# DYNAMIC ANALYSIS OF FLEXIBLE ROTORS SUBJECTED TO TORQUE AND FORCE

# (토오크 및 힘을 받는 탄성 회전체계의 동적 해석)

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## ABSTRACT

The effect of the applied direction and magnitude of loads on the stability and natural frequency of flexible rotors is analyzed, when the rotors are subject to nonconservative torque and force. The stability criterion derived from the energy and variational principle is discussed and a general Galerkin's method which utilizes admissible functions is employed for numerical analysis. Illustrative examples are treated to demonstrate the analytical developments.

## 1.INTRODUCTION

A rotor-bearing system is seldom used alone without being connected to a power transmission system. For example, a pump is often connected to a motor with a cooling fan through coupling and a turbopump in space shuttle main engine is driven by a gas turbine. Thus such rotor-bearing systems are often modeled as the flexible rotors subjected to torque and force acting upon rotating shafts in the longitudinal direction. On the other hand, the torque and force acting upon disks are generated by the impeller fluid resistance and the pressure difference between impellers, respectively. As the rotational speed increases to allow high power, these work loads may cause the rotordynamic instability problems, leading to costly failures in many cases.

For finding the critical speeds of rotating shafts subjected to work loads, Willems and Holzer (1967), and Eshleman and Eubanks (1969) employed the Galerkin's and the analytic methods, respectively; Zorzi and Nelson (1980) used the finite element method to study the effect of axial torque on the dynamics of rotor-bearing-systems; Shieh (1982) studied, using the variational principle, the stability of rotating circular and elliptic shafts subjected to nonconservative loads; Yim, Noah, and Vance(1986) investigated the effect of tangential load torque on the dynamics of flexible rotors by utilizing the transfer matrix method and showed that the applied torque generates positive real parts of eigenvalues of the system. In recent years, Chen and Ku (1992) used the finite element method to study the dynamic stability of a cantilever shaft-disk system subjected to axial periodic forces; Czolczynski and Marynowski (1992) studied the stability of the Laval rotor subjected to a longitudinal force acting on a disk.

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However, the case with the nonconservative torque and force simultaneously applied to rotating shafts has not been investigated and the effects of the change in direction and magnitude of work loads on the stability and natural frequency of flexible rotors have not yet been addressed. In this study, we derive the qualitative stability criterion using the energy and variational principles to gain physical insights into the behavior of a flexible rotor system and develop a general Galerkin's method which efficiently calculates the eigenvalues of the system when the system is subject to nonconservative torque and force with the applied direction and magnitude varied.

## 2.ANALYSIS OF FLEXIBLE ROTORS

In this section, the governing equations and the associated boundary conditions are derived applying the Newton's 2nd law, when a flexible rotor is subject to nonconservative torque and force. Then the qualitative stability analysis technique and a general Galerkin's method are developed.

### 2.1 Equations of Motion of Flexible Rotors

Equations of motion of a rotating uniform shaft with rigid disks, as shown in Fig. 1, including the effects of constant applied, axial or tangential, torque and force, rotary inertia, and gyroscopic moment, can be derived as (Shieh, 1982; Lee, 1993)

$$EI\frac{\partial^4 y}{\partial x^4}+T\frac{\partial^3 z}{\partial x^3}+P\frac{\partial^2 y}{\partial x^2}+m(x)\frac{\partial^2 y}{\partial t^2}-\frac{\partial}{\partial x}[J_T(x)\frac{\partial^3 y}{\partial x\partial t^2}]-\Omega\frac{\partial}{\partial x}[J_P(x)\frac{\partial^2 z}{\partial x\partial t}]=f_y(x,t),$$

$$EI\frac{\partial^4 z}{\partial x^4}-T\frac{\partial^3 y}{\partial x^3}+P\frac{\partial^2 z}{\partial x^2}+m(x)\frac{\partial^2 z}{\partial t^2}-\frac{\partial}{\partial x}[J_T(x)\frac{\partial^3 z}{\partial x\partial t^2}]+\Omega\frac{\partial}{\partial x}[J_P(x)\frac{\partial^2 y}{\partial x\partial t}]=f_z(x,t), \quad (1)$$

where y(x,t) and z(x,t), and $f_y(x,t)$ and $f_z(x,t)$ are the displacements and forces, respectively, in the y and z directions, m(x) is the mass per unit length, $J_T(x)$ the diametral mass moment of inertia per unit length, $J_p(x)$ the polar mass moment of inertia per unit length, EI the flexural rigidity, $\Omega$ the rotational speed, x the position coordinate along the shaft of length L, and T and P the applied torque and compressive force at the boundary x = 0, L. And the mass, and the polar and diametral mass moment of inertia of disks are included in the terms associated with m(x), $J_P(x)$,

