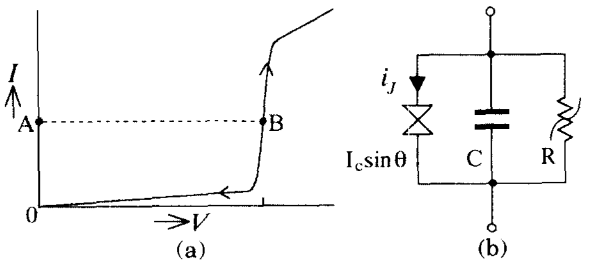
1. Introduction

The superconducting device of Josephson junction have been intensively studied for developing a high performance computer, ultra high frequency devices, magnetic sensor and so on [1],[2]. It is well known in simple Josephson circuit that there exist five types of oscillation modes, that is a periodic, a subharmonic, a quasi-periodic, a chaotic and a relaxation oscillation [3],[4]. Therefore, we have constructed the electronic circuits to simulate the I-V characteristics of a tunnel-type Josephson junction by using an operational amplifier and then have made a chaos generator. The features of this generator are the very simple construction with a simulated Josephson junction, a load resistance and an inductance in parallel connections and capabilities of generating several oscillation modes by the simple way to change the bias current to the junction.

In this paper we describes how to construct the analogue Josephson circuit and what kinds of oscillations are produced in this circuit. This chaos generator is able to display the bifurcation diagram, Poincaré map, phase plane diagram as well as time domain waveforms on the screen to characterize what kind of oscillation occurs. In order to verify the several oscillation modes obtained by the chaos generator, we have made computer simulation. The results of computer simulations agree well with the experimental results.

2. Circuit of Chaos generator

A tunnel type Josephson junction presents nonlinear I-V characteristics as shown in Fig.2.1 (a). When we apply a current to the junction, the current through the junction increases along the Y axis. Once the current reaches to the critical value indicated by the pint A, the junction switches to the voltage state of the point B to produce a voltage with high frequency across the junction. After switching of the junction, the voltage increases or decreases along the nonlinear characteristics of the junction as indicated by the arrow. The equivalent circuit of the junction is shown in Fig.2.1(b), where $I_c \sin\theta$ is the current source, C is the capacitance, R is the nonlinear resistance. In experimental circuit, for simplicity, the nonlinear resistance to present the I-V characteristics has



(a) (b)

Fig.2.1 Josephson Junction:
(a) Current-voltage characteristics
(b) Equivalent circuit

been replaced by the common linear resistance.

The equations describing the behavior of the junction is given by

$$i_J = I_c \sin\theta \qquad (2.1)$$

$$\frac{d\theta}{dt} = \frac{2e}{\hbar}v = \frac{2\pi}{\Phi_0}v \qquad (2.2)$$

where, θ is the phase difference between both superconductors, I_c is the critical current of the junction, e is the charge of an electron, ħ is the Plank constant and Φ_0 is the the quantum flux.

The circuit of Josephson junction described by equ. (2.1) and (2.2) can be constructed by using common electronic elements. The equivalent circuit of the chaos generator with a simulated Josephson junction is shown in Fig.2.2. The equations for this autonomous oscillation circuit are represented by

$$\frac{d\theta}{dt} = \frac{2\pi}{\Phi_0}v \qquad (2.3)$$

$$\frac{dv}{dt} = \frac{1}{C}\left(I_b - I_c \sin\theta - \frac{v}{R} - i_L\right) \qquad (2.4)$$

$$\frac{di_L}{dt} = \frac{1}{L}\left(v - R_L i_L\right) \qquad (2.5)$$

where C and R are the capacitance and the resistance of Josephson junction, R_L and L are the load resistance and the inductance, i_L is the current through load impedance and I_b is the bias current to the junction.

In experiment, the above equations are transformed to the equations normalized by the special factors. According to the block diagram of Phase-Locked Loop[5], we can construct the chaos generator by using commercially available elements. Fig.2.3 shows the block diagram of constructed Josephson Junction , where VCO is the voltage control oscillator, OSC is the external sine-wave source, MIX is the mixer, LPF is the low pass filter to block the high frequency wave component of $\sin(2\omega_0 t+\theta)$, and G is the converter from voltage to current.

