Isotropic Control of Rotor Bearing System
(회전체 베어링계의 등방 제어)

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ABSTRACT

A new rotor control scheme, the isotropic control of anisotropic rotor bearing system in complex state space, is proposed, which utilizes the concepts on the eigen-structure of the isotropic rotor system. Then the control scheme is applied to an active magnetic bearing system and the control performance is investigated in relation to control energy, transient response, and unbalance response. In particular, it is shown that the proposed method is efficient for control of unbalance response.

1. INTRODUCTION

Complex notation has been commonly adopted in dynamic analysis of a rotor bearing system due to the notational convenience and clear physical interpretation, and the dynamic analysis[1,2] and control of isotropic rotor bearing system[3,4] in complex space has been well developed. In this work, an isotropic control of anisotropic rotor bearing system in complex state space is proposed, which utilizes the concepts on the eigenstructure of the isotropic rotor system[5]. Isotropic controller design in complex state space is essentially composed of two steps. Firstly system is decomposed into isotropic and anisotropic parts, and direct cancelling control of the system anisotropy is performed. Secondly an isotropic control scheme such as the optimal control in complex domain[3] and the optimal pole assignment into the specified regions is applied to the resulting isotropic system[4]. Advantages of the proposed method are that the controlled system always retains isotropic eigenstructure, leading to circular whirling due to unbalance response[1] and the controller design is more comprehensive and simpler.

In order to demonstrate the performance of the proposed method, the control of active magnetic bearing system[6] is investigated in relation to control energy, and transient and unbalance responses.

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2. ROTOR BEARING SYSTEM

The equation of motion of a multi-degree-of-freedom rotor bearing system may be written as[1]

$$M\ddot{q} + C\dot{q} + Kq = f \tag{1}$$

where

$$q = \begin{Bmatrix} y \\ z \end{Bmatrix}, \qquad f = \begin{Bmatrix} f_y \\ f_z \end{Bmatrix}$$

$$M = \begin{bmatrix} M_{yy} & M_{yz} \\ M_{zy} & M_{zz} \end{bmatrix}, \quad C = \begin{bmatrix} C_{yy} & C_{yz} \\ C_{zy} & C_{zz} \end{bmatrix}, \quad K = \begin{bmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{bmatrix}.$$

Here M, C, K are the 2nx2n mass, damping including gyroscopic effect, and stiffness matrices, and y (z) and f_y (f_z) are the n-dimensional y-(z-) directional displacement and force vectors.

Assuming that the rotor is axisymmetric and introducing complex notations such that $p = y + jz$ and $g = f_y + jf_z$, we can rewrite Eq.(1) as

$$M_c\ddot{p} + C_c\dot{p} + C_\Delta\dot{\bar{p}} + K_c p + K_\Delta\bar{p} = g \tag{2}$$

where

$$M_c = M_{yy} = M_{zz}, \qquad M_{yz} = M_{zy} = 0$$

$$\begin{aligned} C_c &= \frac{C_{yy} + C_{zz}}{2} + j\frac{C_{zy} - C_{yz}}{2} \\ C_\Delta &= \frac{C_{yy} - C_{zz}}{2} + j\frac{C_{zy} + C_{yz}}{2} \\ K_c &= \frac{K_{yy} + K_{zz}}{2} + j\frac{K_{zy} - K_{yz}}{2} \\ K_\Delta &= \frac{K_{yy} - K_{zz}}{2} + j\frac{K_{zy} + K_{yz}}{2}. \end{aligned} \tag{3}$$

Here j is the imaginary number and '-' denotes the complex conjugate. In Eq.(2), the nxn complex matrices M_c, C_c, K_c represent the isotropic properties of the rotor bearing system whereas the nxn complex matrices C_Δ, K_Δ represent the anisotropic properties of bearings.

