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Fuzzy Logic Based Sliding Mode Control

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Abstract - **A fuzzy logic controller derived from the variable structure control (VSC) theory is designed. Unlike the conventional design of the fuzzy controller, we do not fuzzify the error and the rate of error, but fuzzify the sliding surface. After the fuzzy sliding surface is introduced, the fuzzy rules are defined based on the sliding control theory. It will be shown this sliding mode fuzzy controller is a kind of VSC that introduces the boundary layer in the switching surface and that the control input is continuously approximated in the layer. As a result we can guarantee the stability and the robustness by the help of VSC, which were difficult to insure in the past fuzzy controllers. Simulation results for the inverted pendulum will show the validity.**

1. Introduction

Recently a lot of design efforts to control complex systems or poorly modeled systems using fuzzy logic have been done and these have fruited successfully in many areas [1]. However the design procedure for fuzzy logic controller is not well defined yet and it depends greatly upon the expert's knowledge or trial-and-errors. Further more, due to the linguistic expressions of the fuzzy control we can hardly say the stability or the robustness of the fuzzy controlled system. Thus we need a design method for the fuzzy controller that is more structured and can guarantee the stability and the robustness.

In many cases the fuzzy logic controller is designed using the error and the rate of error [2],[3]. The error and the rate of error are represented graphically in the phase plane, so one can design the fuzzy logic controller systematically if he uses the phase portrait method. If a switching line is introduced and the fuzzy rules are defined according to the switching line, then this fuzzy controller behaves similarly to the sliding mode controller [4].

The sliding mode control method can guarantee the robustness of the system involving the disturbances or noises, but this method uses drastic changes of the control input which make the chattering phenomena. The chattering may excite the high frequency components of the system that is neglected when modelling. In order to avoid the chattering problem, the boundary layer is introduced and the control input is properly approximated in the boundary layer [5].

In this paper, we adopt the advantages of the sliding mode control theory and apply them to design the fuzzy logic controller. Instead of fuzzifying the error and the rate of error, we fuzzify the sliding surface so that we design a fuzzy sliding mode controller. This proposed controller can get rid of the chattering problem and guarantees the stability and the robustness. We will show these results are due to the facts that the fuzzy sliding mode controller has the boundary layer and the control input is continuous approximation in the boundary layer.

2. Sliding Mode Controller

Assume the nth order system as follows

$$x^{(n)}(t) = f(\mathbf{x},t) + u(t) + d(t) \tag{1}$$

where $\mathbf{x}^T = [x \;\; \dot{x} \;\; \cdots \;\; x^{(n-1)}]$ is the state vector, and $d(t)$ and $u(t)$ are the disturbance and the control input, respectively. The nonlinear function $f(\mathbf{x},t)$ is given by

$$f(\mathbf{x},t) = \hat{f}(\mathbf{x},t) + \Delta f(\mathbf{x},t), \tag{2}$$

where $\hat{f}(\mathbf{x},t)$ is the estimation of $f(\mathbf{x},t)$ and $\Delta f(\mathbf{x},t)$ represents the model uncertainty. The disturbance and the model uncertainty are bounded below F and D respectively,

$$|\Delta f(\mathbf{x},t)| \le F(\mathbf{x},t), \tag{3}$$

$$|d(t)| \le D(\mathbf{x},t).$$

Define the tracking error of a state x as follows

$$e(t) = x(t) - x_d(t). \tag{4}$$

Then the control problem is for the state x to track the desired state $x_d(t)$ even under the model uncertainty and the disturbance. Now define the sliding surface $s(\mathbf{x},t) = 0$ by the following equation

$$s(\mathbf{x},t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} e. \tag{5}$$