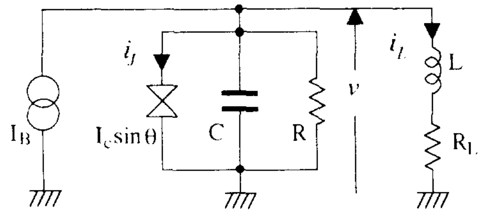


Fig.2.2 Equivalent circuit of the proposed chaos generator

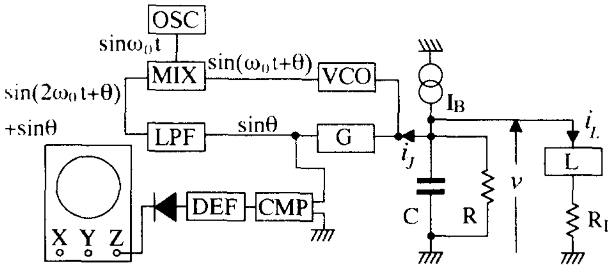


Fig.2.3 The block diagram of the chaos generator to realize the equivarent circuit of Fig 2.2

3. Observation of Chaos and its associated oscillations

The chaotic oscillations can be easily produced by adjusting the DC current to the junction in the chaos generator. Besides the chaotic oscillation, the associated oscillations such as a periodic, a subharmonic, and a relaxation oscillation can be generated by changing the bias currents. Once each oscillation starts at each bias current, the oscillation is stable. The over-view of oscillation modes produced by the chaos generator can be represented by the bifurcation diagram. Fig.3.1(a) shows the bifurcation diagram obtained in the experiment, where the instantaneous terminal voltages across the Josephson junction is taken as the vertical axis while the bias current to the junction is as the horizontal axis. At any point of I_b in the figure, about one hundred points of instantaneous voltages across the junction are plotted at the discrete time of $\theta = n\,2\pi$ where n is the integer.

In order to compare these experimental data with analytical results we have made simulations. Fig.3.1(b) shows the bifurcation diagram by the

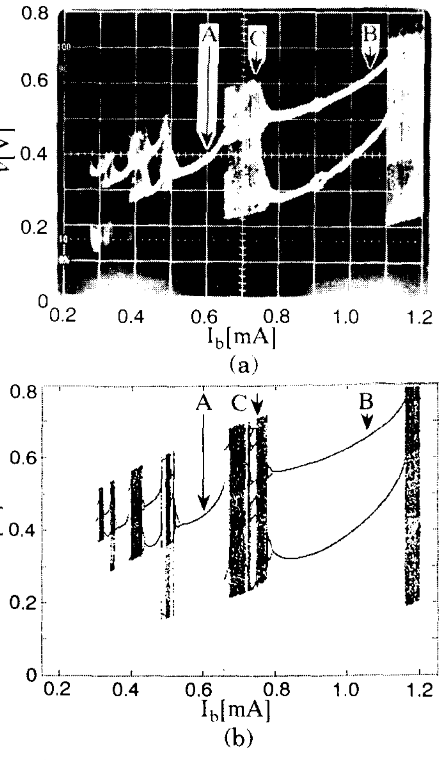


Fig.3.1 A bifurcation diagram obtained by the chaos generator in Comparison with the calculations: (a) Experimental results, (b) Simulated results

simulation for the similar circuit to the experimental circuit. These simulation results agree very well with the experimental results. We can see from these bifurcation diagrams that there exist three types of oscillations, that is a periodic oscillation indicated by the region of A in the figure, a subharmonic oscillation by B and a chaotic oscillation by C, although the quasi-periodic oscillation is not observed. The relaxation oscillation can not be shown by the bifurcation diagram. This mode is generally recognized by the time-domain oscillation waveform.

The oscillation modes obtained by the chaos generator are described in comparison with the simulation results.