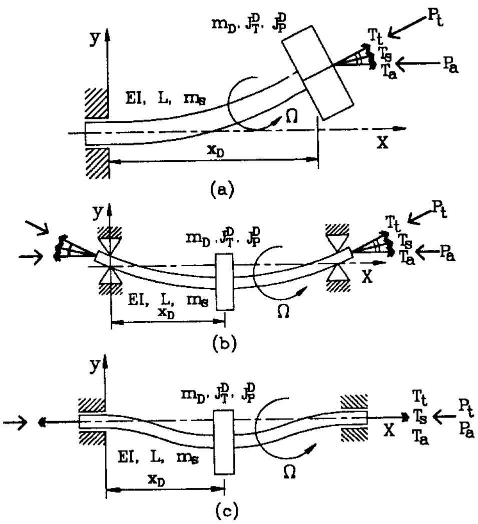

Fig.1 Coordinate, and positive torque and force:
(a) cantilevered overhung rotor,
(b) rotating shaft on rigid short bearings,
(c) rotating shaft on rigid long bearings

and $J_T(x)$, respectively.

The boundary conditions including the terms associated with the torque and force can be derived, at x= 0, L, as (Shieh, 1982; Lee, 1993)

$$y\{EI\frac{\partial^3 y}{\partial x^3}+T\frac{\partial^2 z}{\partial x^2}+(1-\beta)P\frac{\partial y}{\partial x}\}=0,$$
$$z\{EI\frac{\partial^3 z}{\partial x^3}-T\frac{\partial^2 y}{\partial x^2}+(1-\beta)P\frac{\partial z}{\partial x}\}=0,$$
$$\frac{\partial y}{\partial x}\{EI\frac{\partial^2 y}{\partial x^2}+(1-\alpha)T\frac{\partial z}{\partial x}\}=0,$$
$$\frac{\partial z}{\partial x}\{EI\frac{\partial^2 z}{\partial x^2}-(1-\alpha)T\frac{\partial y}{\partial x}\}=0, \tag{2}$$

where α=0 and α=1 correspond to the axial ($T_a$) and tangential torque ($T_t$) loading cases, respectively, and, β=0 and β=1 correspond to the axial ($P_a$) and tangential force ($P_t$) loading cases, respectively. In particular, α=0.5 corresponds to the semi-tangential torque case ($T_s$).

Introducing the complex notations u(x,t) = y(x,t) + j z(x,t) and f(x,t) = $f_y(x,t)$ + j $f_z(x,t)$, and the dimensionless variables,

$$\zeta=\frac{u}{L},\ \tau=\Omega t,\ \chi=\frac{x}{L},\ \Xi(\chi,\tau)=\frac{f(x,t)L^3}{EI},\ C^2(\chi)=\frac{m(x)\Omega^2L^4}{EI},$$
$$H=\frac{TL}{EI},\ \kappa=\frac{PL^2}{EI},\ C_T^2(\chi)=\frac{J_T(x)\Omega^2L^2}{EI},\ C_P^2(\chi)=\frac{J_P(x)\Omega^2L^2}{EI}, \tag{3}$$

we can rewrite Eqs. (1) and (2) as

$$\frac{\partial^4\zeta}{\partial\chi^4}-jH\frac{\partial^3\zeta}{\partial\chi^3}+\kappa\frac{\partial^2\zeta}{\partial\chi^2}+C^2(\chi)\frac{\partial^2\zeta}{\partial\tau^2} \tag{4}$$
$$-\frac{\partial}{\partial\chi}[C_T^2(\chi)\frac{\partial^3\zeta}{\partial\tau^2\partial\chi}]+j\frac{\partial}{\partial\chi}[C_P^2(\chi)\frac{\partial^2\zeta}{\partial\tau\partial\chi}]=\Xi(\chi,\tau),$$

and, at χ = 0, 1,

$$\zeta[\frac{\partial^3\zeta}{\partial\chi^3}-jH\frac{\partial^2\zeta}{\partial\chi^2}+(1-\beta)\kappa\frac{\partial\zeta}{\partial\chi}]=0,$$
$$\frac{\partial\zeta}{\partial\chi}[\frac{\partial^2\zeta}{\partial\chi^2}-j(1-\alpha)H\frac{\partial\zeta}{\partial\chi}]=0. \tag{5}$$

Here when κ > 0 (κ < 0), the force is compressive (tensile).

