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Representing Fuzzy, Uncertain Evidences and Confidence Propagation for Rule-Based System

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ABSTRACT

Representing knowledge uncertainty, aggregating evidence confidences, and propagating uncertainties are three key elements that effect the ability of a rule-based expert system to represent domains with uncertainty. Fuzzy set theory provide a good mathematical tool for representing the vagueness associated with a variable when, as the condition of a rule, it only partially corresponds to the input data. However, the aggregation of ANDed and ORed confidences is not as simple as the intersection and union operators defined for fuzzy set membership. There is, in fact, a certain degree of compensation that occurs when an expert aggregates confidences associated with compound evidence. Further, expert often consider individual evidences to be varying importance, or weight, in their support for a conclusion. This paper presents a flexible approach for evaluating evidence and conclusion confidences. Evidences may be represented as fuzzy or nonfuzzy variables with as associated degree of certainty. Different weights can also be assigned to the individual condition in determining the confidence of compound evidence. Conclusion confidence is calculated using a modified approach combining the evidence confidence and a rule strength. The techniques developed offer a flexible framework for representing knowledge and propagating uncertainties. This framework has the potention to reflect human aggregation of uncertain information more accurately than simple minimum and maxmum operator do.

INTRODUCTION

One of the characteristics of human reasoning is the ability to form useful judgments from uncertain and incomplete evidence.An expert system is an information system which contain the knowledge of an expert and provides the user with a facility for obtaining answers to question relating to the information stored in its knowledge base. Thus, an expert system can be considered a model of the expert's knowledge and reasoning processes. Since much of human experties includes reasonig with fuzzy, incomplete, and uncertain information, an expert system should be able to represent fuzziness in facts and uncertainty associated with aggregating multiple evidences, and be able to establish the level of confidence in its conclusion.Thus,a basic issue in the design of knowledge-based system is the need for a computational approach to estimate the confidences of evidence and the propagation of uncertaity from premises to the final conclusion.

Several mathematical models for dealing with uncertainty bave been proposed[3,4,12,15]. These model typically use minimum and maximum operators to evaluate the aggregation of ANDed and ORed evidences.However, empirical studies have shown that minimum and maximum operators do not adequately represent the nature of the aggregation of evidence confidence [13,18-20]. In fact, human reasoning and evidence aggregation are very complex processes.Defining appropriate representation techniques is an area of continuing research,and simple operators such as minimum and maximum functions may not provide comprehensive and realistic models.

CONFIDENCE OF SIMPLE EVIDENCE

In a rule-based system, uncertainty associated with the conclusion of a rule arises from two sources: evidence uncertainty and rule uncertainty (rule strengh). Evidences may be simple(single),or compound with ANDed and ORed evidence.

1. REPRESENTING FUZZINESS OF SIMPLE EVIDENCE

An evidence, which consists of only one condition, we term simple evidence. Examples of simple evidence are:

(1) Slope of land = 3%.

(2) Weight of body > 10 kg.

(3) Age is old.

Example (1) and (2) are categorical characterizations. The user's response is compared with the value to establish whether the statement is "true" or "false". Example (3) is ambiguous. When a vague linguistic quantifier is presented in a proposition, it induces a fuzzy set in the sense defined by Zadeh[14]. To indentify a member of a fuzzy set, a grade of membership is associated with each element that could potentially be a member. The grade of membership is a number between 0 and 1. The membership function $\mu_A(u)$ defines the grade of membership associated with u, a value in the fuzzy subset. With the fuzzy proposition

X (Age) is A (old)

we associate the strengh of the evidence for the statement "X is A" to be equivalent to the grade of membership $\mu_A(u)$ of u in A:

$$E(x \text{ is } A) = \mu_A(u) \tag{1}$$

For practical application, what is primary importance is to determine the set of values which are completely possible and the set of values which are completely impossible. The remaining subset corresponds to the gradual transition of $\mu_A(u)$ between 0 to 1. Fuzzy interval can be defined conveniently as combinations of the S-membership functions.The membership function S_1 which is adaptable to many cases, is a piecewise quadratic function defined by