(1) Periodic oscillation: This type of oscillation is observed at the bias current of Ib corresponding to the point A in Fig.3.1(a). Fig.3.2(a), (b) and (c) show the time domain waveform, the phase plane and Poincaré map of a periodic oscillation obtained by the chaos generator. Fig.3.3(a), (b) and (c) show the simulation results of the waveform, the phase plane and Poincaré map in the corresponding circuit to the chaos generator.

(2) Subharmonic oscillation: A typical subharmonic oscillation is seen at the bias current

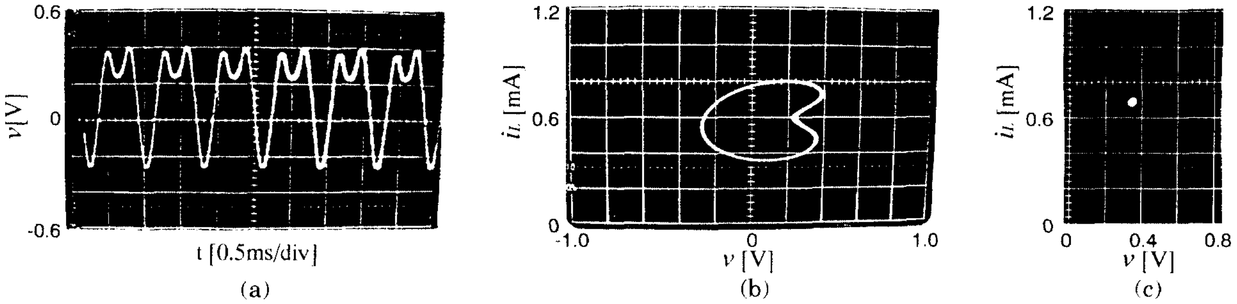


Fig.3.2. Periodic Oscillation produced by the chaos generator:
(a) Time domain map, (b) Phase Plane, (c) Poincaré map