## 2.2 Stability Criterion

A qualitative stability criterion for gyroscopic nonconservative systems, which is formulated in "equivalent energy" term (Shieh, 1982), is derived and demonstrated through the examples of rotating shafts subjected to the applied torque and force.

When Ξ(χ, τ) = 0, substitution of $\zeta(\chi,\tau)=\psi(\chi)e^{\lambda\tau}$ into Eqs. (4) and (5) yields the complex eigenvalue problem in λ and ψ, i.e.,

$$\psi''''-jH\psi'''+\kappa\psi''+C^2(\chi)\psi\lambda^2 \tag{6a}$$
$$-[C_T^2(\chi)\psi']'\lambda^2+j[C_P^2(\chi)\psi']'\lambda=0,$$
$$\psi[\psi'''-jH\psi''+(1-\beta)\kappa\psi']=0,$$
$$\psi'[\psi''-j(1-\alpha)H\psi']=0. \tag{6b}$$

Multiplying Eq. (6a) by $\overline{\psi}$ (the complex conjugate of eigenfunction, ψ) and integrating over 0 < χ < 1, and using the boundary conditions (6b), we obtain

$$K\lambda^2-jG\lambda+(Q-jW)=0, \tag{7}$$

where

$$K=\int_0^1 C^2(\chi)\overline{\psi}\psi d\chi+\int_0^1 C_T^2(\chi)\overline{\psi}'\psi' d\chi,$$
$$G=\int_0^1 C_P^2(\chi)\overline{\psi}'\psi' d\chi,$$
$$Q=\int_0^1\overline{\psi}''\psi''d\chi-\kappa\int_0^1\overline{\psi}'\psi'd\chi+\frac{1}{2}jH[\int_0^1\overline{\psi}'\psi''d\chi-\int_0^1\overline{\psi}''\psi'd\chi]$$
$$+\frac{1}{2}\beta\kappa[\overline{\psi}(1)\psi'(1)+\overline{\psi}'(1)\psi(1)-\overline{\psi}(0)\psi'(0)-\overline{\psi}'(0)\psi(0)],$$
$$W=\frac{1}{2}H[\overline{\psi}'(1)\psi'(1)-\overline{\psi}'(0)\psi'(0)](1-2\alpha)$$
$$+\frac{1}{2}j\beta\kappa[\overline{\psi}(1)\psi'(1)-\overline{\psi}'(1)\psi(1)-\overline{\psi}(0)\psi'(0)+\overline{\psi}'(0)\psi(0)].$$

The real quantities K, G, Q, and W which have the dimension of an energy but are fictitious quantities, are associated with the conservative kinetic energy, conservative gyroscopic energy, conservative potential energy, and nonconservative work due to the applied torque and force, respectively. Also K is symmetric and positive definite; Q is symmetric; -jG and -jW are skew-symmetric.

From Eq. (7), the stability criteria are obtained as follows:

*The state is stable against all types of instability, flutter and divergence instability, if and only if*

$Q > 0,\quad W = 0,\quad for\ G = 0,$ (8a)

$G^2+4KQ > 0,\ W = 0,\quad for\ G \neq 0,$ (8b) *for all eigenfunctions. The flutter instability occurs if and only if, for at least one mode, either*

$$W \neq 0, \qquad (9a)$$

$$\text{or } G^2+4KQ < 0, W = 0, \text{ for } G \neq 0. \qquad (9b)$$

*The divergence instability occurs if and only if*

$$Q < 0, W = 0, \text{ for } G = 0. \qquad (10)$$

From the viewpoint of design of flexible rotors, for system stability, it is necessary that the quantity W be zero. Then the conditions for W to be zero are: $\alpha$=0.5 and $\beta$=0, or $\psi'(1)=\pm\psi'(0)$ and $\psi(1)=\psi(0)=0$, or $\psi'(1)=\psi'(0)=\psi(1)=\psi(0)=0$, etc. Therefore one can conclude that a rotating shaft with disks symmetrically arranged with respect to its midspan supported by rigid short bearings, in which $\psi'(1)=\pm\psi'(0)$ and $\psi(1)=\psi(0)=0$ are satisfied, and a rotating shaft with disks at arbitrary locations supported by rigid long bearings, in which $\psi'(1)=\psi'(0)=\psi(1)=\psi(0)=0$ are satisfied, remain stable regardless of the applied direction and magnitude of torque and force.