The tracking control is to place the error e on the sliding surface. The control input is made to satisfy the following sliding condition,

$$\frac{1}{2}\frac{d}{dt}\left[s^2(\mathbf{x},t)\right] \le -\eta|s|. \tag{6}$$

$$\eta \ge 0$$

Consider the second order system without a loss of generality as follows,

$$\ddot{x}(t) = f(\mathbf{x},t) + u + d(t). \tag{7}$$

The sliding surface is

$$s = \lambda e + \dot{e} \tag{8}$$

and $\dot{s} = \lambda\dot{e} + \ddot{x}(t) - \ddot{x}_d(t)$. From the sliding condition of (6), we get

$$s \cdot \dot{s} = s \cdot (\lambda\dot{e} + f(\mathbf{x},t) + u + d - \ddot{x}_d(t)) \le -\eta|s|. \tag{9}$$

Let the control input be $u = \hat{u} - K(\mathbf{x},t)\cdot\mathrm{sgn}(s)$, then

$$u = \left(-\hat{f}(\mathbf{x},t) - \lambda\dot{e} + \ddot{x}_d(t)\right) - K(\mathbf{x},t)\mathrm{sgn}(s) \tag{10}$$

where

$$\mathrm{sgn}(s) = \begin{cases} 1 & \text{for } s > 0 \\ -1 & \text{otherwise} \end{cases}.$$

We get the range of $K(\mathbf{x},t)$ from the equations (9) and (10),

$$K(\mathbf{x},t) \ge F(\mathbf{x},t) + D(\mathbf{x},t) + \eta. \tag{11}$$

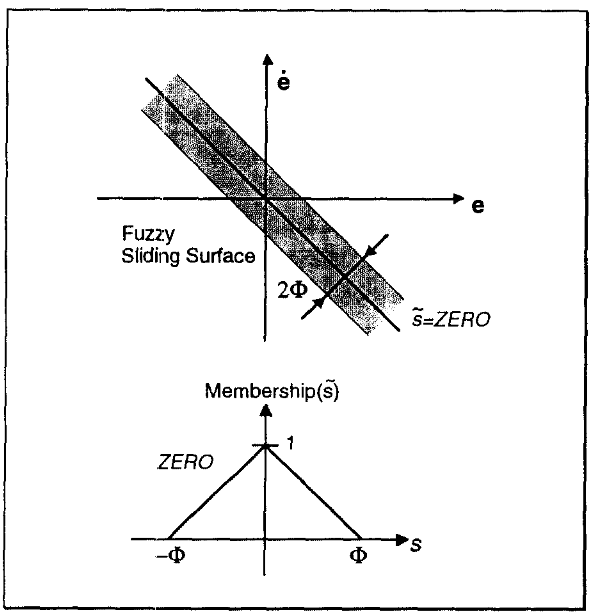


Figure 1. Fuzzy sliding surface and its membership

3. Fuzzy Sliding Mode Controller

Now introduce the fuzziness to the sliding surface defined in previous section. Instead of the crisp sliding surface $s=0$, define a linguistic value *ZERO* and consider the fuzzy sliding surface $\tilde{s}$ =*ZERO* where $\tilde{s}$ denotes the fuzzy variable corresponding to s. This is shown in Figure 1 with the triangular membership function.

We then partition the $e-\dot{e}$ plane into three parts such that $\tilde{s}$ =*ZERO*, $\tilde{s}$ =*NEGATIVE* and $\tilde{s}$ =*POSITIVE*. According to the control input of equation (10), u is partitioned into three parts, $\tilde{u}$ ={*SMALL*, *MEDIUM*, *BIG*} [Figure 2.].

Define fuzzy rules as follows

[Rule 1] If s is *ZERO* then u is *MEDIUM*.
[Rule 2] If s is *NEGATIVE* then u is *BIG*.
[Rule 3] If s is *POSITIVE* then u is *SMALL*.

The inference uses Mamdani's MAX-MIN reasoning and the defuzzification is the center-of-area (COA) method [2]. The result of the inference for every s is shown in Figure 3. We see that the control input is continuously approximated in the boundary layer. For more perspective analysis, the result of the fuzzy rules within the boundary layer is inferred explicitly as follows:

$$u = \begin{cases} \hat{u} + K & \text{for } z < -1 \\ \hat{u} + K\dfrac{z(z+3)}{z^2+z-1} & \text{for } -1 \le z < 0 \\ \hat{u} - K\dfrac{z(z-3)}{z^2-z-1} & \text{for } 0 \le z < 1 \\ \hat{u} - K & \text{for } z > 1 \end{cases} \tag{12}$$

where $z = s/\Phi$.