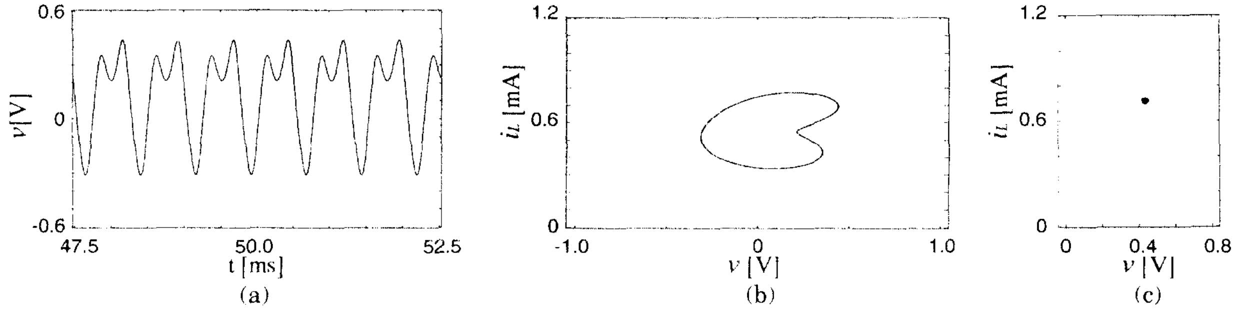


Fig.3.3. Periodic Oscillation produced by simulation

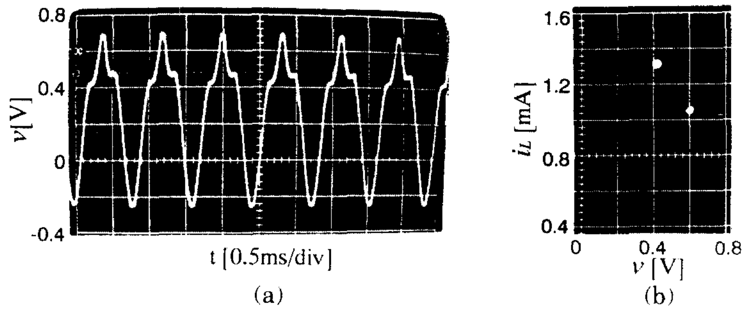


Fig.3.4. Subharmonic Oscillation by the chaos generator

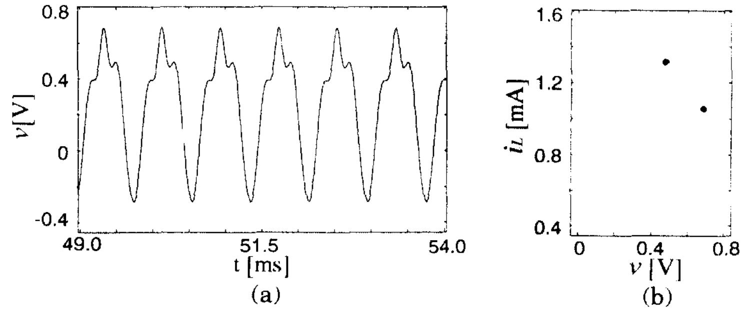


Fig.3.5. Subharmonic Oscillation by simulation

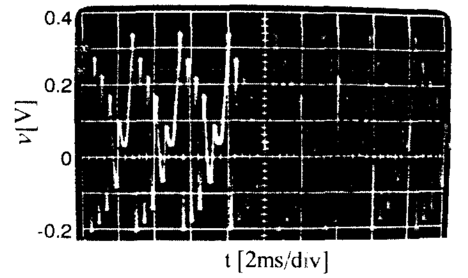


Fig.3.6. Relaxation Oscillation by the chaos generator

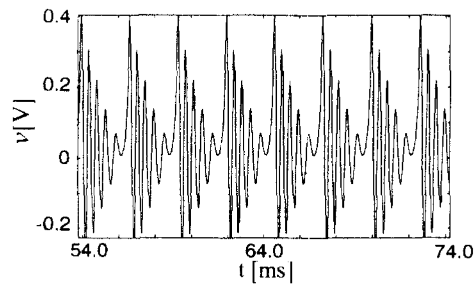


Fig.3.7. Relaxation Oscillation by simulation

of Ib corresponding to the point B in Fig.3.1(a). Fig.3.4(a) and (b) show the time domain waveform, and the Poincaré map for this oscillation, while Fig.3.5(a) and (b) are simulation results corresponding to the experimental results.

(3) Relaxation Oscillation[6]: This type of oscillation can be produced in the circuit by decreasing the load inductance while other circuit parameters are kept constant. Fig.3.6 shows the time domain waveform of one relaxation oscillation obtained in the experiment. Fig.3.7 indicates the corresponding time domain waveform by simulation.

(4) Chaotic oscillation: One example of the chaotic oscillation are shown in Fig.3.8(a),(b) and (c) which indicate the time domain waveform, the phase plane and the Poincaré map. Fig.3.9(a),(b) and (c) show the corresponding simulation results to Fig.3.8(a),(b) and (c).

Thus, we can generate the four types of oscillations by the proposed chaos generator and easily characterize which type of oscillation occurs by showing the time domain waveform, the phase plane, the Poincaré map and the bifurcation diagram.

4. Conclusion

We have proposed a novel chaos generator using a model of the Josephson junction. This generator has been able to produce several kinds of oscillations such as the chaotic, the periodic, the subharmonic and the relaxation oscillations by only adjusting the bias currents in the generator. In experiments, the time domain waveforms, Poincaré map, Phase plane diagram as well as the bifurcation diagram of the oscillation were illustrated in detail. Furthermore, computer simulations have been made in order to verify the illustrated waveforms obtained by the chaos generator. The quite satisfactory agreement between the simulation results and the experimental results has been obtained. Although this generator is used for the oscillations generated in an autonomous system of Josephson junction circuit, we can easily develop the chaos generator for the forced oscillation system by installing the external sine-wave source in place of the DC bias source in this autonomous system.

