MODELING STATIC STABILITY OF AGRICULTURAL TRACTOR

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ABSTRACT

A mathematical model has been developed for the simulation of static stability of agricultural tractors. The model was based on Reichmann's model but modified to improve its shortcomings for practical applications. Two examples of model simulation were presented and showed its usefulness in evaluating the control and stability loss boundaries.

Key Word: Agricultural Tractor, Static Stability, Computer Simulation

INTRODUCTION

Stability is one of the primary factors limiting work performance or field capacity of agricultural tractors. Particularly, on sloping grounds stability determines safety and, in many cases, even their productivity.

Traditionally, tractor stability has been concerned mostly with overturns: either rearward or sideways. The stability defining the likelihood of rearward overturning is called longitudinal stability and the lateral stability defines the likelihood of sideways overturns. Both of these stabilities are based on the concept that if wheels on the same side about tipping axis are off the ground the tractor loses its stability and overturns with respect to that tipping axis. Therefore, any factors causing the wheel to lose the contact with the ground are considered tending to decrease the stability.

Most widely used method for evaluating the criteria of such overturning stabilities has been the moment equation or phase plane analysis. Stability analysis by the moment equation is simple and straight forward, but should take into account all the factors, both statically and dynamically, generating the moments about overturning axis (Goering at al.(1967) and Smith et al.(1972)). Phase plane analysis, however, determines the stability indirectly by plotting a phase plane plot. The phase plane plot depicts the relationship between the tipping velocity and the angle of tip and it shows that in unstable or overturning conditions the tipping velocity attaining a peak value does not return to zero as the angle of tip increases (Larson et al.(1971) and Mitchell et al.(1972)). The phase plane approach, thus, separates the plane into two regions: safe and unsafe. This is particularly useful for developing stability criteria in dynamic conditions.

A concept of stability, developed in the European countries, defines the stability by the interacting forces at the wheel-ground interface without any reference to the longitudinal or lateral stabilities. This concept is that the loss of

wheel-ground adhesion force limits the stability of tractor rather than its likelihood of any actual overturns. Therefore, with this approach, tractor stability is expressed as control loss when the wheel force on the ground plane is no longer sufficient to maintain the static equilibrium and as stability loss when the radial wheel force becomes zero.

By applying the concept of control and stability losses, the work reported in this paper was intended to develop a mathematical model for tractor stability simulation. The model was originally developed by Reichmann(1972) and used lately by Spencer(1978), but has been considered inappropriate in applications for simulation purposes because the reference coordinate system used in the modeling is uncommon in most vehicle dynamics studies. In addition, the parameters, heading and slope angles, by which the original model describes the attitude of a tractor on sloping lands seem less advantageous than the angles of pitch and roll in experimental or practical applications. Improvement of these two inappropriateness were major modifications made from the original model.

DEVELOPMENT OF STABILITY MODEL

Notations

Notations used in the development of stability model were defined as follows (Fig. 1):

lows (Fig. 1):	
X-Y-Z	: inertia coordinate system fixed in space.
х-у-г	: reference coordinate system fixed in tractor body with its origin at the mass center of tractor, and its positive x, y, and z axis directions as shown in Fig. 1. All the vectors, defined hereafter, are expressed with respect to this coordinate system unless stated otherwise.
i, j, k	: unit vectors in the directions of positive x, y and z axes, respectively.
C.G.	: mass center of tractor.
G_1	: mass center of the front axle assembly.
G_2	 mass center of tractor excluding the front axle assembly.
a, b, c, d	ground contact points of the left front, right front, left rear, and right rear wheels, respectively.
p	: drawbar hitch point.
h	: front axle hinge point.
	position vectors from C.G. to a, b, c, and d respectively [L].
p p	: position vector from C.G. to p, [L].
$\vec{\mathbf{h}}$: position vector from C.G. to h, [L].
\overrightarrow{h} $\overrightarrow{s_1}$, $\overrightarrow{s_2}$	position vectors from C.G. to G ₁ and G ₂ respectively [L].
è	unit vector representing hinge axis of the front axle, [L].