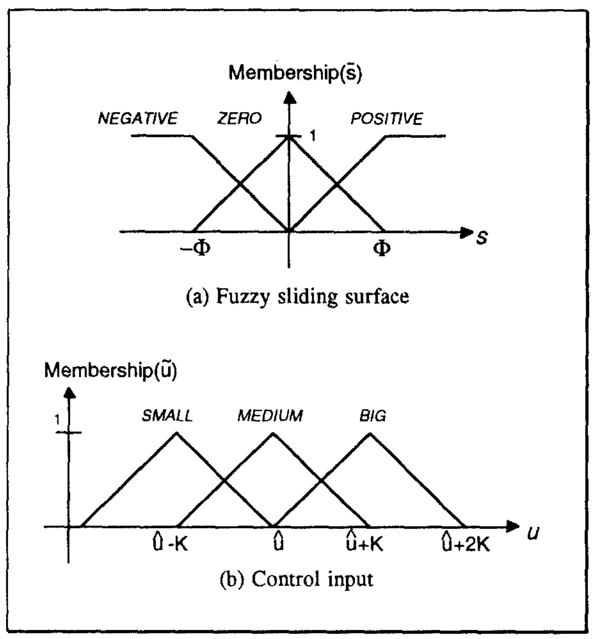


Figure 2. Membership functions for case of three rules

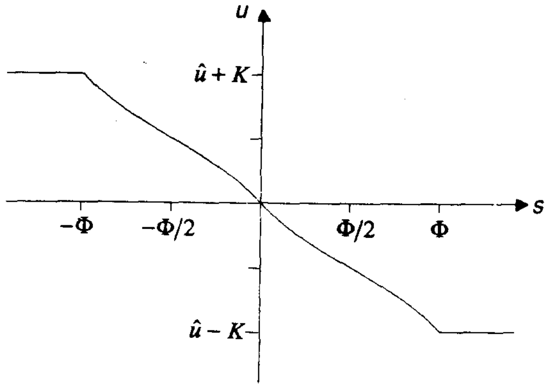


Figure 3. The result of the inference

4. Simulation

The inverted pendulum is considered to verify the performance of the fuzzy controller with the fuzzy sliding line [Figure 4]. The nonlinear dynamics of this systems is governed by

$$m\ell^2\ddot{x}+b\dot{x}+mg\ell\cos(x)=u+d(t,x,\dot{x}) \qquad (13)$$

where x and $d(t,x,\dot{x})$ denote the joint angle and the disturbance, respectively. For the simplicity of the simulation, assume the parameters of (13) are

$$m=\ell=b=g=1 \qquad (14)$$

and the disturbance satisfies that

$$d(t,x,\dot{x})=\sin(3t)+\dot{x}. \qquad (15)$$

Then equation (13) results in

$$\ddot{x}=-\dot{x}-\cos(x)+u+d(t,x,\dot{x}). \qquad (16)$$

Comparing (13) with (1) and (2), we get the following,

$$f(x,t)=0, \qquad F(x,t)=D(x,t)=|x|+1. \qquad (17)$$

From equation (10) with $\lambda=20, \eta=0.1$, the control input is

$$u=\ddot{x}_d-20\dot{e}-(2|\dot{x}|+2.1)\cdot\mathrm{sgn}(e+20\dot{e}) \qquad (18)$$

where the desired trajectory is given by $x_d(t)=\sin(\pi t/2)$.

In analogy with the rules in Section 3, define the following 5 rules and their membership functions [Figure 5].

[Rule 1] If s is *ZR* then u is *MEDIUM.*

[Rule 2] If s is *NB* then u is *BIGGER.*

[Rule 3] If s is *NM* then u is *BIG.*

[Rule 4] If s is *PM* then u is *SMALL.*

[Rule 5] If s is *PB* then u is *SMALLER.*

We use this five rules for following simulation. The result of the inference using the new five rules are very similar to that of three rules, so the result is not printed here.

