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FUZZY ADAPTIVE CONTROL ENVIRONMENT USING LYAPUNOV FUNCTIONS: FACE

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ABSTRACT: Adaptive Control is used in order to improve close loop dynamics with a fuzzy controller when process parameters are unknown or fluctuate form an initial value. The way in which the adaptive control environment may be applied is the following. First we obtain a *linear* fuzzy controller. Second, we apply the adaptive rules by means of actuating directly over fuzzy variables which change their value. The techniques are based on Lyapunov functions. Third, we comment about extending this method to non-piecewise linear controllers using the *contrast* definition for a fuzzy controller.

1. Introduction

Adaptive control and self-organizing techniques are used in order to improve the close-loop dynamic of a system. But, although for linear systems several adaptive algorithms have been developed, for fuzzy systems few studies are known because its complexity, while self-organizing controllers have become more popular.

Linguistic self-organizing controllers [4, 6, 9, 10] use a performance table which gives a cost index depending on the deviation of the system output from a desired model. The quantity in which the rules must be modified depends on this index [11].

In the other hand, fuzzy adaptive control is able to modify directly fuzzy variables, for example, they can change their value from *small* to *very small*. This is useful when the parameters of the process to be controlled are unknown, or when dynamic changes in them are frequent.

Another case in which fuzzy adaptive control is useful is in the design procedure of fuzzy controllers explained in [7]: when no control rules are supplied by the expert, nor process model is present, a good idea is to obtain a first *linear* fuzzy controller from the information about a PID controller already functioning. First, using for example Ziegler-Nichols techniques, we design a PID controller. Second, we obtain an equivalent fuzzy controller. And third, we let the adaptive algorithm to improve the controller fuzzy variables.

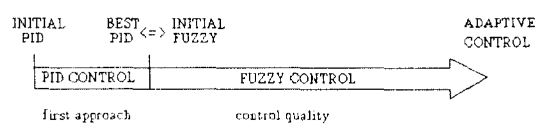

Figure 1. Design Steps.

In this sense, Adaptive Control may complete the design steps of a fuzzy controller.

2. Fuzzy Control Systems (FCS)

Let there consider a Fuzzy Controller [5] with rules expressed in linguistic terms like: IF (E is E_i) AND (CE is CE_j) THEN (CU is CU_{ij}), where E means the Error, CE the Change in Error and CU the Change in the Control Action. This is what we will call a FPI controller, since it has the same inputs and output as a PI one. For the following analysis, the fuzzy sets defined over E and CE are represented with triangular membership functions μ_{Ei} and μ_{CEi} as it is shown in Figure 2, with the particularity that they are overlapped by pairs, they are perfectly symmetrical, and the corresponding fuzzy sets are normal [2]. We will call this one a *normal controller*. For the change in the control action, we do not need in a fuzzy controller to define corresponding fuzzy sets (as we do not have a chain of rules) being sufficient to establish their center of gravity c_{ij} and their area a_{ij}. Let there have N linguistic rules and suppose $a_{ij} = 1$ for all i, j. The controller output is obtained calculating the center of gravity:

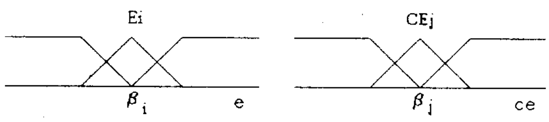

Figure 2. Input Membership Functions.

$$cu(ce) = \frac{\sum_i \sum_j c_{ij} \cdot \min(\mu_i(e), \mu_j(ce))}{\sum_i \sum_j \min(\mu_i(e), \mu_j(ce))} \quad (1)$$

We will call β_{Ei} and β_{CEj} to the values of e and ce, respectively, which match with the maximum value of each membership function. For those points, we only use the rule ij, so the output value cu is equal to c_{ij}. We will call *guide points* to these values (β_{Ei}, β_{CEj}, c_{ij}). Between each two guide points, the fuzzy control function is piecewise-linear only for e = β_{Ei} or for ce = β_{CEj}, as it is shown in Figure 3. Figure 4 shows a linear approach around each guide point.