 $\overrightarrow{F_{a}}$, $\overrightarrow{F_{b}}$, $\overrightarrow{F_{c}}$, $\overrightarrow{F_{d}}$: force vectors acting at a, b, c, and d, respectively, [F].

: force vector of drawbar loading acting at p, [F].

: force vectors representing weights of the front axle assembly and tractor excluding the front axle respectively, [F].

: force vector representing total weight of tractor, [F]. θ : pitch angle. ϕ : roll angle.

: frictional coefficient.

Basic Assumptions

The control and stability loss concept can be applied most properly to the evaluation of static stability in which tractors are considered in a uniform velocity or in very slow motions. Such tractor motions are expected when tractors are operating, particularly, on sloping grounds. Thus, bearing the static stability in mind, the following assumptions were made for the model development:

- 1. Tractor is two wheel drive and wide front end type.
- 2. Rolling resistance is negligible.
- 3. Traction forces of driving wheels are equal.
- 4. Wheel-ground contact points of wheels are on the same plane.
- Wheel forces act through a single contact point between the wheel and ground.

Determination of Wheel Loads

When a tractor has angular orientations of pitch angle θ , and roll angle ϕ with respect to the space fixed reference coordinate system as shown in Fig. 1, its weight components w^x , w^y , and w^z in the directions of x, y, and z axes are given respectively as follows:

$$\mathbf{w}^{x} = -\mathbf{w} \sin \theta,$$

 $\mathbf{w}^{y} = \mathbf{w} \cos \theta \cdot \sin \phi$
 $\mathbf{w}^{z} = \mathbf{w} \cos \theta \cdot \cos \phi.$

and

Then, the force vector representing the total tractor weight becomes

$$\overrightarrow{\mathbf{w}} = -\mathbf{w} \sin \theta \mathbf{i} + \mathbf{w} \cos \theta \cdot \sin \phi \mathbf{j} + \mathbf{w} \cos \theta \cdot \cos \phi \mathbf{k}. \tag{1}$$

Similarly, weights of the front axle assembly and rear body of tractor are given by force vectors as follows:

$$\overrightarrow{\mathbf{w}_1} = -\mathbf{w}_1 \sin\theta \mathbf{i} + \mathbf{w}_1 \cos\theta \cdot \sin\phi \mathbf{j} + \mathbf{w}_1 \cos\theta \cdot \cos\phi \mathbf{k}, \qquad (2)$$

and

$$\overrightarrow{\mathbf{w}_2} = -\mathbf{w}_2 \sin\theta \,\mathbf{i} + \mathbf{w}_2 \cos\theta \cdot \sin\phi \,\mathbf{j} + \mathbf{w}_1 \cos\theta \cdot \cos\phi \,\mathbf{k}. \tag{3}$$

In order to satisfy the static equilibrium conditions, all the force vectors acting on the tractor must be summed to yield a zero vector, and so should all the moment vectors taken about the center of gravity of a tractor. That is;

$$\overrightarrow{F_a} + \overrightarrow{F_b} + \overrightarrow{F_c} + \overrightarrow{F_d} + \overrightarrow{F_p} + \overrightarrow{w} = \overrightarrow{0}, \tag{4}$$

and

$$\vec{a} \times \vec{F}_a + \vec{b} \times \vec{F}_b + \vec{c} \times \vec{F}_c + \vec{d} \times \vec{F}_d + \vec{p} \times \vec{F}_p = \vec{0}.$$
 (5)

Two additional equations necessary to determine the wheel loads can be obtained by establishing moment equations for the front and rear bodies of a tractor about the hinge axis of the front axle. Taking the moments about the hinge axis of the forces acting on the front axle assembly and equating it to zero for the moment equilibrium give

$$\vec{e} \cdot [(\vec{a} - \vec{h}) \times \vec{F_a} + (\vec{b} - \vec{h}) \times \vec{F_b} + (\vec{s_1} - \vec{h}) \times \vec{w_1}] = 0.$$
 (6)

Similarly, the moment equation for the rear body of a tractor becomes

$$\vec{e} \cdot [(\vec{c} - \vec{h}) \times \vec{F_c} + (\vec{d} - \vec{h}) \times \vec{F_d} + (\vec{p} - \vec{h}) \times \vec{F_p} + (\vec{s_2} - \vec{h}) \times \vec{w_2}] = 0.$$
 (7)

Equations (4) through (7) are the basic force and moment equations from which the wheel loads are to be determined.