## 2.3 General Galerkin's Method

In the present work , a general Galerkin's method (Leipholz, 1987) is employed in order to find approximate solutions. The restricted Galerkin's method assumes that the response $\zeta(\chi,\tau)$ can be represented in the form of an infinite series

$$\zeta(\chi,\tau)=\sum_{n=1}^{\infty}\phi_n(\chi)q_n(\tau), \qquad (11)$$

where the base functions, $\phi_n(\chi)$, are normally taken as the comparison functions, which satisfy all the boundary conditions and $q_n(\tau)$ are the generalized coordinates. However, the comparison functions are not easily found due to the complexity involved with the boundary conditions. The restricted Galerkin's method can be generalized such that the requirement for the base functions is relaxed to approximate $\zeta$ with a truncated expansion

$$\zeta(\chi,\tau)\approx\hat{\zeta}(\chi,\tau)=\sum_{n=1}^{N}\phi_n(\chi)q_n(\tau), \qquad (12)$$

where the base functions, $\phi_n(\chi)$, are taken as the admissible functions, which satisfy only the geometric boundary conditions.

When $\Xi(\chi,\tau)=0$, the residuals in the domain ($0<\chi<1$) and on the boundary shear force and bending moment, at $\chi=0,1$, are

$$R_A=\hat{\zeta}''''-jH\hat{\zeta}'''+\kappa\hat{\zeta}''+C^2(\chi)\ddot{\hat{\zeta}}-[C_T^2(\chi)\ddot{\hat{\zeta}}']'+j[C_P^2(\chi)\dot{\hat{\zeta}}']', \qquad (13a)$$

and

$$R_{\Gamma_F}=\hat{\zeta}'''-jH\hat{\zeta}''+(1-\beta)\kappa\hat{\zeta}',$$
$$R_{\Gamma_M}=\hat{\zeta}''-j(1-\alpha)H\hat{\zeta}'. \qquad (13b)$$

The weighted sum of the residuals in the domain and on the boundary for variational formulation can then be written as

$$\int_A w_m R_A dA+\int_{\Gamma_F} w_m^F R_{\Gamma_F} d\Gamma+\int_{\Gamma_M} w_m^M R_{\Gamma_M} d\Gamma=0,$$
$$(m=1, 2,\ldots, N), \qquad (14)$$

where the weighting functions $w_m, w_m^F$, and $w_m^M$ are chosen as $\phi_m(\chi)$, $-\phi_m(\chi)$, and $\phi'_m(\chi)$, respectively. Clearly, if Eq. (14) is satisfied for a large number of functions $w_m, w_m^F$, and $w_m^M$, then the expansion $\hat{\zeta}(\chi,\tau)$ must approach the exact solution $\zeta(\chi,\tau)$. Integration by parts of Eq. (14) yields

$$M\ddot{q}+C\dot{q}+Kq=f, \qquad (15)$$

where **M, C** and **K** are the complex matrices of the order NxN and **q** is the N-dimensional vectors.

## 3.NUMERICAL ANALYSIS AND RESULTS

The stability and the modal frequency of three models of a cantilevered overhung rotor, and rotating shafts on rigid short bearings (simple supports) and rigid long bearings (clamped supports), are examined when the direction and magnitude of the applied torque and force are changed.

For the three models with one disk, shown in Fig. 1, the rotary inertia and gyroscopic moment of the shaft can be neglected in comparison with the disk, but the shaft mass is considered. The equation of motion is the same as Eq. (4) with the dimensionless parameters

$$C^2(\chi)=C_S^2+C_D^2\delta(\chi-\mu),$$
$$C_T^2(\chi)=C_T^2\delta(\chi-\mu), C_P^2(\chi)=C_P^2\delta(\chi-\mu), \qquad (16)$$

where

$$C_S^2=\frac{m_S\Omega^2L^4}{EI}, C_D^2=\frac{m_D\Omega^2L^3}{EI},$$
$$C_T^2=\frac{J_T^D\Omega^2L}{EI}, C_P^2=\frac{J_P^D\Omega^2L}{EI}.$$

Here $m_S$ is the shaft mass per unit length, $m_D$ the disk mass, and $J_T^D$ and $J_P^D$ the diametral and polar mass moment of inertia of the disk. And $\delta(\chi)$ is the Dirac delta function of $\chi$ defined as

$$\delta(\chi)=0 \text{ if } \chi\neq0; \quad \int_{-\infty}^{\infty}\delta(\chi)d\chi=1, \qquad (17)$$

and $\mu$ is the dimensionless position of disk located at $x = x_D$.

