Cognitive Mapping with Multiple Participants

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ABSTRACT

Cognitive map is used for structuring and analyzing complex and unstructured decision environments, and it is a tool for graphically representing interrelationships among a variety of factors. It is a representation of the subjective perception of individual decision maker. Usually it is constructed by a number of experts at group level. This paper presents the technique of building an accurate cognitive map through several tests about the preliminary generated one. This paper also proposes an approach for aggregating the causal intensities of multiple experts.

I. Introduction

Strategic issues tend to be more complex and unstructured and not readily quantifiable. It is recognized that purely analytical techniques that can handle well–defined problems are not adequate to deal with such problems, and the need for the development of a more general tools for decisions has been identified.

In order to cope with the complex problem, this paper describes a cognitive map. The cognitive map is useful tool for mapping strategic issues made complex by interactions among a variety of technological, social and political concepts or factors. It is reported that the cognitive map techniques have been employed for decision making in the fields of international relations, administrative sciences, management sciences and operational researches [3, 4, 6, 9].

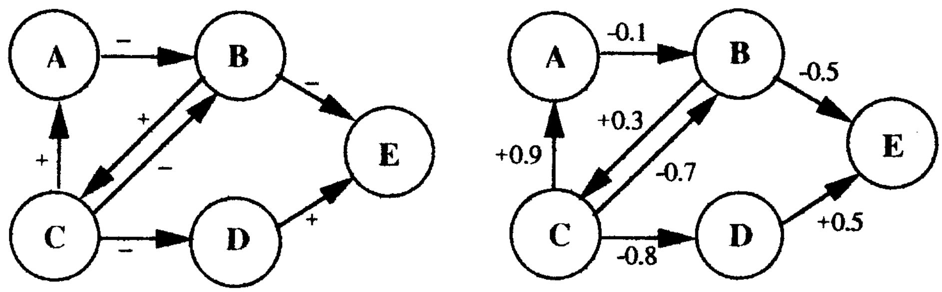
Cognitive map is a representation of the perceptions and beliefs of an individual decision maker about his/her own subjective world, rather than objective reality. Usually the cognitive mapping, *i.e.*, constructing cognitive maps, is performed by a number of experts for gathering information about a decision environment, and then the knowledge is aggregated.

The pooled cognitive map is potentially stronger than an individual cognitive map because the knowledge is derived from multiple sources, making point errors less likely. Thus it is important that we develop a suitable means of structuring and combining the preliminary considerations of individuals.

This paper considers the construction of cognitive map models under the participation of multiple persons. Especially, we present the technique of generating an accurate model through several tests about the preliminary generated cognitive map of decision domain, and we propose an optimization approach for aggregating the causal relationships among the factors of the cognitive map of multiple experts.

II. Cognitive Maps

Cognitive map is an intuitive framework in which to formulate decision problems as perceived by decision participants and to incorporate the knowledge of experts. Fig. 1 shows hypothetical cognitive maps as an example.



(a) Crisp cognitive map (b) Fuzzy cognitive map

Fig. 1. Hypothetical cognitive maps as an example.

	A	B	C	D	E
A		–			
B			+		–
C	+	–		–	
D					+
E					

(a)

	A	B	C	D	E
A		-0.1			
B			+0.3		-0.5
C	+0.9	-0.7		-0.8	
D					+0.5
E					

(b)

Fig. 2. The adjacency matrix representation in Fig. 1.

These maps consist of the factors relevant to the decision environment with arrows linking them and pluses and minuses on the arcs. The arcs represent causal relationships which the decision maker believes to exist between the factors. The relationships of the map are summarized as:

1) If **A** affects **B** directly, we put an arrow from **A** to **B** (**A** –> **B**).
2) If an increase in **A** causes an increase in **B**, and a decrease in **A** causes a decrease in B, then the arrow has a plus sign (if $\Delta A \uparrow \rightarrow \Delta B \uparrow$ and $\Delta A \downarrow \rightarrow \Delta B \downarrow$, then $A \xrightarrow{+r} B$, where r (>0) is the strength of causal relationship).
3) If an increase in **A** causes a decrease in B, and a decrease in **A** causes an increase in B, then the arrow has a minus sign (if $\Delta A \uparrow \rightarrow \Delta B \downarrow$ and $\Delta A \downarrow \rightarrow \Delta B \uparrow$, then $A \xrightarrow{-r} B$).

Fig. 2 depicts the adjacency matrix representation of the corresponding map. Matrix powering may be applied to adjacency matrices to determine paths of various lengths between nodes. For instance, if **M** is the adjacency matrix, then $\mathbf{M}^l$ gives all paths of length l in the matrix. A total connection matrix **T** gives the total number of paths between each node pair and is given by: $\mathbf{T} = \Sigma\mathbf{M}^l$ for l=1, 2, ..., p, where p is maximum power to which **M** is raised.