3. CONTROL OF ROTOR BEARING SYSTEM

3.1 Conventional Optimal Control

The state space form of Eq.(1) can be written, on the assumption of the positive definiteness of mass matrix $\boldsymbol{M}$, as

$$\dot{x} = Ax + Bu \tag{4}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},\quad x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix},\quad u = f = \begin{Bmatrix} f_y \\ f_z \end{Bmatrix}.$$

Here $\boldsymbol{A}$ is the real valued 4n×4n system matrix and $\boldsymbol{x}$ is the 4n×1 state vector. Consider the quadratic performance index given by

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \tag{5}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the positive semidefinite and positive definite weighting matrices, respectively. Then solution to the minimization of J is the optimal control law given by

$$u = -R^{-1} B^T P x \tag{6}$$

where $\boldsymbol{P}$ is the solution of the algebraic matrix Riccati equation

$$PA + A^T P - PBR^{-1} B^T P + Q = 0. \tag{7}$$

Here the positive definite solution matrix $\boldsymbol{P}$ always exists and the controlled system is asymptotically stable, if $[\boldsymbol{A},\boldsymbol{B}]$ is controllable and $[\boldsymbol{A},\boldsymbol{D}]$ is completely observable, where $\boldsymbol{D}$ is any matrix such that $DD^T = Q$[7]. As the result, optimal control gain matrix, K_{opt}, can be written as

$$K_{opt} = \begin{bmatrix} K_{po} & K_{do} \end{bmatrix} \equiv R^{-1} B^T P \tag{8}$$

where K_{po} and K_{do} are the proportional and derivative gain matrices. From Eqs.(6) and (8), control force vector can be written using the feedback gain matrices as

$$f = u = -\left\{ K_{po} q + K_{do} \dot{q} \right\} \tag{9}$$

Substituting Eq.(9) into Eq.(1), we can write the controlled system as

$$M\ddot{q} + \left[C + K_{do} \right] \dot{q} + \left[K + K_{po} \right] q = f_e \tag{10}$$

where f_e is the external force vector.

In general, conventional optimal controlled rotor bearing system (10) retains the characteristics of uncontrolled system. Therefore if the original system is anisotropic, controlled system is also likely to be anisotropic, leading to the backward whirling in unbalance response.

3.2 Isotropic Optimal Control in Complex State Space

The essence of the *isotropic optimal control* of rotor bearing system in complex domain is that the control efforts is twofold. The first part of the control action is solely devoted to make the system isotropic and then the second part of control action is applied to the resulting isotropic system, which utilizes an optimal control in complex domain. Complex control force $\boldsymbol{g}$ in Eq.(2) can be decomposed into two parts as

$$g = g_c + g_\Delta \tag{11}$$

where

$$g_\Delta = C_\Delta \dot{\bar{p}} + K_\Delta \bar{p} \tag{12}$$

and g_c is determined from the equation of motion associated with the isotropic system resulting from the control action of g_Δ, i.e.

$$M_c \ddot{p} + C_c \dot{p} + K_c p = g_c \tag{13}$$

and then the optimal controller in complex state space is to be designed.

The state space form of Eq.(13) is

$$\dot{x}_c = A_c x_c + B_c u_c \tag{14}$$

where

$$A_c = \begin{bmatrix} 0 & I \\ -M_c^{-1} K_c & -M_c^{-1} C_c \end{bmatrix},\quad B_c = \begin{bmatrix} 0 \\ M_c^{-1} \end{bmatrix},\quad x_c = \begin{Bmatrix} p \\ \dot{p} \end{Bmatrix},\quad u_c = g_c$$

and A_c is the 2n×2n complex system matrix and x_c is the 2n×1 complex state vector. Consider the quadratic performance index in complex domain given by

$$J_c = \int_0^\infty (x_c^* Q_c x_c + u_c^* R_c u_c)\, dt \tag{15}$$

where Q_c and R_c are the positive semidefinite and positive definite Hermitian matrices, respectively, and '*' denotes the conjugate transpose. Then the solution to the minimization of J_c is the complex optimal control law given by

$$u_c = -R_c^{-1} B_c^* P_c x_c = -\begin{bmatrix} K_{pc} & K_{dc} \end{bmatrix} x_c \tag{16}$$

where the positive definite Hermitian matrix P_c is the solution of a complex valued algebraic Riccati equation

$$P_c A_c + A_c^* P_c - P_c B_c R_c^{-1} B_c^* P_c + Q_c = 0. \tag{17}$$

Form Eq.(16), the complex control force vector becomes, using feedback gain matrix,

$$g_c = u_c = -\left\{ K_{pc} p + K_{dc} \dot{p} \right\} \tag{18}$$

and the final control law is the superposition of the two control actions in Eqs. (12) and (18), i.e.