$$\mu=\frac{x_D}{L}(0<\mu\leq1). \qquad (18)$$

### (1) Cantilevered Overhung Rotor

As shown in Fig. 1(a), a cantilevered overhung rotor is considered. The associated dimensionless boundary conditions are given by

$$\zeta(0,\tau)=\zeta'(0,\tau)=0,$$
$$[\frac{\partial^3\zeta}{\partial\chi^3}-jH\frac{\partial^2\zeta}{\partial\chi^2}+(1-\beta)\kappa\frac{\partial\zeta}{\partial\chi}]_{\chi=1}=0,$$
$$[\frac{\partial^2\zeta}{\partial\chi^2}-j(1-\alpha)H\frac{\partial\zeta}{\partial\chi}]_{\chi=1}=0. \qquad (19)$$

The residuals in the domain and on the boundary are written as

$$R_A=\hat{\zeta}''''-jH\hat{\zeta}'''+\kappa\hat{\zeta}''+C_S^2(\chi)\ddot{\hat{\zeta}}+C_D^2\delta(\chi-\mu)\ddot{\hat{\zeta}}$$
$$-[C_T^2\delta(\chi-\mu)\ddot{\hat{\zeta}}']'+j[C_P^2\delta(\chi-\mu)\dot{\hat{\zeta}}']',$$
$$R_{\Gamma_F}=[\hat{\zeta}'''-jH\hat{\zeta}''+(1-\beta)\kappa\hat{\zeta}']_{\chi=1},$$
$$R_{\Gamma_M}=[\hat{\zeta}''-j(1-\alpha)H\hat{\zeta}']_{\chi=1}. \qquad (20)$$

Typical base functions , $\phi_n$, satisfying only the geometric boundary conditions $\zeta(0, \tau) = \zeta'(0, \tau) = 0$, are the eigenfunctions of the non-rotating clamped-free uniform beam satisfying the equation

$$\phi_n'''' - \lambda_n^4 \phi_n = 0, \qquad (21)$$

in which the dimensionless natural frequencies and $\phi_n$ are given by

$$\omega_n^* = \pm \sqrt{\frac{EI}{m_s \Omega^2 L^4}}\, \lambda_n^2 \qquad (22)$$

$$\phi_n(\chi) = \cosh(\lambda_n \chi) - \cos(\lambda_n \chi) - \sigma_n\{\sinh(\lambda_n \chi) - \sin(\lambda_n \chi)\}.$$

Here $\lambda_n$ and scaling factors $\sigma_n$ are available in (Blevins, 1979).

To investigate the effect of torque and force on the stability and natural frequency of the system, the following numerical values have been used : shaft length L=0.254 m ; shaft diameter D=0.0381 m ; rotational speed $\Omega$=7000 rpm ; shaft density $\rho$=7833 Kg/m$^3$ ; Young's modulus E=2x10$^{11}$ N/m$^2$; disk polar mass moment of inertia $J_P^D$=1.808x10$^{-1}$ Kg-m$^2$ ; disk diametral mass moment of inertia $J_T^D$=9.040x10$^{-2}$ Kg-m$^2$; disk mass $m_D$=31.77 Kg ; disk location $x_D$=0.254 m . The related dimensionless parameters are calculated to be $C_S^2$ = 0.965, $C_D^2$ = 13.52, $C_T^2$ = 0.596, $C_P^2$ = 1.192, and $\mu$ = 1. The data represents one stage of a typical modern high-speed turbomachine. In the general Galerkin's method, twenty base functions are used.