$$S_1(u;\alpha_1,\gamma_1)=\begin{cases}0, & u\leqslant\alpha_1,\\ 2\left(\dfrac{u-\alpha_1}{\gamma_1-\alpha_1}\right)^2, & \alpha_1\leqslant u\leqslant\beta_1,\\ 1-2\left(\dfrac{u-\gamma_1}{\gamma_1-\alpha_1}\right)^2, & \beta_1\leqslant u\leqslant\gamma_1,\\ 1, & u\geqslant\gamma_1,\end{cases} \tag{2}$$

where the parameters α_1 and γ_1 are the lower and upper fuzzy subset interval bounds, respectively, and the parameter $\beta_1=(\alpha_1+\gamma_1)/2$ is the crossover point, the value of u at which $S_1(\beta;\alpha_1,\gamma_1)=0.5$. The function S1 increases progressively between α_1 and β_1, a form useful for representing the grade of membership of a proposition such as "Age is old".

On the other hand, a decreasing from of the S-membership function may sometimes be needed for describing the grade of membership. such a function can be constructed simply as $S_2(u;\alpha_2,\gamma_2)=1-S_1$. These two forms of the S-function, S_1 and S_2, can be combined to express a fuzzy interval. When $\gamma_1=\alpha_2$ and $\gamma_1-\alpha_1=\gamma_2-\alpha_2$, the function S_3 reduces to a fuzzy relationship expressing "around" or "close to" u.

It should be noted that nonfuzzy conditions can also be expressed in the framework of fuzzy membership function. The membership grade of a nonfuzzy condition is zero everywhere except at a point, where it is egual to 1. In short, whether a simple evidence involves a fuzzy or a nonfuzzy variable, the evidence confidence can be represented by the grade of membership.

$$E(e) = \mu_A(u) \tag{3}$$

2. CONFIDENCE OF SIMPLE EVIDENCE WITH FUZZINESS AND UNCERTAINTY

In some case, an user may not have complete confidence in the source information when asked to provide data for a variable. In this case the evidence can be expressed in the form

X is A with certainty c,

where c (a value in the unit interval) is the degree of certainty associated with the evidence. A premise like this is not only fuzzy but also somewhat uncertain,and the evidence confidence must then depend on both the certainty c and the grade of membership, $\mu_A(u)$. The evidence confidence can be expressed using the notation

$$E(\mu, c) = f[\mu_A(u), c] \tag{4}$$

The proposition " X is A with certainty c " can be considered as two subpropositions

X = u with certainty c

u is A with membership grade $\mu_A(u)$.

These two subpropositions can also be considered as conditons connected by AND.Before introducing the approach which we propose for combining these subpropositions to determine the evidence confidence, we note corresponding methods used in probability and in fuzzy set theory. In probability theory, under the assumption that evidences are mutually independent, ANDed conditions are represented as

$$P(A_1,A_2) = P(A_1)\times P(A_2) \quad (5)$$

while in fuzzy set theory, this union is represented as

$$\pi\,[A_1,A_2] = \min[\mu_{A1}(u_1), \mu_{A2}(u_2)] \quad (6)$$

It seems reasonable to assume that the evidence confidecnce, $E(\mu,c)$, should be dependent upon both the belief degree of the source information and the grade of membership. Based on this consideration, we assume the product as a low bound and the minimum as an upper bound. Then, the evidence confidence of a fact which is not only fuzzy but also uncertain can be expressed as

$$\mu_A(u)\times c \leqslant E(\mu,c) \leqslant \min[\mu_A(u),c] \quad (7)$$

This can be written as

$$E(\mu,c) = \mu_A(u)\times c + \delta, \quad (8)$$

where δ is a compensation. From Eq.(7) and (8) it can be determined that

$$\delta = \xi\,\{\min[\mu A(u),c] - \mu A(u)\times c\}$$

where ξ is between 0 to 1,and defined as a compensation coefficient.Eq.(8) can then be rewritten as

$$E(\mu,c) = \mu_A(u)\times c + \xi\,\{\min[\mu_A(u),c] - \mu_A(u)\times c\} \quad (9)$$

We proposed a value for the compensation coefficient of $\xi = c$. In general, the belief degree of source information is more important than the grade of membership for an evidence, and for assigning a value to the conclusion. The evidence confidence $E(\mu,c)$ will approach the upper bound if the belief degree c of the source data is high; otherwise, it will be nearer the lower bound. The higher the belief degree, the greater the compensation.