$$g = g_c + g_\Delta = -\left\{K_{pc}p + K_{dc}\dot{p}\right\} + \left\{C_\Delta \dot{\bar{p}} + K_\Delta \bar{p}\right\} \quad (19)$$

The controlled system becomes then

$$M_c\ddot{p} + \left[C_c + K_{dc}\right]\dot{p} + \left[K_c + K_{pc}\right]p = g_e \quad (20)$$

where g_e is the external force.

Unlike the conventional optimal control, the isotropic optimal control ensures that the controlled system remains always isotropic, leading to circular whirling due to unbalance response. Since the order of the matrices treated in the complex domain approach is half of that in the real approach, the system analysis and controller design is more comprehensive and simpler. In real application of this control scheme, however, some cautions are necessary for good system performance such that the total control gain matrix in real domain must be at least positive semidefinite, if not, some control energy may be consumed to degrade the control performance.

4. APPLICATION TO ACTIVE MAGNETIC BEARING SYSTEM

The equation of motion for an axisymmetric rigid rotor-active magnetic bearing system shown in Figure 1 can be written, using complex notations, as

$$M_c\ddot{p} + C_c\dot{p} + K_c p + K_\Delta \bar{p} = K_i i \quad (21)$$

where

$$M_c = \begin{bmatrix} ml_2^2 + i_d & ml_1l_2 - i_d \\ ml_1l_2 - i_d & ml_1^2 + i_d \end{bmatrix}$$

$$C_c = -j\Omega i_p \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_c = \begin{bmatrix} K_{10} & 0 \\ 0 & K_{20} \end{bmatrix}, \quad K_\Delta = \begin{bmatrix} K_{1\Delta} & 0 \\ 0 & K_{2\Delta} \end{bmatrix}$$

$$K_i = \begin{bmatrix} K_{i1} & 0 \\ 0 & K_{i2} \end{bmatrix}$$

$$p = \begin{Bmatrix} y_1 + jz_1 \\ y_2 + jz_2 \end{Bmatrix}, \quad i = \begin{Bmatrix} i_{y1} + ji_{z1} \\ i_{y2} + ji_{z2} \end{Bmatrix}$$

and

$$i_d = {}^{I_d}\!/_{b_t^2}, \quad i_p = {}^{I_p}\!/_{b_t^2}$$

$$l_1 = {}^{b_1}\!/_{b_t}, \quad l_2 = {}^{b_2}\!/_{b_t}, \quad b_t = b_1 + b_2$$

$$K_{10} = \frac{1}{2}\left(K_{y1} + K_{z1}\right), \quad K_{1\Delta} = \frac{1}{2}\left(K_{y1} - K_{z1}\right)$$

$$K_{20} = \frac{1}{2}\left(K_{y2} + K_{z2}\right), \quad K_{2\Delta} = \frac{1}{2}\left(K_{y2} - K_{z2}\right).$$

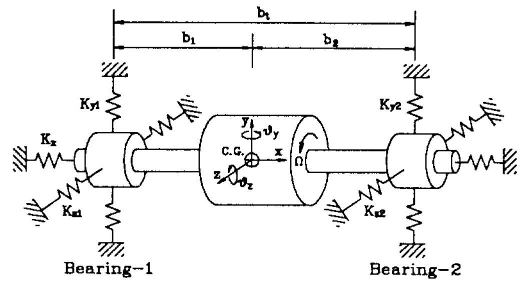

Fig. 1 Active magnetic bearing system.

Here m, I_d and I_p denote the mass, the diametral and the polar mass moment of inertia of the rotor respectively, b_1 and b_2 are the distances of two magnetic bearings from the mass center of the rotor, Ω is the rotating speed of the rotor, K_{i1} and K_{i2} are the current stiffnesses of magnetic bearings, and K_q, i_q, $q=y_1,y_2,z_1,z_2$, are the negative stiffness of the uncontrolled magnetic bearings and the control current of each magnet, respectively.

