상수도 관망개선의 최적설계

Optimal Rehabilitation Model for Water Distribution Systems

김 중 훈 *

요지: 이 논문의 목적은 기존의 上水道 管網을 改善하는데 있어 어느 관을 交替 또는 更生할 것인가와 펌프용량을 얼마나 늘릴 것인가를 결정함으로써 관망내 각 급수지점에서의 요구유량및 수압을 만족시킴은 물론 그에 드는 비용을 最小化시키는 모델을 개발하는데 있다. 이 논문은 管交替 費用, 洗管및 更生 費用, 管補修 費用, 펌핑 費用, 펌프施設 擴充費用 등의 다섯가지 비용들을 비교 검토함으로써 의사결정을 하게된다. 制約條件式으로는 급수 조건식, 에너지 방정식, 수리학적 방정식, 결정 조건식, 한계 조건식, 정수 조건식 등이 있다. 이 모델을 數式化하면 整數混合 非線形計劃法 (mixed-integer nonlinear programming, MINLP) 問題가 된다. 이 문제를 풀기 위해 非線形解法의 GRG (generalized reduced gradient) 방법과 分岐와 限界 (branch and bound) 기법을 통한 implicit enumeration 기법을 接木시키는 방법을 제안하였다.

1. Introduction

The primary goal of all water distribution systems is the delivery of water to meet demands on quantity and pressure. Unfortunately, as a system ages, its ability to transport water diminishes and the demands placed upon it are increased. In addition to unsatisfactory performance of a deteriorated network, there are direct economic impacts of a failing system. It has been reported that for some utilities 90% of the total budget is for energy required for pumping. Older systems have reduced carrying capacity due to corrosion and tuberculation and are more susceptible to leaks and breaks resulting in losses of water and requiring time and money to repair. Thus, the rehabilitation, replacement, and/or expansion of an existing system to meet current and future demands of flowrate and pressure head has become a topic of interest.

* 고려대학교 공과대학 토목환경공학과 조교수

Since it is unrealistic and not necessary to rehabilitate and/or replace all pipes in an existing water distribution system, a methodology should decide which pipes in the system should be rehabilitated and/or replaced so that the demand and pressure requirements can be satisfied and the total cost becomes the minimum. Basically, the problem is to determine the trade-offs among the following decisions: 1) replace pipes, 2) rehabilitate pipes, 3) increase pumping power, and 4) do nothing. The associated costs are evaluated for each decision so as to find the optimal combination of these decisions for an entire pipe network accounting for the optimal trade-offs. The objective of the model is to minimize the overall cost associated with the decisions. The overall cost includes the replacement cost (f_1), the rehabilitation cost (f_2), the expected reparation cost (f_3), the energy cost (f_4), and the increasing pumping equipment cost (f_5), some of which are expressed by nonlinear functions. The major constraints in the model are the conservation of mass and energy constraints, the water quantity demand constraints, the nodal pressure head requirement constraints, the pump characteristic constraints, and the non-negativity constraints. Many of those constraints are expressed as nonlinear functions. The decision of whether to replace (or rehabilitate) a pipe or not is represented by a zero-one integer variable. Basically, two integer variables are required for each pipe of the system for replacement and rehabilitation, respectively. The proposed model formulation is a mixed-integer nonlinear programming (MINLP) problem.

2. Model Formulation

The cost functions for pipe replacement, rehabilitation, and repair are regression equations from the U. S. Army Corps of Engineers (1983). The cost function for pipe replacement is expressed as

$$f_1 = \sum_j \left(25.26 DN_j^{1.578} + 33.043\right) L_j N_j \tag{1}$$

where f_1 is in dollars, DN_j and L_j are in feet. The cost function for pipe rehabilitation is expressed as

$$f_2 = \sum_j \left(0.418 DO_j^{2.767} + 23.742\right) L_j R_j \tag{2}$$

where f_2 is in dollars, DO_j and L_j are in feet. The present value of the expected repair cost is given by

$$f_3 = \sum_j \sum_p \left\{ \left[\frac{\left(1300\, DO_j^{0.62}\right)\left(0.819\, e^{-0.1363 DO_j}\right) L_j}{(1+r)^p \;\; 5280} \right] \left(1 - N_j - R_j\right) \right\} \qquad (3)$$

where r is the interest rate and p is the planning period in years. The present value of the energy cost for single loading is given by

$$f_4 = \sum_k \sum_p \left[\frac{6534.96 \; M_k \; HP_k}{(1+r)^p \;\; E_k} \right] \qquad (4)$$

where M_k is the unit cost (\$/kwhr) of electricity for pump k and E_k is the efficiency of pump k. The cost of electrical equipment for pump stations is given by

$$f_5 = \sum_k \left[792.46 \left(HP_k - \underline{HP}_k \right)^{0.71} \right] \qquad (5)$$

in which $\underline{HP}_k$ is pumping power of the existing pump k.