When the torque and force are applied simultaneously, the applied direction of torque has a significant effect on the stability of the system, whereas the applied direction of force has a significant effect on the natural frequency. Figure 2 shows how the logarithmic decrement is changed with the applied direction and magnitude of torque and the applied direction of constant compressive force ($\kappa$=2). When the force is of axial type ($\beta$=0), the values of W for $\alpha$=1 (tangential torque; Fig. 2(a)) and $\alpha$=0 (axial torque; Fig. 2(c)) are the same in magnitude but reversed in sign. Thus the logarithmic decrements are reversed in sign. On the other hand, when the force is of tangential type ($\beta$=1), the values of W for $\alpha$=1 (Fig. 2(d)) and $\alpha$=0 (Fig. 2(f)) are not the same in magnitude but reversed in sign, leading to reversal in the unstable modes. For the special case with $\alpha$=0.5 (semi-tangential torque) and $\beta$=0 (axial force), since W=0 irrespective of the applied torque and thus the system is conservative, all modes are in the stability limit, as can be seen from Fig. 2(b) unless H, the torque, exceeds the critical value determined from $G^2$+4KQ=0. Beyond the critical torque, it holds $G^2$+4KQ < 0 and thus flutter instability takes place. On the other hand, for the case with $\alpha$=0.5 (semi-tangential torque) and $\beta$=1 (tangential force), as shown in Fig. 2(e), W≠0 and thus flutter instability takes place as the torque is applied. It can be concluded here that forward (backward) modes become unstable when the axial (tangential) type torque with 0<$\alpha$<0.5 (0.5<$\alpha$<1) is applied. In other words, the stable and unstable mode switching occurs at $\alpha$=0.5. It implies that, in practical rotors where it is difficult to define the type of applied torque, unstable modes may not be clearly identified.

Figure 3 shows the results with applied force only. When $\beta$=0 (axial force) and H=0, W=0 and thus the system is conservative, leading to pure imaginary eigenvalues. Figure 3(a) shows that all modal frequencies tend to decrease in magnitude with the compressive force ($\kappa$) increased until flutter instability is encountered at the critical $\kappa$ satisfying

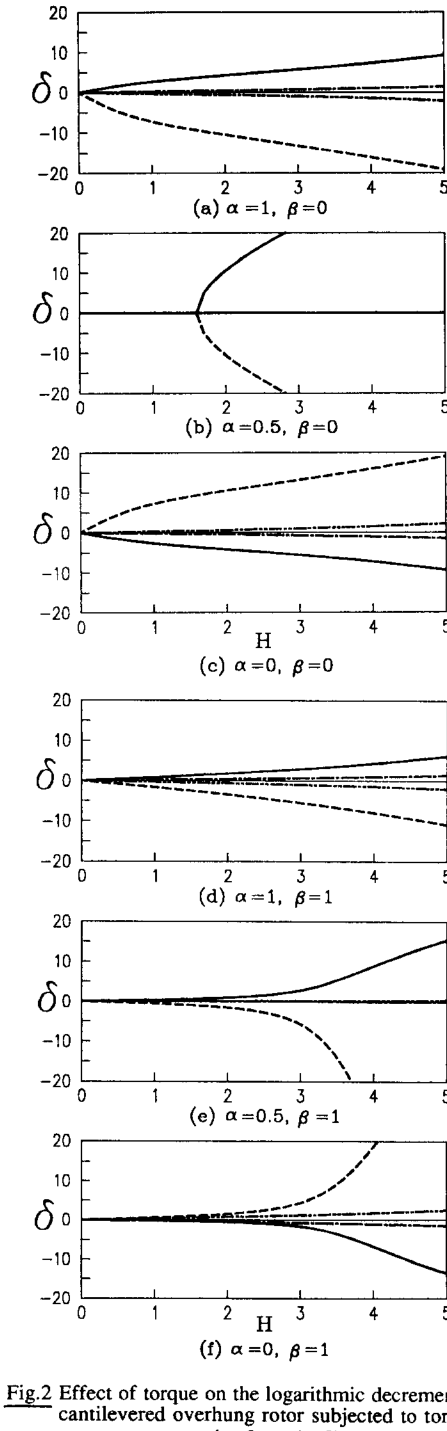


Fig.2 Effect of torque on the logarithmic decrement of the cantilevered overhung rotor subjected to torque and constant compressive force ($\kappa$=2):
———1F; – – – –1B; ——- - —2F, —— - - —2B

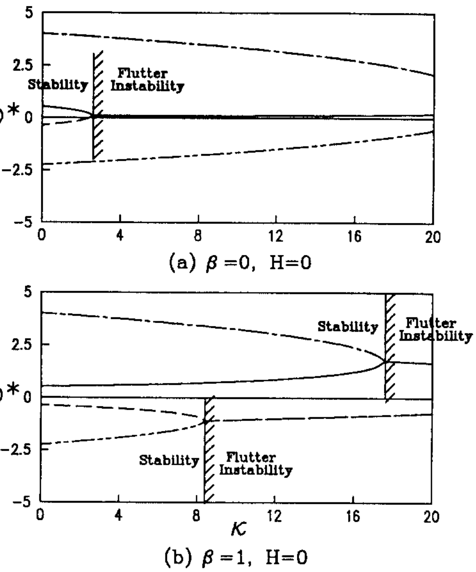


Fig.3 Effect of force on the stability and the dimensionless natural frequency of the cantilevered overhung rotor subjected to force only (H=0):
——— 1F; – – – 1B; —— – —— 2F; —— - - —— 2B