Note that the open loop system (21) is inherently unstable by the negative stiffness K_q which is generated by attractive magnetic force

$$f_q \approx -K_q q + K_{iq} i_q, \qquad q = y_1, y_2, z_1, z_2 \qquad (22)$$

so that the stabilization control is always required. Controller of magnetic bearing can be analog, digital or hybrid types. In this example, we select a hybrid type controller. First, stabilizing control using four single-axis analog PD control is performed. Next, coupled 4-d.o.f. digital controller is designed for the stabilized system using the proposed isotropic control method.

The parameter values used in this illustrative example are as follows :

m=9.41kg, I_p=0.00927, I_d=0.128kg-m^2
b_1=0.159, b_2=0.118m, Ω=12000rpm
$\{K_q\}=\{-5.5, -5.0, -4.5, -4.0\}^T \times 10^5$ N/m
$\{K_{iq}\}=\{480, 460, 480, 460\}^T$ N/A

And the controlled stiffness and damping by the decoupled analog controller are

K_a=diag[9.6, 9.2, 9.6, 9.2] × 10^5 N/m
C_a=diag[480, 460, 480, 460] N s/m.

Here the stabilized bearing parameters can be rewritten in complex space as

$$C_c = \begin{bmatrix} 480 & 0 \\ 0 & 460 \end{bmatrix} + j\begin{bmatrix} -152 & 152 \\ 152 & -152 \end{bmatrix} \text{ N s/m}$$

$$K_c = \begin{bmatrix} 4.6 & 0 \\ 0 & 4.7 \end{bmatrix} \times 10^5, \; K_\Delta = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} \times 10^5 \text{ N/m}.$$

First, the conventional optimal control is performed using the procedure in section 3.1 with

TABLE 1 Eigenvalues of Magnetic Bearing System.

Mode	Original	Conven.	Isotropic
1B	-45.68±283.1j	-215.4±346.3j	-210.4-352.4j
1F	-48.41±315.2j	-206.8±370.0j	-211.5+363.8j
2B	-67.48±336.8j	-279.0±412.6j	-276.6-414.4j
2F	-83.28±425.9j	-281.4±495.6j	-283.4+494.1j

TABLE 2 Eigenvectors of Controlled Systems.

Mode	1B	1F	2B	2F
	Conventional Optimal Control			
y_1	1.00	1.00	1.00	1.00
y_2	3.31∓.124j	0.470±1.06j	-.263∓.022j	-.4828±.089j
z_1	-.302∓.160j	0.963∓1.77j	-.107±.701j	-.2368∓1.35j
z_2	-.289±.805j	2.20±4.54j	0.147∓.160j	0.165±.589j
	Isotropic Optimal Control			
y_1	1.00	1.00	1.00	1.00
y_2	3.93+1.67j	1.97+.372j	-.245+.080j	-.461+.064j
z_1	-j	-j	-j	-j
z_2	1.67-3.93j	0.372-1.97j	0.080+.245j	0.064+.461j

$$\boldsymbol{Q} = \mathrm{diag}[10^7, 10^7, 10^7, 10^7, 4, 4, 4, 4]$$
$$\boldsymbol{R} = \mathrm{diag}[1, 1, 1, 1].$$

Secondly, the isotropic optimal control law is calculated with

$$\boldsymbol{Q}_c = \mathrm{diag}[10^7, 10^7, 4, 4]$$
$$\boldsymbol{R}_c = \mathrm{diag}[1, 1].$$

The eigenvalues of the uncontrolled and controlled systems are listed in Table 1. It shows that both controlled systems have similar eigenvalues and the control forces act mainly for increasing damping. Table 2 shows the eigenvectors associated with the conventional and isotropic optimal controlled systems. It shows that the eigenstructure of the conventional optimal system is not isotropic since the relations, such as $z_1=-jy_1$ and $z_2=-jy_2$, do not hold, unlike the isotropic optimal controlled system.

The typical transient responses and control forces are depicted in Figures 2 and 3 for the conventional and isotropic optimal controls, respectively. The performance indices calculated for the conventional optimal control are $J(x)$=6.81×10^{-11}, $J(u)$=21.2 and those for the isotropic optimal control are $J(x)$=6.79×10^{-11}, $J(u)$=21.4. These results indicate that both controlled systems have very similar characteristics, but one method is slightly better in response performance but worse in control energy than the other.