Writing in scalar component form, Equation (4) can be expressed as a set of three scalar equations as follows

$$F_a^x + F_b^x + F_c^x + F_d^x + F_p^x + w^x = 0,$$
 (8-1)

$$F_a^y + F_b^y + F_c^y + F_d^y + F_p^y + w^y = 0,$$
 (8-2)

and

$$F_a^z + F_b^z + F_c^z + F_d^z + F_p^z + w^z = 0.$$
 (8-3)

Expanding the cross product of Equation (5) and rearranging terms in scalar components give

$$(a_y F_a^z - a_z F_a^y) + (b_y F_b^z - b_z F_b^y) + (c_y F_c^z - c_z F_c^y) + (d_y F_d^z - d_z F_c^y) + (p_y F_b^z - p_z F_b^y) = 0,$$
(9-1)

$$(a_z F_a^x - a_x F_a^z) + (b_z F_b^x - b_x F_b^z) + (c_z F_c^x - c_x F_c^z) + (d_z F_d^x - d_x F_d^z) + (p_z F_p^x - p_x F_p^z) = 0,$$
(9-2)

and

$$(a_x F_a^y - a_y F_a^x) + (b_x F_b^y - b_y F_b^x) + (c_x F_c^y - c_y F_c^x) + (d_x F_d^y - d_y F_d^x) + (p_x F_p^y - p_y F_p^x) = 0.$$
(9-3)

of position vectors \overrightarrow{a} , \overrightarrow{b} , \overrightarrow{c} , \overrightarrow{d} and \overrightarrow{p} and force vectors $\overrightarrow{F_a}$, $\overrightarrow{F_b}$, $\overrightarrow{F_c}$, $\overrightarrow{F_p}$ and $\overrightarrow{F_p}$ in the directions of x, y, and z axes respectively.

From the tractor geometry, it may be assumed that

$$\mathbf{a_x} = \mathbf{b_x}, \tag{10-1}$$

$$c_x = d_x, \qquad (10-2)$$

and

$$\mathbf{a}_{\mathbf{z}} = \mathbf{b}_{\mathbf{z}} = \mathbf{c}_{\mathbf{z}} = \mathbf{d}_{\mathbf{z}}. \tag{10-3}$$

It is also assumed that traction forces acting on the wheels of the same driving axle are equal. That is,

$$F_a^x = F_b^x, (11-1)$$

and

$$\mathbf{F_c}^{\mathbf{x}} = \mathbf{F_d}^{\mathbf{x}}. \tag{11-2}$$

Substitution of Equations (10-1) through (11-2) into Equation (9-3) and rearrangement of terms give

$$a_{x}(F_{a}^{y} + F_{b}^{y}) - (a_{y} + b_{y})F_{a}^{x} + c_{x}(F_{c}^{y} + F_{d}^{y}) - (c_{y} + d_{y})F_{c}^{x} + (p_{x}F_{p}^{y} - p_{y}F_{p}^{x}) = 0.$$
(12)

From Equations (8-1) and (8-2), we obtain

$$F_{c}^{x} = -F_{a}^{x} - F_{b}^{x} - F_{d}^{x} - F_{d}^{x} - F_{d}^{x} - W^{x}$$
(13-1)

and

$$F_c^y + F_d^y = -F_a^y - F_b^y - F_b^y - w^y.$$
 (13-2)

Equation (13-1) can be rewritten using the tractive force relations of Equations (11-1) and (11-2) as follows

$$F_{c}^{x} = -F_{a}^{x} - \frac{1}{2}(F_{p}^{x} + w^{x})$$
 (14)