The problem formulation is given as following:

Minimize ($f_1 + f_2 + f_3 + f_4 + f_5$) (6)

subject to

(1) Demand requirement: The amount of water being supplied to each demand node should be greater than or equal to the required demand, expressed as

$$\sum_{j \in I_i} q_j \geq Q_i \qquad \text{for all demand nodes i} \qquad (7)$$

in which I_i is the set of pipes connected to demand node i; q_j is a continuous variable representing the resulting flow rate in pipe j; and Q_i is the required nodal demand at node i.

(2) Pressure head requirement: At each demand node of the water distribution system, the pressure head (h_i) being supplied at the node should be greater than or equal to the minimum allowable pressure head ($\underline{H}_i$) and less than or equal to the maximum pressure head ($\overline{H}_i$), i.e.,

$$\underline{H}_i \leq h_i \leq \overline{H}_i \qquad \text{for all demand nodes i} \qquad (8)$$

(3) Conservation of energy constraints: For each primary loop, i.e., an independent closed path in a water distribution system, if J_s and K_s represent the sets of pipes and pumps in primary loop S, respectively, then the energy conservation equation can be written for pipe sections in the loop as follows:

$$\sum_{j \in J_s} \Delta h_j = \sum_{k \in K_s} PH_k \qquad \text{for all } J_S, K_S \qquad (9)$$

in which Δh_j is a continuous variable representing the resulting head loss in pipe j and PH_k is a continuous variable representing the pumping head of pump k. In case of no pump in a loop, the energy equation (9) states that the sum of the energy losses around the loop equals zero. If there are F fixed grade nodes, F-1 independent energy conservation equations can be written for the pipe sections along the path between any two fixed grade nodes as follows:

$$\Delta E_R = \sum_{j \in J_R} \Delta h_j - \sum_{k \in K_R} PH_k \qquad \text{for all } J_R, K_R \qquad (10)$$

where ΔE_R is the difference in total grade between the two fixed grade nodes, and J_R and K_R represent the sets of pipes and pumps in path R connecting the two fixed grade nodes, respectively. Eq. (9) is a special case of Eq. (10) where the difference in total grade (ΔE) is zero for a path which forms a closed loop.

(4) Decision constraints: These constraints are applied to the model to eliminate the possibility of simultaneously replacing and rehabilitating the same pipe.

$$N_j + R_j \le 1 \qquad \text{for all pipe j} \qquad (11)$$

in which N_j is a binary variable with either a value of one representing the replacement of pipe j or zero otherwise, and R_j is a binary variable with a value of one representing that pipe j is rehabilitated or zero otherwise.

(5) Hydraulic constraints: The Hazen-Williams equation for each pipe in a water distribution system is given by

$$q_j = \frac{AC_j \; AD_j^{2.63} \; \Delta h_j^{0.54}}{(4.73 \, L_j)^{0.54}} \qquad \text{for all pipe j} \qquad (12)$$

in English unit where AD_j is a continuous variable representing the actual diameter of pipe j after the decision is made and L_j is the length of the pipe j. The actual roughness coefficient for pipe j after decision, AC_j, is given by the following equation:

$$AC_j = CO_j \, [1 - (N_j + R_j)] + CR_j R_j + CN_j N_j \qquad \text{for all pipe j} \qquad (13)$$

in which CO_j is the Hazen-Williams roughness coefficient in old pipe j, CR_j is the roughness coefficient in the rehabilitated pipe j, and CN_j is the roughness coefficient in the replaced pipe j. Eq. (13) states that, if N_j or $R_j = 1$, then AC_j equals CN_j or CR_j respectively, otherwise, $AC_j = CO_j$. In other words, if either a replacement or a rehabilitation is performed for pipe j, then the roughness of pipe j is the value for the replaced pipe or the relined value. Otherwise, the value of the roughness is the same as that of the existing old pipe. Similarly, the value of the actual pipe diameter after decision, AD_j, is given by the following equation:

$$AD_j = DO_j (1 - N_j) + DN_j N_j \text{for all pipe j} \tag{14}$$

with a bound constraint

$$DN_j \geq 0 \qquad \text{for all pipe j} \tag{15}$$

where DO_j is the diameter of original pipe j and DN_j is a continuous variable representing diameter of the replaced pipe j.