Figures 4 and 5 show unbalance responses and control forces of each bearing. In many practical rotor bearing systems, the major whirl radius and maximum control force are the important factors to be minimized. Table 3 shows the forward and backward circular whirl radii, the major whirl radii, and the maximum control forces of each bearing at the rotating speed of Ω = 3600 rpm.

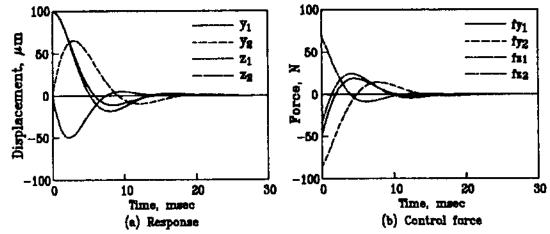

Fig. 2 Transient responses and control forces for conventional optimal control: q(0) = {100, 0, 0, 100} μm, $\dot{q}$(0) = {0, 0.05, -0.05, 0} μm.

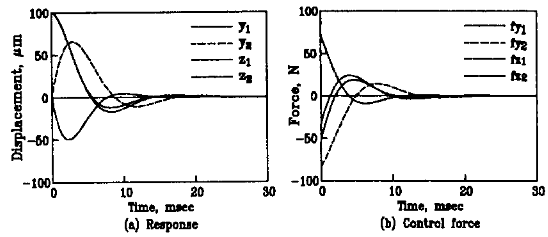

Fig. 3 Transient responses and control forces for isotropic control: p(0) = {100, 100j} μm, $\dot{p}$(0) = {-0.05j, 0.05} μm.

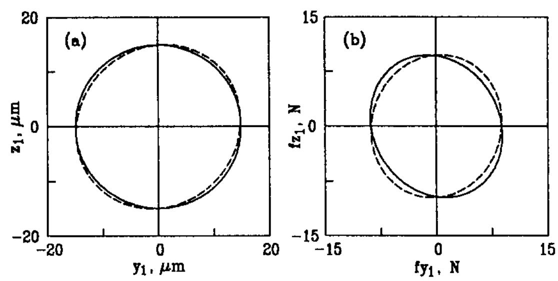

Fig. 4 Unbalance response(a) and control force(b) at bearing-1 for e_1=20μm ∠0°, e_2=20μm ∠90°, Ω= 3600rpm:
—— Isotropic control , ----- Conventional control

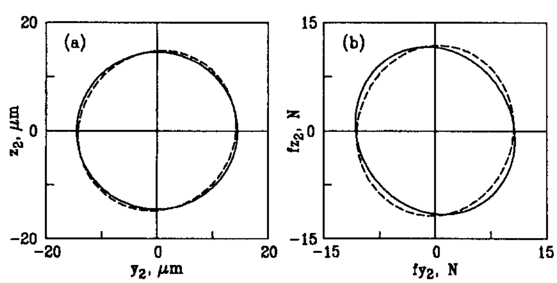

Fig. 5 Unbalance response(a) and control force(b) at bearing-2 for e_1=20μm ∠0°, e_2=20μm ∠90°, Ω= 3600rpm:
—— Isotropic control , ----- Conventional control

TABLE 3 Whirl radii and control forces for unbalance response.

* Bearing-1/ Bearing-2

Response Comp.	Whirl radius, μm		Control force, N	
	Conven.	Isotropic	Conven.	Isotropic
Forward	14.89/14.51	14.93/14.53	9.29/11.18	9.293/11.16
Backward	0.627/0.552	0/0	0.57/0.690	0.747/0.727
Major	15.52/15.06	14.93/14.53	9.86/11.87	10.04/11.90