Substituting Equation (13-2) and (14) into Equation (12), and solving for $F_a{}^y + F_b{}^y$ give

$$F_{a}^{y} + F_{b}^{y} = \frac{1}{a_{x} - c_{x}} \{ (a_{y} + b_{y} - c_{y} - d_{y}) F_{a}^{x} + \frac{1}{2} (c_{y} - d_{y}) (F_{p}^{x} + w^{x}) + (c_{x} - p_{x}) F_{p}^{y} - (c_{y} - p_{y}) F_{p}^{x} + c_{x} w^{y} - c_{y} w^{x} \}.$$
 (15)

Once $F_a^y + F_b^y$ is determined, $F_c^y + F_d^y$ can be obtained from Equation (8-2). Thus,

$$F_{c}^{y} + F_{d}^{y} = -(F_{a}^{y} + F_{b}^{y}) - F_{p}^{y} - w^{y},$$
or
$$F_{c}^{y} + F_{d}^{y} = -\frac{1}{a_{x} - c_{x}} ((a_{y} + b_{y} - c_{y} - d_{y})F_{a}^{x} + \frac{1}{2}(c_{y} - d_{y})(F_{p}^{x} - w^{x}) + (a_{x} - p_{x})F_{p}^{y} - (c_{y} - p_{y})F_{p}^{x} + a_{x}w^{y} - c_{y}w^{x}).$$
(16)

Similarly, substituting Equations (10-1) through (11-2) into Equation (9-2) and rearranging terms give

$$a_{z}(F_{a}^{x} + F_{b}^{x} + F_{c}^{x} + F_{d}^{x}) - a_{x}(F_{a}^{z} + F_{b}^{z}) - c_{x}(F_{c}^{z} + F_{d}^{z}) + (p_{z}F_{p}^{x} - p_{x}F_{p}^{z}) = 0.$$
(17)

Rearranging Equations (8-1) and (8-3), we obtain

$$F_a^x + F_b^x + F_c^x + F_d^x = -F_p^x - w^x,$$
 (18-1)

and

$$F_c^z + F_d^z = -F_a^z - F_b^z - F_p^z - w^z$$
. (18-2)

Substituting Equations (18-1) and (18-2) into Equation (17) and solving for $F_a{}^z + F_b{}^z$ give

$$F_a^z + F_b^z = \frac{1}{a_x - c_x} \{ (c_x - p_x) F_p^z - (c_z - p_z) F_p^x + c_x w^z - c_z w^x \}.$$
 (19)

Again, $F_c^z + F_d^z$ is determined from Equations (8-3) and (19).

$$F_{c}^{z} + F_{d}^{z} = -(F_{a}^{z} + F_{b}^{z}) - F_{p}^{z} - w^{z}$$
or
$$F_{c}^{z} + F_{d}^{z} = -\frac{1}{a_{x} - c_{x}} \{(a_{x} - p_{x})F_{p}^{z} - (c_{z} - p_{z})F_{p}^{x} + a_{x}w^{z} - c_{z}w^{x}\}.$$
 (20)

In order to determine F_a^z , F_b^z , F_c^z and F_d^z Equations (6) and (7) are expanded in scalar forms. First, expanding Equation (6) yields

$$\{(a_x - h_z)e_y - (a_y - h_y)e_z\} F_a^x$$

$$+ \{(a_x - h_x)e_z - (a_z - h_z)e_x\} F_a^y$$

$$+ \{(a_y - h_y)e_x - (a_x - h_x)e_y\} F_a^z$$

$$+ \{(b_z - h_z)e_y - (b_y - h_y)e_z\} F_b^x$$

$$+\{(b_{x} - h_{x})e_{z} - (b_{z} - h_{z})e_{x}\} F_{b}^{y}$$

$$+\{(b_{y} - h_{y})e_{x} - (b_{x} - h_{x})e_{y}\} F_{b}^{z}$$

$$+\{(s_{1z} - h_{z})e_{y} - (s_{1y} - h_{y})e_{z}\} w_{1}^{x}$$

$$+\{(s_{1x} - h_{x})e_{z} - (s_{1z} - h_{z})e_{x}\} w_{1}^{y}$$

$$+\{(s_{1y} - h_{y})e_{x} - (s_{1x} - h_{x})e_{y}\} w_{1}^{z} = 0.$$
(21)