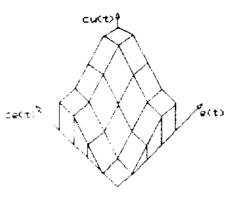

Figure 3. Fuzzy Controller Function

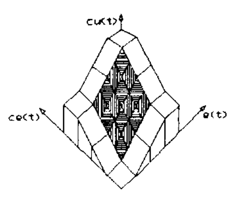

Figure 4. Linear approximation

3. The Controller Contrast

Some important remarks on the previous comments may be advised. If the controller is not normal, the control function, in general, will not pass through the guide points. Also, if the membership functions shape is not piecewise-linear, neither will be the control function. In general, we can define a membership function as

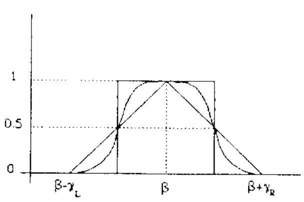

Figure 5. Generic Membership Function

$$\mu_{CE}(ce) = \begin{cases} 0 & , \ if\ ce \leq \beta-\gamma_L \\ \frac{1}{2}\left[2\left(1+\frac{ce-\beta}{\gamma_L}\right)\right]^{\lambda+1} & , \ if\ \beta-\gamma_L \leq ce \leq \beta-\gamma_L/2 \\ 1+\frac{1}{2}\left[\frac{2(ce-\beta)}{\gamma_L}\right]^{\lambda+1} & , \ if\ \beta-\gamma_L/2 \leq ce \leq \beta \\ 1-\frac{1}{2}\left[2\frac{ce-\beta}{\gamma_R}\right]^{\lambda+1} & , \ if\ \beta \leq ce \leq \beta+\gamma_R/2 \\ \frac{1}{2}\left[2\left(1-\frac{ce-\beta}{\gamma_R}\right)\right]^{\lambda+1} & , \ if\ \beta+\gamma_R/2 \leq ce \leq \beta+\gamma_R \\ 0 & , \ if\ ce \geq \beta+\gamma_R \end{cases} \quad (2)$$

with $\lambda > 0$. If $\lambda = 0$, we obtain a triangular membership function. For $\lambda = 1$, we have a polynomial membership function. From $\lambda = 0$ to $\lambda = 1$, we have an increasing derivative for ce = β-γ_L/2 and decreasing for ce = $\beta + \gamma_R$/2. When λ goes to ∞, these derivatives go to $\pm\infty$, reaching a crisp set.

We will call λ the *contrast* of the membership function. We will call *first canonical form* of a fuzzy controller to a normal fuzzy controller with null contrast. From this follows the definition of the *second canonical form* of a fuzzy controller as a normal fuzzy controller with unitary contrast.

4. The Fuzzy Control Function Model

Going back to the definition of the control function, and in the case of the first canonic form, for the linear approach around each guide point we will use the following fuzzy control function model (MISO case):

$$cu(e, ce) = c_{ij} + K_{Eij}(e-\beta_{Ei}) + K_{CEij}(ce-\beta_{CEj}) \quad (3)$$
for $\beta_{Ei} <= e <= (\beta_{Ei}+\beta_{Ei+1})/2$
and $\beta_{CEj} <= ce <= (\beta_{CEj}+\beta_{CEj+1})/2$.

Similar expressions follow for other intervals in e and ce. K_{Eij} and K_{CEij} are the integral and proportional gains of the controller in each interval (β_{Ei}, β_{Ei+1}) and (β_{CEj}, β_{CEj+1}) respectively, and may be obtained as:

$$K_{Eij} = (c_{i+1j}-c_{ij})/(\beta_{Ei+1}-\beta_{Ei}) \quad (4)$$
$$K_{CEij} = (c_{ij+1}-c_{ij})/(\beta_{CEj+1}-\beta_{CEj}) \quad (5)$$

and β_{Ei}, β_{CEj}, c_{ij} depend on the fuzzy sets position.