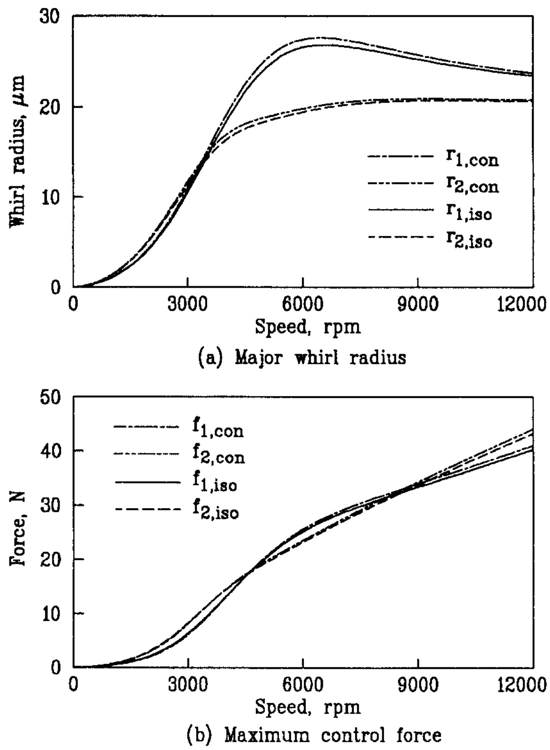

Fig. 6 Major whirl radius and maximum control force of magnetic bearing for e_1=20μm∠0°, e_2=20μm∠90°

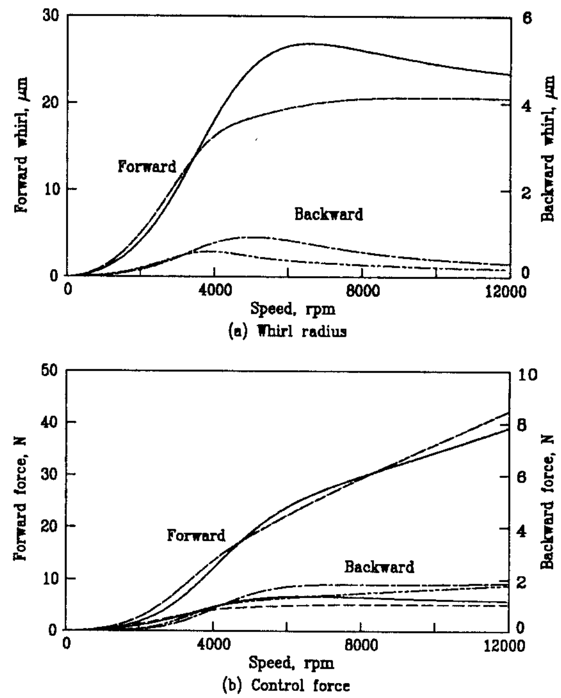

Fig. 7 Forward and backward components of whirls and control forces for e_1=20μm∠0°, e_2=20μm∠90°: —— iso.-1, — — con.-1, - - - - iso.-2, — - - con.-2.

In isotropic optimal controlled systems, unbalance response is characterized by a forward synchronous circular whirl; there exists no backward whirling component. Thus the major whirl radii of isotropic optimal controlled systems tend to be smaller than those of conventional optimal controlled systems. Figure 6 shows the major whirl radius and the maximum control force at each bearing for unbalance of $e_1 = 20\mu m\angle 0^o$ and $e_2 = 20\mu m\angle 90^o$, as the rotational speed is varied. Figure 7 is the plot for the backward and forward components of the unbalance response and control force. Figures 5, 6 and 7 clearly indicate that the major whirl radius remains to be smaller for the isotropic optimal control than the conventional optimal control as the rotational speed varies. In addition, the maximum required control force for the isotropic optimal control also tends to be slightly less than that for the conventional optimal control except over the low rotational speed range. This is mainly due to the fact that, in the isotropic optimal control, the backward whirling component is eliminated by changing the phase and/or radius of backward control force component. The phase shift alone, unlike the radius change, does not affect the maximum control force.

5. CONCLUSION

Isotropic control of anisotropic rotor bearing system in complex state space is proposed, which assigns isotropic eigenstructure to the controlled system. The isotropic optimal control scheme is applied to an active magnetic bearing system and the control performance is compared with the conventional optimal control method. It can be concluded that the isotropic optimal control method, which essentially eliminates the backward unbalance response component, is more efficient than the conventional optimal control in that it gives smaller major whirl radius and yet it often requires less control effort.

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