Figure 6 and Figure 7 show the simulation results. The former is for the conventional VSC and the latter is for the sliding mode fuzzy controller. We see that the chattering is eliminated while the tracking error still remains small.

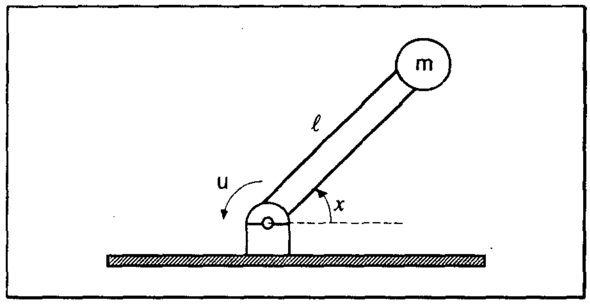


Figure 4. Inverted Pendulum System

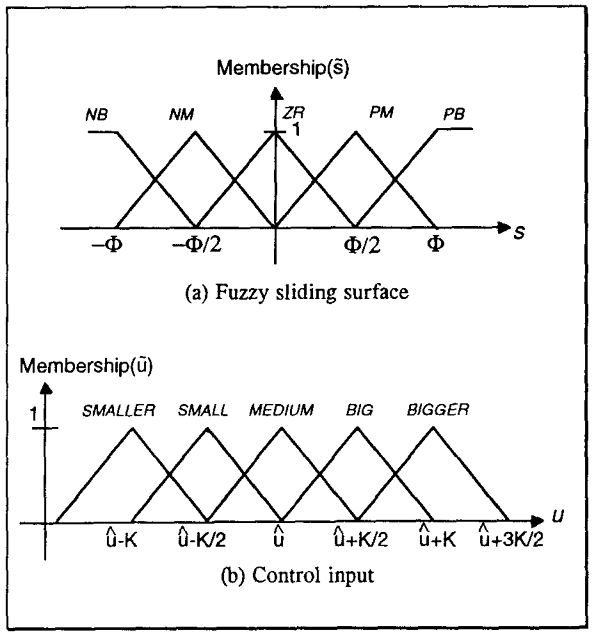
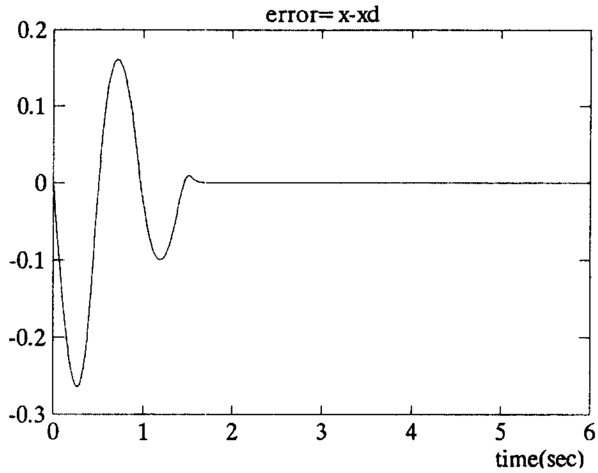


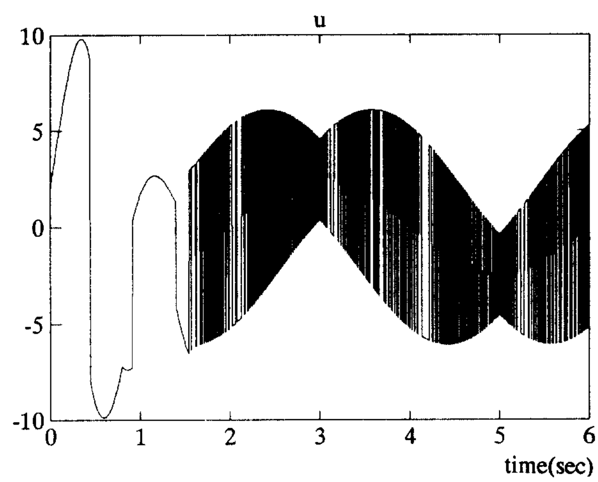
Figure 5. Membership functions for the case of five rules