Substitution of Equations (10-1) through (11-2) into Equation (21) and rearrangement of terms give

$$\{(a_{z} - h_{z})e_{y} - (a_{y} - h_{y})e_{z} + (a_{z} - h_{z})e_{y} - (b_{y} - h_{y})e_{z}\} F_{a}^{x}$$

$$+ \{(a_{x} - h_{x})e_{z} - (a_{z} - h_{z})e_{x}\} (F_{a}^{y} + F_{b}^{y})$$

$$+ \{(a_{y} - h_{y})e_{x} - (a_{x} - h_{x})e_{y}\} F_{a}^{z}$$

$$+ \{(b_{y} - h_{y})e_{x} - (a_{x} - h_{x})e_{y}\} F_{b}^{z} + K = 0,$$

$$K = \{(s_{1z} - h_{z})e_{y} - (s_{1y} - h_{y})e_{z}\} w_{1}^{x}$$

$$+ \{s_{1x} - h_{x})e_{z} - (s_{1z} - h_{z})e_{x}\} w_{1}^{y}$$

$$+ \{(s_{1y} - h_{y})e_{x} - (s_{1x} - h_{x})e_{y}\} w_{1}^{z}.$$

where

Substituting Equation (19) into Equation (22) and solving for Fa² give

$$F_{a}^{z} = \frac{1}{(b_{y} - a_{y})e_{x}} \{2(a_{z} - h_{z})e_{y} - (a_{y} + b_{y} - 2h_{y})e_{z}\} F_{a}^{x} + \{(a_{x} - h_{x})e_{z} - (a_{z} - h_{z})e_{x}\} (F_{a}^{y} + F_{b}^{y})$$

$$+ \frac{1}{a_{x} - c_{x}} \{(b_{y} - h_{y})e_{x} - (b_{x} - h_{x})e_{y}\} \{(c_{x} - p_{x})F_{p}^{z} - (c_{z} - p_{z})F_{p}^{x} + c_{x}w^{z} - c_{z}w^{x}\} + K.$$

$$(23)$$

Solving Equation (19) for Fbz yields

$$F_{b}^{z} = \frac{1}{a_{x} - c_{x}} \{ (c_{x} - p_{x}) F_{p}^{z} - (c_{z} - p_{z}) F_{p}^{x} + c_{x} w^{z} - c_{z} w^{x} \} - F_{a}^{z}, \qquad (24)$$

where F_a^z is obtained by Equation (23).

Secondly, expanding Equation (7) and following the same procedures for substitution of Equations (10-1) through (11-2) into the expanded equation and arrangement of terms give

$$\{2(c_z - h_z)e_y - (c_y + d_y - 2h_y)e_z\} F_c^x \\ + \{(c_x - h_x)e_z - (c_z - h_z)e_x\} (F_c^y + F_d^y) \\ + \{(c_y - h_y)e_x - (c_x - h_x)e_y\} F_c^z \\ + \{(d_y - h_y)e_x - (c_x - h_x)e_y\} F_d^z + L + M = 0.$$
 where
$$L = \{(p_z - h_z)e_y - (p_y - h_y)e_z\} F_p^x \\ + \{(p_x - h_x)e_z - (p_z - h_z)e_x\} F_p^y \\ + \{(p_y - h_y)e_x - (p_x - h_x)e_y\} F_p^z$$
 and
$$M = \{(s_{2z} - h_z)e_y - (s_{2y} - h_y)e_z\} w_2^x \\ + \{(s_{2x} - h_x)e_z - (s_{2z} - h_z)e_x\} w_2^y \\ + \{(s_{2y} - h_y)e_x - (s_{2x} - h_x)e_y\} w_2^z.$$