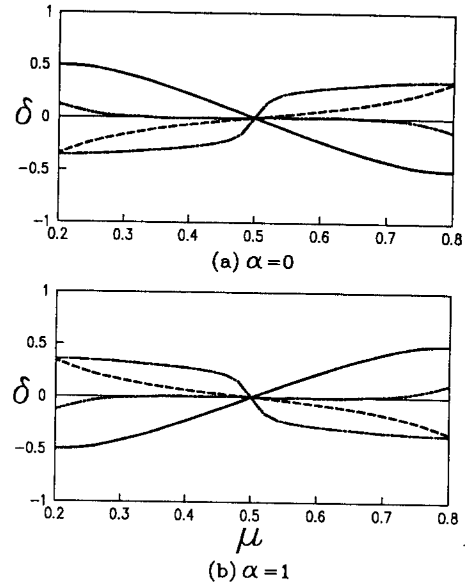

Fig.4 Effect of disk location on the logarithmic decrement of the rotating shaft on rigid short bearings (H=2, κ=2):
——— 1F; – – – 1B; —— – —— 2F; —— - - —— 2B

$G^2+4KQ = 0$. When β=1 (tangential force), as shown in Fig. 3(b), the first (second) forward and backward modal frequencies tend to increase (decrease) with κ increased, leading to flutter instability beyond critical value of κ.

### (2) A Rotating Shaft on Rigid Short Bearings (Simple Supports)

Consider a flexible rotating shaft with a disk on rigid short bearings (simple supports), as shown in Fig. 1(b). The related dimensionless boundary conditions, which hold for both the axial and tangential force cases, become

$$\zeta = 0, [\frac{\partial^2 \zeta}{\partial \chi^2} - j(1-\alpha)H\frac{\partial \zeta}{\partial \chi}] = 0, \quad at \quad \chi = 0,1. \tag{23}$$

To apply the general Galerkin's method, the eigenfunctions of the non-rotating uniform Euler-Bernoulli beam with simply-supported boundary conditions are selected as the required set of admissible functions, that is,

$$\phi_n(\chi) = \sqrt{2}\sin(\lambda_n \chi) \; ; \quad \lambda_n = n\pi. \tag{24}$$

The residuals in the domain and on the boundary are written as

$$R_A = \hat{\zeta}'''' - jH\hat{\zeta}''' + \kappa\hat{\zeta}'' + C_S^2(\chi)\ddot{\hat{\zeta}} + C_D^2\delta(\chi-\mu)\ddot{\hat{\zeta}}$$
$$-[C_T^2\delta(\chi-\mu)\ddot{\hat{\zeta}}']' + j[C_P^2\delta(\chi-\mu)\dot{\hat{\zeta}}']', \tag{25}$$
$$R_{\Gamma_M} = [\hat{\zeta}'' - j(1-\alpha)H\hat{\zeta}']_{\chi=0,1}.$$

To investigate the effect of torque and force on the eigenvalues of the system, the following numerical values simply assumed have been used: shaft length L=2.0 m ; shaft diameter D=0.2 m ; rotational speed Ω=35952.6 rpm ; shaft density ρ=7833 $Kg/m^3$ ; Young's modulus $E=2\times10^{11}$ $N/m^2$; disk polar mass moment of inertia $J_P^D$=25.34 $Kg\text{-}m^2$ ; disk diametral mass moment of inertia $J_T^D$=12.67 $Kg\text{-}m^2$ ; disk mass $m_D$=2583.85 Kg . In the numerical simulations, twenty base functions are used with the related dimensionless parameters , $C_S^2$ = 3553, $C_D^2$ = 18653, $C_T^2$ = 22.87, and $C_P^2$ = 45.74 .

For the case of the simply supported rotating shaft with a disk at its midspan (μ=0.5), W becomes zero because either ψ′(1) = ψ′(0) or ψ′(1) = -ψ′(0) holds due to the geometric symmetry with respect to the midpoint. Thus the applied torque and force do not induce instability. Figure 4 shows the results with constant torque (H=2) and constant compressive force (κ=2). The logarithmic decrements vary as the applied direction of torque and disk location (μ) are changed. In particular, flutter instability takes place unless μ=0.5 where switching between the stable and unstable modes occurs. The switching also occurs with the type of axial force.