Finally, an appointment that will be useful later. If one gain changes its value (K_{Eij} or K_{CEij}), some of the rules may be recalculated as follows:

$$c_{ik+1} = c_{ik} + K_{CEik}(\beta_{CEk+1}-\beta_{CEk}),\ \forall k \geq j \quad (6)$$
$$\text{if } 0 <= \beta_{CEj} <= ce <= \beta_{CEj+1}$$

and

$$c_{k+1l} = c_{kl} + K_{CEkl}(\beta_{Ek+1}-\beta_{Ek}), \quad (7)$$
$$\forall k \geq i,\ \forall l \geq j, \text{ if } 0 <= \beta_{Ei} <= e <= \beta_{Ei+1}$$

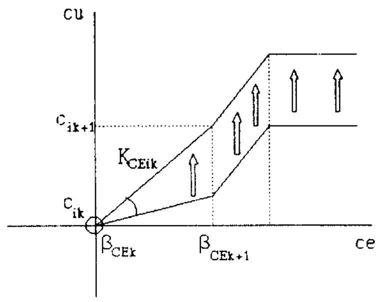

Figure 6. Rule Recalculation

This is the same to adapt the output fuzzy variables:

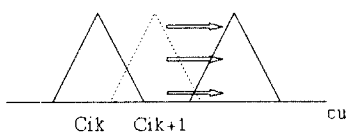

Figure 7. Fuzzy Variables Adaptation

5. The FACE Method

We will call a Fuzzy Adaptive Control Environment (FACE), to a software structure which includes a Model-Reference Adaptive Control (MRAC) structure [8] and uses the the following methodology.

The adaptive algorithm will try to adjust the controller gains K_{Eij} and K_{CEij}, searching for minimizing the difference between the system output y and the output y_M of a reference model G_M. In order to guarantee the convergence of the dynamic behaviour of the feedback system M, we use the Lyapunov stability theorem [1, 3]. The idea is to improve the dynamic in just two steps:

FACE I: tries to adapt fuzzy variables.
FACE II: works over the contrast in each interval, searching for a fine adjustment.

This paper centres in FACE I, appointing some general guidelines about FACE II ideas.

6. Choosing Lyapunov Functions

The Lyapunov function depends on the system to be controlled G, on the controller F and on the reference model G_M. Following we show a very simple example for a linear case for G, a PI controller and a linear G_M:

Process:	$G(s) = 1/(s+a)$	(8)
Controller:	$F(s) = K_E/s + K_{CE}$	(9)
Ref. Model:	$G_M(s) = K/(s^2+T\cdot s+K)$	(10)

with K, T > 0. From the process and controller equations we get:

$$y''(t) + (a+K_E)\cdot y'(t) + K_{CE}\cdot y(t) = K_{CE}\cdot r \quad (11)$$

with r the set point, and

$$y_M''(t) + T\cdot y'_M(t) + K\cdot y_M(t) = K\cdot r \quad (12)$$

so

$$e_0''(t) + T\cdot e_0'(t) + K\cdot e_0(t) = (K_{CEj}-K)\cdot e(t) + (a+K_{Ei}-T)\cdot e'(t) = \phi_E(t)\cdot e(t) + \phi_{CE}(t)\cdot e'(t) \quad (13)$$

being $e_0 = y-y_M$, and $e = r-y$. Now we choose the following Lyapunov function:

$$V(t) = e_0'^2(t) + K\cdot e_0^2(t) + \lambda_{CE}\cdot\phi_{CE}^2 + \lambda_E\cdot\phi_E^2 \quad (14)$$

with λ_E and $\lambda_{CE} > 0$. This means

$$V(t) > 0 \quad (15)$$

and

$$V'(t) = -2T\cdot e_0'(t) < 0, \quad (16)$$

which are the two Lyapunov required conditions, if and only if we choose:

$$\phi_E'(t) = -e(t)\cdot e_0'(t)/\lambda_E \quad (17)$$
$$\phi_{CE}'(t) = -e'(t)\cdot e_0'(t)/\lambda_{CE} \quad (18)$$

so the adaptive rules are

$$K_E'(t) = -e(t)\cdot e_0'(t)/\lambda_E \quad (19)$$
$$K_{CE}'(t) = -e'(t)\cdot e_0'(t)/\lambda_{CE} \quad (20)$$