Substituting F_d² obtained by Equation (20) into Equation (25) and solving

for F_c give

$$F_{c}^{z} = \frac{1}{(d_{y} - c_{y})e_{x}} \{2(c_{z} - h_{z})e_{y} - (c_{y} + d_{y} - 2h_{y})e_{z}\} F_{c}^{x}$$

$$+ \{(c_{x} - h_{x})e_{z} - (c_{z} - h_{z})e_{x}\} (F_{c}^{y} + F_{d}^{y})$$

$$- \{(d_{y} - h_{y})e_{x} - (c_{x} - h_{x})e_{y}\} (F_{a}^{z} + F_{b}^{z} + F_{p}^{z} + w^{z})$$

$$+ I_{c} + M_{c}$$

$$(26)$$

Finally, F_d^z is determined from Equation (20) as follows

$$F_{d}^{z} = -(F_{h}^{z} + F_{h}^{z}) - F_{c}^{z} - F_{h}^{z} - W^{z}$$
(27)

where F_c^z is obtained by Equation (26).

In the above equations, the values of scalar components of position vectors are determined directly from the tractor geometry. Weight components \overline{W}_1^* and \overline{W}_2^* , and drawbar loadings are also obtained from the given type of tractor and loading conditions. One parameter whose value must be calculated before the computation is performed is F_a^x or F_b^x , the force acting in x direction at the wheel-ground contact point of the front wheel. This force depends on the traction force of the front wheel and rolling resistance acting on it. Assuming that the traction force of the front wheel is T_f and rolling resistance R_f , then F_a^x is

$$F_a^x = T_f - R_f. \tag{28}$$

In this study, since tractor was assumed as rear wheel drive and rolling resistance of wheels as negligible, F_a^x becomes zero. In cases where the front wheels are powered, however, the traction and rolling resistance forces must be determined depending on the wheel-ground interacting conditions.

Determination of Stability Boundary

Because the stability loss occurs when the normal to ground wheel load vanishes, the stability criteria can be defined by

$$F_i^2 \le 0$$
, $i = a, b, c, d$. (29)

If any of F_i^z becomes zero or positive, tractor lose their stability. the control loss occurs when the wheel load on the ground plane is no longer sufficient to maintain the static equilibrium. In such situations, the following may take place (Spencer, 1978)

 Steering control is lost, i.e., the front wheels are no longer capable of producing the lateral force necessary for directional control. The criterion for steering control, then, can be defined by

$$\mu (F_a^z + F_b^z) \le F_a^y + F_b^y. \tag{30}$$

2) Rear wheel slips sideways. The criterion for control in this situation is

$$\mu (F_c^z + F_d^z) \le F_c^y + F_d^y.$$
 (31)

3) The wheel-ground adhesion is insufficient to withstand the braking force so that uncontrolled down hill acceleration occurs or up hill travel ceases. If the left rear wheel loses adhesion, criterion for control is defined by

$$2 \mu F_c^z \leq \mathbf{w}^x. \tag{32}$$

Contrary, if the right rear wheel loses adhesion, the criterion becomes

$$2 \mu F_d^z \le \mathbf{w}^z. \tag{33}$$

The friction coefficient, μ , in the criterion equations can be evaluated using the friction circle concept (Spencer, 1972).

COMPUTER PROGRAMMING

A computer program has been developed to determine the boundary points of the control and stability losses i.e., the combination of the pitch and roll angles at which the control or stability losses occur. At a given pitch angle, the roll angle of the tractor was increased until the conditions for the control and stability boundaries were failed.

In the programming, rolling resistance was assumed to be zero. Another assumption made in the programming was that the hinge axis of the front axle is always pointed to the positive x axis direction. This can be acceptable in most agricultural tractors. Thus, \vec{e} was expressed as a unit vector representing the positive x axis direction. The pitch and roll angles were varied in both positive and negative directions. The program was written using MS-FORTRAN language executable by the IBM PC compatible computers.