(6) Pump characteristic constraints: For each pump in a water distribution system, the pumping power is defined by:

$$HP_k = \frac{\gamma \; q_k \; PH_k}{550} \qquad \text{for all pump k} \tag{16}$$

where HP_k is a continuous variable representing the useful pumping power of pump k; γ is the specific weight of water; and q_k is the flow rate through pump k. The constant pump horsepower, HP_k, can not be smaller than the existing constant horsepower, $\underline{HP}_k$, expressed as

$$HP_k \geq \underline{HP}_k \qquad \text{for all pump k} \tag{17}$$

which defines a lower bound on the decision variable HP_k.

(7) Integer (binary) variable characteristics:

$$N_j = [0, 1]; \quad R_j = [0, 1] \qquad \text{for all pipe j} \tag{18}$$

3. Solution Procedure

The proposed model formulation is a mixed-integer nonlinear programming problem. Kim (1992) developed an implicit enumeration algorithm to find the optimal combination for the 0-1 integer variables which represent the optimal rehabilitation plan for the existing water distribution system. Figure 1 shows the incorporation of the program solvers. It shows the connection between the branch and bound (integer) master problem and the nonlinear subproblem. The proposed algorithm solves a nonlinear programming subproblem for each branch node in the enumeration procedure. The integer master problem provides fixed values of the binary variables to the nonlinear subproblem, and the nonlinear subproblem provides the optimal objective values to the master problem. Figure 1 also shows the interfacing of the hydraulic simulation model, KYPIPE by Wood (1980), and the nonlinear solver, GRG2 by Lasdon (1983), in an optimal control framework. Based on a set of initial values for the continuous control variables, DN_j and HP_k, the simulation model is executed and the resulting values of nodal pressure heads, h_i, are transferred back to the nonlinear model.

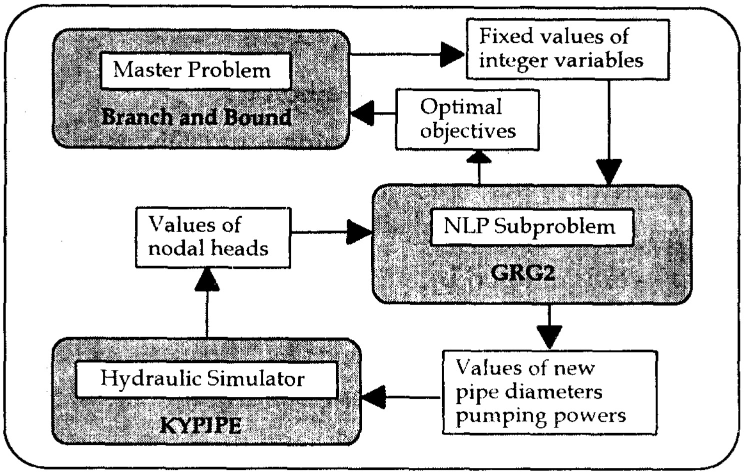

Figure 1 Incorporation of the Program Solvers

4. Application to an Example

Figure 2 shows an example network which consists of 17 pipes and 12 junction nodes. The Hazen-Williams roughness coefficients in all the pipes were assumed to be 50 in order to simulate an old system which does not meet the water demand and pressure head requirements. The Hazen-Williams coefficient in a new pipe was assumed to be 130, while that of a rehabilitated (relined) pipe is 100. Pressures at demand nodes were constrained to be between 40 and 120 psi. Table 1 lists the optimal decisions for all the pipes and Table 2 is a list of the minimum costs for example two. It should be noted that the pipe repair cost and the pumping energy cost are for a 20 year time period with all costs converted to present worth.