Now, if we select a fuzzy control function model as shown in point 4,

$$cu(e, ce) = c_{ij} + K_{Eij}(e-\beta_{Ei}) + K_{CEij}(ce-\beta_{CEj}) \quad (21)$$

we will have the following adaptive rules:

$$K_{Eij}'(t) = -e(t)\cdot e_0'(t)/\lambda_E \quad (22)$$
$$K_{CEij}'(t) = -ce(t)\cdot e_0'(t)/\lambda_{CE} \quad (23)$$

and the adaptation will stop when $K_{Ei}=K_i$ and $K_{CEj}=T_j$-a. Note that, when one gain is modified, we must recalculate all the rules hanging at the right/left hand of c_{ij}, depending on the sign of e and ce. More general cases might be also studied using a fuzzy function model for the reference model G_M and for the process model.

7. Examples

The first example uses a classic PI controller with a sample period equal to 1 second, $K_E = 0.08$ and $K_{CE} = 0.4$, and a first order process with a = 2, but unknown. The reference model is also linear and has K = 0.12 and T = 2.6. At the end of the simulation, with $\lambda_E = 1$ and $\lambda_{CE} = 0.1$, we obtained $K_E = 0.12$ and $K_{CE} = 0.4$, which corresponds with $K_E = K$ and $K_{CE} = T\text{-}a$.

The second simulation was carried out for a fuzzy PI controller (FPI) with the following table of rules

e \ ce	-1	0	1
-1	-0.48	-0.08	0.32
-0.5	-0.44	-0.04	0.36
0	-0.40	0.00	0.40
0.5	-0.36	0.04	0.44
1	-0.32	0.08	0.48

this means $K_{E_i} = 0.08$ $\forall i = 1..4$ and $K_{CE_j} = 0.4$ $\forall j = 1..2$. The reference model is the same as in the last case, K = 0.12 and T = 2.6, and starting from the last table we obtain:

e \ ce	-1	0	1
-1	-0.69	-0.10	0.50
-0.5	-0.65	-0.06	0.54
0	-0.59	0.00	0.60
0.5	-0.53	0.05	0.65
1	-0.46	0.12	0.72

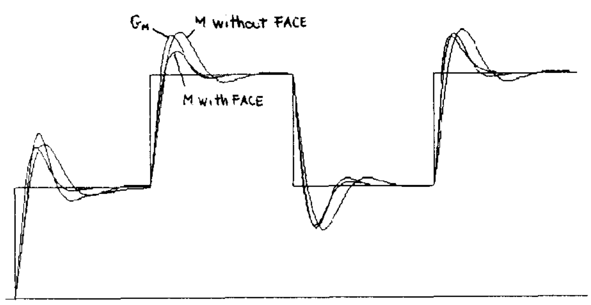

Figure 8. Simulation

Note that the table differs from an ideal PI table. A longer adaptation seems necessary.

8. Future Research

Once we have seen that the adaptive algorithms have success for different regions, future research will center on the following ideas.

The first one is to keep linear process models in order to demonstrate that a fuzzy proportional controller (FP) is sufficient in many cases to reach dynamic specification where P controllers can not.

Non piecewise-linear models, as Sugeno fuzzy model [12], will be the next case to analyze, trying to apply these same ideas. The controller contrast would be useful to simplify the models coping with the canonical forms. The shown algorithm is only appropiate if we use the first canonical form of fuzzy controllers. However, we might try to research deeply on the influence of the controller contrast in order to develop a FACE II algorithm. First experiences have shown a very few influence of this parameter.

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