Input data to the program include the weights of a tractor with and without the front axle assembly, position vectors of the tractor geometry, frictional coefficient between the wheel and ground and increments of pitch and roll angles for the computation. Output forms can be selected from three different modes. Mode 1 prints out all the wheel loads at every combination of pitch and roll angles until the control and stability losses occur. It also prints out the diagnostic statements at each boundary point and finally the polar diagram depicting the control and stability loss boundaries. Mode 2 lists the boundary points, at the given pitch angle, with their diagnostic statements and the polar diagram. Mode 3 gives only the polar diagram.

APPLICATION FOR STABILITY SIMULATION

The computer program developed in this study was used, as a demonstration, to evaluate the control and stability loss boundaries of two example tractors. Input data of the example tractors were obtained from reference (Reichmann, 1972) and summarized in Table 1. Figure 2 shows the respective polar diagram produced by the computer simulation.

The control loss boundary varied significantly with variations of the frictional coefficient. The smaller the value of frictional coefficient became, the more the region for safe operation decreased. At an infinite coefficient of friction, the safe operation is limited only by the stability loss.

Application of the stability model also can be made for four-wheel drive tractors if traction forces of the front wheels are defined.

CONCLUSIONS

A mathematical model has been developed for the simulation of static stability of agricultural tractors. The model was based on Reichmann's model but modified to improve its shortcomings for practical applications. Two examples of model simulation were presented and showed its usefulness in evaluating the control and stability loss boundaries.

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Table 1. Input data of example tractors

	Tractor A	Tractor B
→ a	(0.72, -0.76, 1.18)m	(1.61, -0.75, 0.86)m
→ b	(0.72, 0.79, 1.18)m	(1.61, 0.75, 0.86)m
→ c	(-1.09, -0.76, 1.18)m	(-0.92, -0.82, 0.86)m
d d	(-1.09, 0.79, 1.18)m	(-0.92, 0.82, 0.86)m
→ p	(-1.25, 0.00, 0.72)m	(-1.20, 0.00, 0.50)m
→ h	(0.72, 0.02, 0.28)m	(1.61, 0.00, 0.18)m
→ S ₁	(0.72, -0.07, 0.41)m	(1.61, 0.00, 0.31)m
S ₂	(-0.08, 0.01, -0.05)m	(-0.05, 0.00, -0.01)m
$\overrightarrow{\mathbf{w}}_{1}$	425 Kg _f	105 Kg _f
$\overrightarrow{\mathbf{w}_2}$	3820 Kg _f	3325 Kg _f
$\overrightarrow{\mathbf{F}}_{p}$	(0.00, 0.00, 0.00)m	(-150.00, 0.00, 30.00)Kg _i
μ	0.6	0.7

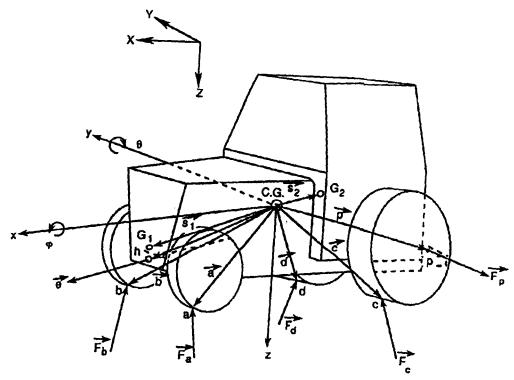


Fig. 1 Coordinate axes and vector definition for stability model of tractor

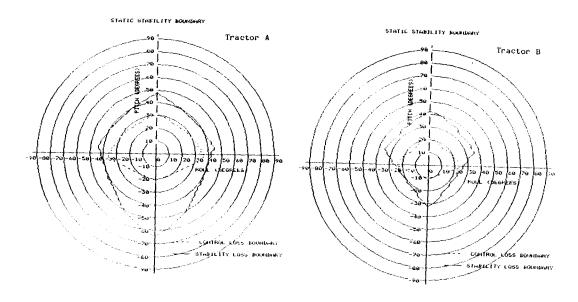


Fig. 2 Stability boundaries of example tractors