### (3) A Rotating Shaft on Rigid Long Bearings (Clamped Supports)

Consider a flexible rotating shaft with a disk supported by rigid long bearings (clamped supports), as shown in Fig. 1(c). The related dimensionless boundary conditions, which hold for any types of torque and force, are given by

$$\zeta = 0, \frac{\partial \zeta}{\partial \chi} = 0, \quad at \quad \chi = 0,1. \tag{26}$$

In this case, we use base functions as the eigenfunctions of the non-rotating clamped-clamped uniform beam given by

$$\phi_n(\chi) = \cosh(\lambda_n\chi) - \cos(\lambda_n\chi) - \sigma_n\{\sinh(\lambda_n\chi) - \sin(\lambda_n\chi)\}, \tag{27}$$

in which $\lambda_n$ and scaling factors $\sigma_n$ are available in (Blevins, 1979). Since the selected base functions satisfy all the boundary conditions, the residual is defined only in the domain as

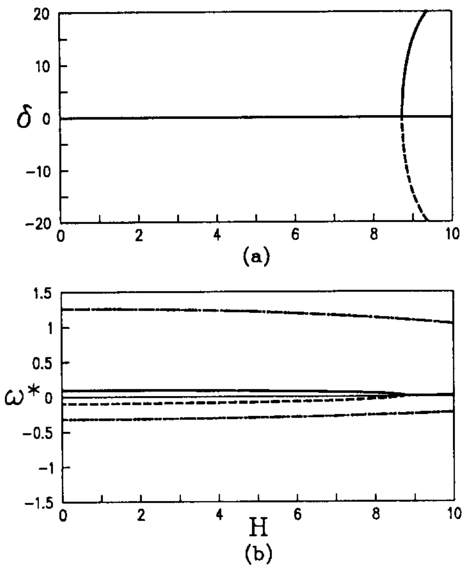


Fig.5 Effect of torque on the logarithmic decrement and the dimensionless natural frequency of the rotating shaft on rigid long bearings (κ=2, μ=0.5):
(a) logarithmic decrement,
(b) dimensionless natural frequency;
——— 1F; – – – 1B; —— – — 2F; —— - - — 2B

$$R_A = \hat{\zeta}'''' - jH\hat{\zeta}''' + \kappa\hat{\zeta}'' + C_S^2(\chi)\ddot{\hat{\zeta}} + C_D^2\delta(\chi-\mu)\ddot{\hat{\zeta}}$$
$$-[C_T^2\delta(\chi-\mu)\ddot{\hat{\zeta}}']' + j[C_P^2\delta(\chi-\mu)\dot{\hat{\zeta}}']'. \quad (28)$$

To investigate the effect of torque and force on the stability and the natural frequencies of this system, we use the numerical values identical to the case of the rotating shaft on rigid short bearings with $C_S^2 = 3553$, $C_D^2 = 18653$, $C_T^2 = 22.87$, $C_P^2 = 45.74$, and twenty base functions. For the case of the clamped supported rotating shaft subjected to torque and force, the system is conservative, leading to W = 0, regardless of the disk location. Therefore the eigenvalues become pure imaginary unless $G^2+4KQ < 0$. Figure 5 shows the results with constant compressive force (κ=2) and disk location (μ=0.5). All modal frequencies tend to decrease in magnitude with the torque (H) increased until flutter instability is encountered at the critical H satisfying $G^2+4KQ = 0$.

## 4.CONCLUSIONS

The stability and the natural frequency of flexible rotors under nonconservative loads such as axial or tangential torque and force acting upon rotating shafts, were investigated qualitatively and quantitatively with respect to the applied direction and magnitude of loads. A qualitative stability analysis for the flexible rotors subjected to nonconservative loads was performed by introducing the concepts of energy and variational principle, and the stability criterion was derived and demonstrated through the illustrative examples. As an efficient approximate method, the general Galerkin's method which utilizes admissible functions was developed and its validity was tested by comparing with the previous results by the transfer matrix method.

The applied direction of torque has a significant effect on the stability of the system, whereas the applied direction of force affects the natural frequency. Tangential torque may induce flutter instability contrary to axial torque and the stable and unstable mode switching occurs at α=0.5. For the system to be stable it is necessary that the nonconservative part of work by external loads be zero. In practical rotors where it is difficult to define the type of applied torque and force, unstable modes may not be clearly identified.

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