One way to calibrate the results of this example is to examine all possible decision combinations of the integer variables. However, there are more than 129 million combinations (3^{17} = 129,140,163) of possible decisions for this example with 17 pipes. A procedure which selects a possible system configuration utilizing a random number generation. One thousand possible system configrations were examined by randomly selecting 0 or 1 for the integer (binary) variables. The minimum total cost obtained from the 1000 cases is $12,208,387. This cost is $248,342 higher than the cost $11,960,045 obtained from the new methodology presented in this paper. Table 3 shows the comparison among those costs.

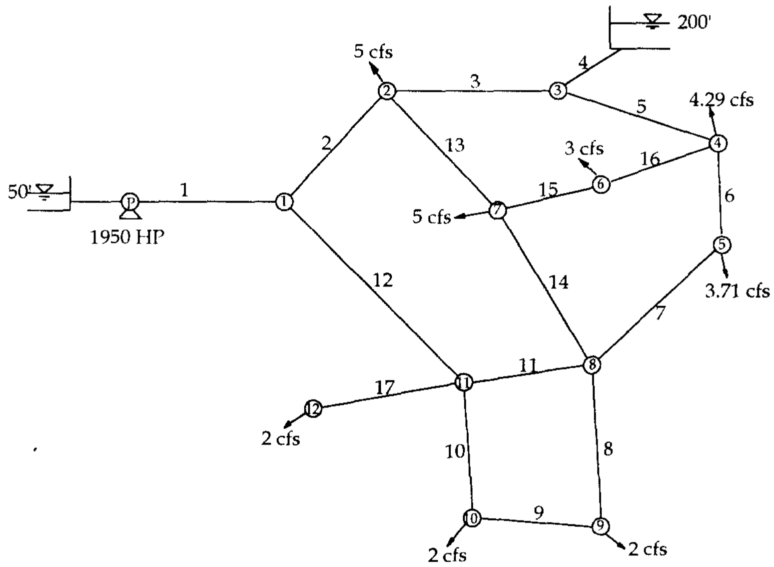

Figure 2 Example Water Distribution System

Table 1 Optimal Decisions for All Pipes and Pump

Component	Decision	Pipe Diameter (inches)	
		Before Decision	After Decision
pipe 1	reline	24	24
pipe 2	reline	18	18
pipe 3	as is	18	
pipe 4	as is	6	
pipe 5	as is	15	
pipe 6	as is	15	
pipe 7	as is	15	
pipe 8	as is	12	
pipe 9	as is	9	
pipe 10	as is	12	
pipe 11	reline	12	12
pipe 12	replace	15	17.22
pipe 13	replace	15	10.59
pipe 14	as is	15	
pipe 15	as is	15	
pipe 16	as is	15	
pipe 17	as is	9	
pump	as is	1950 HP	

Table 2 List of the Optimal Costs

	Cost ($)
Replacement Cost	774,061
Rehabilitation Cost	444,153
Repair Cost	154,624
Energy Cost	10,587,207
Pump Equipment Cost	0
Total Cost	11,960,045

Table 3 Comparison of the Optimal Costs

Total Costs ($) out of the 1000 Random Number Generation		Total Cost ($) from the New Methodology
		11,960,045
Best	12,208,387	
Second Best	12,281,031	
Third Best	12,312,632	

5. Conclusions

A new methodology has been presented for determining the minimum cost rehabilitation and replacement of pipes in water distribution systems. The developed model is able to determine which pipes in an existing water distribution system should be rehabilitated and/or replaced so that overall cost is minimized and all constraints are satisfied. The problem is formulated as a mixed-integer nonlinear programming problem. An implicit enumeration procedure using a branch and bound algorithm was used to find an optimal combination of the binary variables, which represent the optimal rehabilitation plan for the existing water distribution system. The nonlinear subproblem size is reduced by interfacing a hydraulic simulator with a nonlinear solver and by using a penalty method.

Although global optimality cannot be guaranteed, the results from the applications demonstrate the model's ability to find optimal solutions. The comparison of an optimal cost obtained from the model with the minimum cost obtained from the 1,000 random system configurations supports the optimality of the solution. The concept of the solution methodology in this paper is new for solving the optimal rehabilitation and replacement problem for water distribution system components. However, the fact that there can be numerous alternate optima should raise concern among practicing engineers and prompt more research efforts.

6. References

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