It should be pointed out that c^k is an alternative value for ξ. Then Eq.(9) can be written

$$E(\mu,c) = \mu_A(u)\times c + c^k\{\min[\mu_A(u),c] - \mu_A(u)\times c\} \quad (10)$$

The resulting evidence confidence is the product of the grade of memdership and the belief degree plus a compensation factor. Eq.(10) has the following features:

1. When $\mu_A(u)$ or c is zero, the evidence confidence is zero.
2. When $\mu A(u)$ or c is one, the evidence confidence

$$E(\mu,c) = \mu_A(u) \qquad \text{if } c = 1;$$
$$E(\mu,c) = c \qquad \text{if } \mu_A(u) = 1.$$

3.The degree of compensation can be adjusted by changing the power k in order to represent the requirements of the domain. Thus,when k =0,the evidence will get full compensation, and

$$E(\mu,c) = \min[\mu_A(u),c]$$

4. When $k\rightarrow\infty$, the evidence confidence will get no compensation, and

$$E(\mu,c) = \mu_A(u)\times c$$

CONFIDENCE OF COMPOUND EVIDENCE

1. CONFIDENCE OF ANDed EVIDENCE

Compound evidence in a rule-based system is composed of simple evidences joined with AND and/or OR operators.Consider a rule which has ANDed evidence in the form

IF A_1 AND A_2 AND...AND A_n THEN B.

As stated in the above section,in fuzzy set theory the confidence factor of an ANDed evidence is evaluated using the minimum operator, so that

$$E(A_1 \ldots A_n) = \min[\mu_{Ai}(ui)] \quad (11)$$

With this relationship, the confidence of an ANDed evidence depends only on the minimum value. The validity of this function as a model of human aggregation of evidences is questionable for many applications. So when using fuzzy set theory to model real-world problems,a purely mathematical justification of the operator is not sufficient. The àdequacy and the suitability of the operator should be established empirically for each problem domain.

In probability theory, the confidence factor of an ANDed evdence is evaluated as the product

$$E(A_1 \cdots A_n) = \prod_{i=1}^{n} \mu_{A_i}(u_i). \quad (12)$$

A feature of Eq.(12) is that the membership values of all elementary conditions affect the evaluation of the evidence confidence.However,the confidence of the evidence may become unacceptably low if the evidence consists of many elementary conditions, even if all of the elemantary conditions have high degrees of confidence.

Several researchers have presented experimental results on experts aggregating evidence [13,18].Tab.1 shows their results together with the evidence strengths computed using minimum and product opertors. The results of their experiment indicated that:

(a) Neither the minimum opertor nor the product operator fits the data sufficiently well.

(b) Most empirical membership grades are greater than $\min[\mu_m, \mu_n]$.

(c) All empirical membership grades are greater than the product of μ_m and μ_n.

Thus it can be seen that interpreting human aggregation of fuzzy sets as the set intersection is questionable, because it implies no positive compensation between the degrees of membership in the fuzzy sets.If either the minimum or the product is used as an opertor, each of them yields degrees of membership of the resulting fuzzy set which are at or below the lowest degree of membership of all intersecting fuzzy sets. From the data in Tab.1 on human aggregation of fuzzy sets it can be seen that there is almost always some compensation between different grades of membership. The compensatory tendencies in human aggregation are responsible for the

TABLE 1

Empirical Results Compared with Predicted Grade of Membership [13]

Stimulus	μ_M	μ_c	Minimum	Product	$\mu_{M \cap C}$
1	.000	.985	.000	.000	.007
2	.908	.419	.419	.380	.517
3	.215	.149	.149	.032	.170
4	.552	.804	.552	.444	.674
5	.023	.454	.023	.010	.007
6	.501	.437	.437	.219	.493
7	.629	.400	.400	.252	.537
8	.847	1.000	.847	.847	1.000
9	.424	.623	.424	.264	.460
10	.318	.212	.212	.067	.142

failure of the classical opertors(min,product) in empirical inverstigation.[13,18] A new approch which includes some degree of compensation is needed for modeling human aggregation of fuzzy evidences.

2. CONFIDENCE OF WEIGHTED, ANDed EVIDENCE

An expert, making a decision based on the evidence, may not consider every elementary condition to have equal importance for the outcome. Some conditions may be more important than others.

The weight, or importance, of each contributing evidence thus can significantly affect the outcome of aggregating evidences.For this reason, we introduce a weight coefficient associated with each elementary condition of a compound evidence of a rule.The consideration of different weight will play an important role in modeling real-world situations using fuzzy evidences, and will make an expert system more reliable and more valid.