5. Conclusions

In this paper, we design the fuzzy logic controller based on the sliding mode control theory. Unlike the conventional fuzzy logic controller, we do not fuzzify the error and the change of error but the sliding surface s. This makes the controller design more structured and simpler. We can guarantee the stability and the robustness of the proposed fuzzy controller since it is actually a variable structure controller whose input is continuously approximated in the boundary layer. The fuzzy sliding mode controller eliminates the chattering problem. We verified the validity of the proposed idea by simulating for an inverted pendulum with the disturbance.

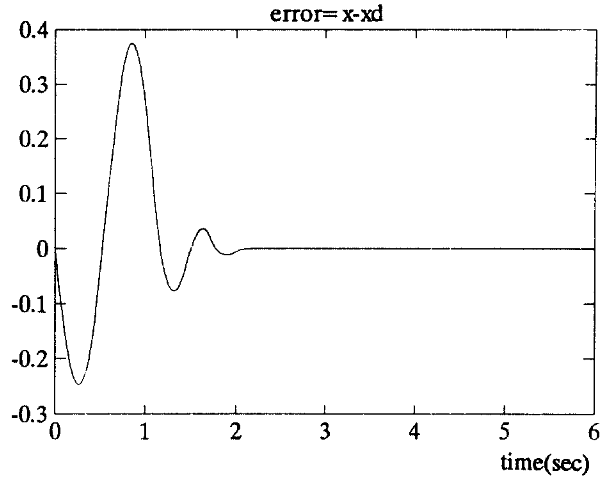


(a) Tracking error

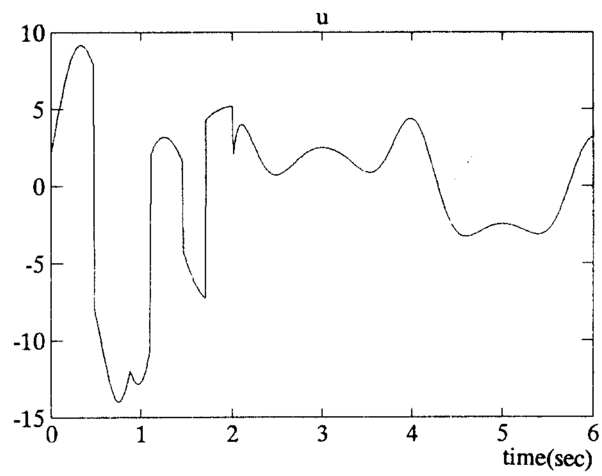


(b) Control input

Figure 6. Case of the conventional VSC



(a) Tracking error



(b) Control input

Figure 7. Case of fuzzy sliding mode control

References

[1] M. Sugeno, *Industrial Application of Fuzzy Control*, Elsevier Science Publishers B. V., 1985

[2] C. C. Lee, "Fuzzy Logic in Control Systems: Fuzzy Logic Controller -- Part I and II," *IEEE Trans. on Sys., Man and Cyb.*, Vol. 20, No. 2, pp. 404-435, 1990

[3] S. Kawaji and N. Matsunaga, "Design of Fuzzy control System based on PD control Schem," *IFSA '91 Brussels, July 1991*, Vol: engineering, pp. 77-80

[4] R. Palm, "Sliding Mode Fuzzy Control," *IEEE Int. Conf. on Fuzzy Systems*, pp. 519-526, 1992

[5] J-J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, 1991

[6] S. -W. Kim and J. -J. Lee, "Design of a Variable Structure Controller using the Fuzzy Sliding Line," *1993 Korea Fuzzy Mathematics Systems Society Spring Conf.* Vol. 3, No. 1, pp. 125-132 (in Korean)