References

[1]. S. Kosaka, A. Shoji, M. Aoyagi, F. Shinoki, S. Tahara, H. Ohigashi, H. Nakagawa, S. Takada and H. Hayakawa, "An integration of all refractory Josephson logic LSI circuit", *IEEE Trans. Magn.*, MAG-21, pp.102-109, 1985

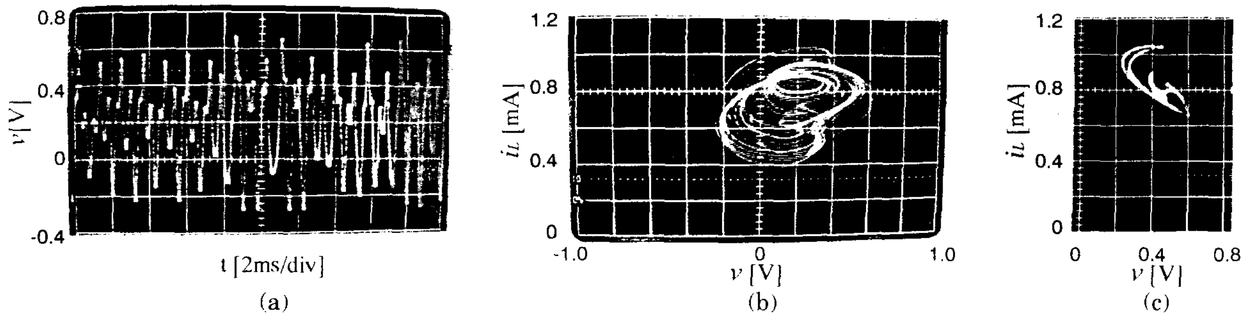
[2]. M. Morisue and K. Suzuki, "A proposal of Josephson ternary fuzzy processor", *Extended Abstracts of Int. Superconductivity Electronics Conference,* pp.259-263, 1989

[3] R. L. Kautz, "Chaos in Josephson Circuits", *IEEE Trans. Magn.*, MAG-19, 3, 1983

[4] M. Morisue, H. Fukuzawa and Y. Marushima, "Chaos in Superconducting Element Circuit", *Proc. ISCAS'85*, pp. 855-858, 1985

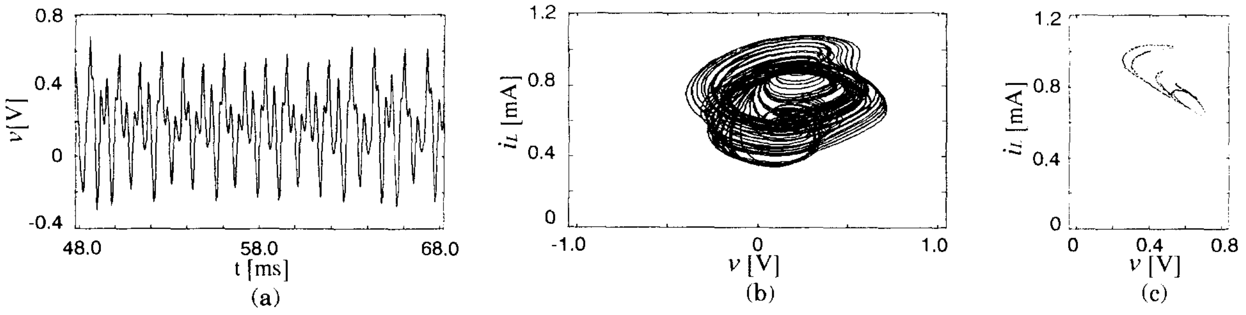
[5] M. Ishikawa, T. Hara, F. Shiota and K. Hara, "Electrical Analogue of Josephson Double Junction", *Oyo Buturi, 47, 7,* pp 25-32 , 1978

[6] H. W. Chan and T. Van Duzer, "Josephson Nonlatching Logic Circuits", *IEEE J. Solid-State Circuits*, SC-12, 1, pp. 73-79, 1977



(a) (b) (c)

Fig.3.8. Chaotic Oscillation by the chaos generator



(a) (b) (c)

Fig.3.9. Chaotic Oscillation by simulation