Based on the data presented in Tab.1,we assume the confidence of ANDed evidence is greater than the product of the membership grades of all elementary evidences and less than their average, i.e.,

$$\prod_{i=1}^{n} \mu_{A_i}(u_i) \leqslant E_{\text{AND}} \leqslant \frac{1}{n} \sum_{i=1}^{n} \mu_{A_i}(u_i). \tag{13}$$

Considering a weight w_i for each elementary condition,

$$\prod_{i=1}^{n} w'_i \mu_i \leqslant E_{\text{AND}} \leqslant \sum_{i=1}^{n} w'_i \mu_i, \tag{14}$$

where μ_i stands for $\mu_{Ai}(ui)$, and $w'_i = w_i/\sum w_i$. In order to adjust the effect of weights on the aggregation of evidence confidence, we propose that the kth power of the weights be substituted for the weights in Eq. (14). By defining

$$w''_i = w_i^k / \sum w_i^k \qquad (k \geqslant 1)$$

$$e_i = w''_i \mu_i.$$

$$\bar{e} = \sum_{i=1}^{n} e_i,$$

Eq.(14) can be rewritten as

$$\prod_{i=1}^{n} e_i \leqslant E_{\text{AND}} \leqslant \bar{e}. \tag{15}$$

Assuming

$$E_{\text{AND}} = \prod_{i=1}^{n} e_i + \delta$$

and

$$\delta = \xi \left(\bar{e} - \prod_{i=1}^{n} e_i \right),$$

then we get

$$E_{\text{AND}} = \prod_{i=1}^{n} e_i + \xi \left(\bar{e} - \prod_{i=1}^{n} e_i \right), \tag{16}$$

where ξ is a compensation coefficient. We propose $\bar{e}$ as one reasonable value for the compensation coefficient. Then Eq.(16) can be writeen

$$E_{AND} = \bar{e}^2 + (1-\bar{e}) \prod_{i=1}^{n} e_i, \tag{17}$$

and choosing k = 2, then

$$E_{AND} = \left(\sum_{i=1}^{n} W_i\mu_i\right)^2 + \left(1-\sum_{i=1}^{n} W_i\mu_i\right) \prod_{i=1}^{n} W_i\mu_i, \tag{18}$$

where

$$W_i = \frac{w_i^2}{\sum_{i=1}^{n} w_i^2}.$$

Some example of confidence of weighted, ANDed evidence are shown in Tab.2. As can be seen from the values in Tab.2, the second term on the right side of Eq.(18) is negligible and the Eq.(18) can be simplified to

$$E_{AND} = \left(\sum_{i=1}^{n} W_i\mu_i\right)^2. \tag{19}$$

TABLE 2

Combination of ANDed evidences with individual weights (Evidence value used: μ_1 = 0.8, μ_2 = 0.6, μ_3 = 0.4, μ_4 = 0.2)

Evidence weights							
w_1	w_2	w_3	w_4	Πe_i	$\bar{e}$	$\bar{e}^2$	E_{AND}
1.0	1.0		1.0	.004	.533	.284	.286
1.0	0.8		0.6	.003	.628	.394	.396
1.0	0.6		0.4	.002	.689	.475	.476
1.0	0.4		0.2	0	.753	.567	..568
0.6	0.8		1.0	.003	.436	.190	.192
0.4	0.6		1.0	.002	.358	.128	.130
0.2	0.4		1.0	0	.273	.075	.074
1.0	1.0	1.0	1.0	0	.500	.250	.250
1.0	0.8	0.6	0.4	0	.630	.397	.397
1.0	0.6	0.4	0.2	0	.697	.486	.486
0.4	0.6	0.8	1.0	0	.370	.137	.137
0.2	0.4	0.6	1.0	0	.303	.092	.092

3. CONFIDENCE OF WEIGHTED ORed EVIDENCE

To develop a relationship for combining weighted, ORed evidences, we use the following definition of the OR operator:

$$E[OR(A_1,A_2,\ldots,A_n)] = E(NOT\{AND[(NOT\ A_1),(NOT\ A_2),\ldots,(NOT\ A_n)]\}) \tag{20}$$

and the definition of the NOT operator from fuzzy set theory,

$$\mu\,(NOT\ A) = 1 - \mu\,(A)$$

Then Eq.(20) can be written as

$E[OR(A_1,A_2,\ldots,A_n)$
$= 1-E\{AND[(NOT\ A_1),(NOT\ A_2),\ldots,(NOT\ A_n)]\}$ (21)

Combining Eq.(19) and (21), the confidence of weighted, ORed evidence can be evaluated as

$$E_{OR} = 1 - \left[\sum_{1}^{n} W_i (1 - \mu_i) \right]^2 \qquad (22)$$

An example of evidence aggregation computed with Eq.(23) is shown in Tab.4

TABLE 4

Combination of ORed evidences with individual weights
(Evidence value used μ_1 = 0.8, μ_2 = 0.6, μ_3 = 0.4, μ_4 = 0.2)

Evidence weights				
w_1	w_2	w_3	w_4	E_{OR}
1.0	1.0	1.0	1.0	.750
1.0	0.8	0.6	0.4	.863
1.0	0.6	0.4	0.2	.908
0.4	0.6	0.8	1.0	.603
0.2	0.4	0.6	1.0	.514

4. CHARACTERISTICS OF WEIGHTED COMPOUND EVIDENCE FUNCTIONS

As techniques for propagating the confidence degree of ANDed and ORed evidences,Eq.(19) and (22) demonstrate some desirable characteristics.

(1). For ANDed evidences:

a. $E_{AND} > \min[\mu_i]$ when an elementary condition which has a higher grade of membership has greater weight.For this case the confidence of ANDed evidence has a positive compensation.

b. $E_{AND} < \min[\mu_i]$ when an elementarty condition which has a low grade of membership has geater weight.In this case,the confidence of ANDed evidence receives negative compensation.

c. $E_{AND} = 0$ if and only if all $\mu_i = 0$.

d. the greater the value of the weighted average of the μ_i, the greatter the cofidence of E_{AND}.

(2). For ORed evidences:

a. E_{OR} is frequently greater than $\max[\mu_i]$, although this is dependent on the values of the i, and strongly affected by the weights.

b. E_{OR} is less than $\max[\mu_i]$ if the elementary condition whose grade of membership is larger has a low weight. The lower the weight of $\mu_j=\max[\mu_i]$, the lower the value of E_{OR}.

c. $E_{OR} = 1$ and only if all $\mu_i = 1$.

(3). The values computed with Eq.(19)and (22) are conceptualyy more reasonable than values based on max and min operators, and have the structure and flexibility to more appropriately model the processes of human decision making.It is clear that when individuals aggregate elementary conditions and evaluate the confidence of compound evidence, the process is not as simple as minimum, prodect, and maximum operations.

CONCLUSION CONFIDENCE

In a rule-based system, the general form of a rule is

IF E THEN C with CF = r,

where E is the evidence, whether simple or compound, C is the conclusion of the rule, and r is a certainty factor of the rule. The factor r, also called the rule strength,represents the maximum certainty with which the conclusion C can be asserted,given that the evidence is completely certain.

What must be specified is the relationship for determining the confidence degree of the conclusion, based on the evidence confidence and the rule strength.The product of evidence confidence and rule strength is the commonly used relationship. A disadvantage of this algorithm is that a very low confidence degree for the final conclusion may result if the final conclusion is achieved through a long chain of rules and the supporting-evidence confidences and rule strength are less than unity.

We set the product of the evidence cofidence and rule strength as a lower bound, and the min as as upper bound for the rule conclusion confidence cc(e,r):

$$E(e) \times r \leqslant \mathrm{cc}(e, r) \leqslant \min[E(e), r]. \tag{23}$$

From this we derive the expression

$$\mathrm{cc}(e, r) = E(e) \times r + \xi\{\min[E(e), r] - E(e) \times r\}. \tag{24}$$

where ξ is a compensation coefficient. We propose the rule strength r as the value of the compensation coefficient. This seems reasonable in that the confidence of the conclusion will approach the upper bound as the rule strength increase. Then Eq.(24) can be rewritten as

$$\mathrm{cc}(e, r) = E(e) \times r + r\{\min[E(e), r] - E(e) \times r\} \tag{25}$$

This function has the following features:

1. When the confidence of the evidence E(e) is 0, then cc(e,r) = 0.
2. When the rule strength r = 1, then cc(e,r) = E(e).

CONCLUSION

The representstion of knowledge uncertainty, the aggregation of evidence confidences, and the propagation of uncertainty are crucial for the performance of an expert system, and must realistically model the expert in these areas.

Human reasoning is not a simple process, and minimum operators for ANDed evidence and maximum operators for ORed evidence do not adequately represent the characteristics of human aggregation of evidence confidence. In fact, there is a certain degree of compensation incorporated in human aggregation. In addition, the elementary evidences supporting a conclusion frequently vary in their degree of importance. By introducing a weight coefficient for individual evidences, the reliability and validity of the representation scheme and system results will be improved.

The equations developed here expand on currently used uncertainty representation techniques. New techniques are presented for evaluating the confidence of simple evidence with inaccuracy of source information.

These relationships are an attempt to provide a more appropriate and flexible model than minimum and maximum operators for propagating uncertainties.With these algrithms,evidence can be readily represented as fuzzy or as nonfuzzy variables, with or without individual weights. The relationships presented here are simple enough to be implemented in existing expert system development shells. Although lower execution speeds can be expected, the benefit will be an improved capability for representing characteristics of the human reasoning process.

REFERENCES

1. Basu, A. and A.Dutta,1986, Computer based support of reasoning in the presence of fuzziness, Decision Support System 2:235-25
2. Buisson, J.C., H.Farreny, and H.Prade, 1985, The developmant of a medical expert system and the treatment of imprecision in the framework of possibility theory, Inform. Sci. 37:221-226
3. Dubois, D. and H.Prade, 1988, Representation and combination of uncertainty with belief functions and possibility measure, Comput. Intell. 4:244-264
4. Duda, R.O., P.E.Hart, and N.J.Nilsson, 1976, Subjective bayesian methods for rule-based inference system, Amer. Fed. Inform. Process, 45:1075-1082
5. Dutta, A.,1985, Reasoning with imprecise knowledge in expert systems,Inform. Sci. 37:3-24
6. Dutta, A. and A.Basu, 1986, Computer based support of reasoning in the presence of fuzziness and uncertainty, Decision Support Systems, 2:331-346
7. Lesmo, L.,L.Saitta and P.Torasso, 1985, Evidence combination in expert system, Inter. J. Man-Machine Stud. 22:307-326
8. Mizumoto, M., 1989, Pictoral representations of fuzzy connections, part 1:Cases of t-norms, t-conorms and averaging operators, Fuzzy Set and Systems, 31:217-242
9. Prade, H., 1985, Reasoning with fuzzy default value, Proceedings of the 15th IEEE Inter. Symp. on Multiple-Valued Logic, Inst. of Electrical and Electronics Engineers, New York, pp191-197.
10. Prade, H.,1985, A computational approach to approximate and plausible reasoning with applications to expert system, IEEE Trans. Pattern Anal. and Mach. Intell., 3:260-283
11. Sanchez, E.,1986, Medical applications with fuzzy sets, Fuzzy Set Theory and Applications, Reidel, Boston, pp331-347
12. Shortlife, E.H. and B.G.Buchanan, 1975, A model of inexact reasoning in medicine, Math. Biosci., 23:351-379
13. Thole, U., H.J.Zimmerman and P.Zysno, 1979, On the suitability of minimum and product operators for the intersection of fuzzy sets, Fuzzy Sets and Systems, 2:167-180
14. Zadeh, L.A., 1978, Fuzzy sets as a basic for a theory of possiblity, Fuzzy Sets and Systems, 1:3-28
15. Zadeh, L.A., 1979, A theory of approximate reasoning, Mach. Intell. 9:149-194
16. Zadeh, L.A., 1983, The role of fuzzy logic in the management of uncertainty in expert systems, Fuzzy Sets and Systems, 11:199-227
17. Zadeh, L.A., 1983, Fuzzy logic, Computer, 21(4):83-92
18. Zimmermann, H.J. and P.Zysno, 1980, Latent connectives in human decision making, Fuzzy Sets and Systems, 4:37-51
19. Zimmermann, H.J. and P.Zyson, 1983, Decision and evaluation by hierarchical aggregation of information, Fuzzy Sets and Systems, 10:243-260
20. Zimmermann, H.J.,1986, Fuzzy set theory and mathematical programming, Fuzzy Set Theory and Applications, Reidel, Boston, pp99-144