Axelrod [1] has developed crisp cognitive map (CCM) for representing social scientific knowledge. However, the intensities of the causal relations are ignored in CCM, only their directions and signs are represented. Hence the CCM refers only to whether or not the effects are increased (+) by the cause or decreased (−) by it.

To settle such a problem, the strength of cause–effect relationship need to be found. Kosko [4] has developed fuzzy cognitive map (FCM) and introduced a causal algebra of fuzzy number [0,1] for propagating causality. Zhang *et al.* [9] has proposed Pool2 as a generic system for FCM development and decision analysis.

III. The Overall Process of Cognitive Mapping

Common to most problem solving approaches is to first some kind of model for representing the decision situation. In a model building, it is recognized that the model will never be *complete*, *i.e.*, there is always approximation and/or subjective bias involved. The completeness of a cognitive map model may depend upon human creativity and intuition to provide judgment, ideas, and information.

There exist techniques that have been developed with the specific purpose of stimulating individuals as group(s) for improving what we might call creative functioning. The typical methods, for example, are such as Brainstorming, Nominal Group Technique, Synectics, Delphi, and Surveys. Such methods can be used for the deriving of cognitive maps.

The processes for constructing cognitive maps are consist of three steps as:

Step 0: **Define the Decision Problem**

It can be performed by data collection using interviews and/or content analysis of published documents. Note that the decision purpose has to be identified clearly. If the purpose is not well–defined, the finding relevant factors is likely to lack direction and the model can grow to an unmanageable size.

Step 1: **Search for Relevant Factors**

Based on the defined problem, the factors of cognitive map is determined. This step is performed by two substeps. 1) It is listed all the key factors relevant to the problem without redundancy and relative order of importance. 2) This step refines the listed factors. Namely the number of factors is limited by deleting the relatively trivial to the decisions through the comparison of importance rating. The rating can be obtained by questionnaires or interviews for experts. In this step a Delphi type iteration may be used with limited interaction.

It is noted that the definition of the factors must be clear. If the definition is well–defined, the rationale causalities can be captured

Step 2: **Identify Interrelationships among the Factors**

This step draws arcs with causal relationships between the factors that influence others. The causalities could be obtained by several individuals independently at group level. In the latter, the individual responses have to aggregate for constructing the cognitive map as a final version. The aim of this study lies in constructing FCM, thus we generate the FCM after the CCM is builded and tested.

IV. Test of Crisp Cognitive Map

Although the preliminary map is generated by the aggregation of group opinion, it may exist unacceptable or incorrect parts caused by a mistake. The relevant nodes and the important relationships among them can not be represented.

we describe the techniques of test with the definitions of some key parts which may include the incorrectness portion of the model.

Definition 1) **Isolated nodes**: *An isolated node is a factor not linked to any other factor.*

Definition 2) **Only affecting nodes**: *An only affecting node is a factor that influences other factor, but is not subject to influence of other factors itself.*

Definition 3) **Only affected nodes**: *An only affected node is a factor that is influenced by another factor, but does not influence any factors itself.*

If the the isolated nodes and/or only affecting/affected nodes are found, these should prompt following questions: Is the isolated node importantly relevant to the decision situation ? Have all important arcs or factors been identified in the model ? Is the node not actually influenced by one of the other nodes in the map ? Has a node been omitted that would affect the subject node and in turn be affected by one or more of the other nodes ?

V. Aggregation of Causal Intensities

In this section, an optimization approach for aggregating the casual strength of the FCM of individuals is proposed.

Let $\mathbf{M}_k$ be the adjacency matrix of the FCM of an individual k and r_{ij}^k be the element of the $\mathbf{M}_k$, *i.e.*, causal intensity of interval [–1,1] between ith factor and jth factor. Let a subset $\mathbf{r}_{ij} = \{r_{ij}^1, r_{ij}^2, ..., r_{ij}^n\}$ be to collect the causality of individuals in factor i and factor j, where n is total number of individuals. Then the following equation performs a weighted sum of the set $\mathbf{r}_{ij}$:

$$r_{ij} = \sum_{k=1}^{n} r_{ij}^k w_k \Big/ \sum_{k=1}^{n} w_k \qquad (1)$$

where w_k is a credibility weight of expert k about the intensity of the causal relationship between factor i and factor j.

We attempt here to determine the w_k^* of an intuitively acceptable value. Up to the present, the weights are easier assumed than estimated. The equal weight $w_k = 1/n$ for all k is commonly used. The assertion of equal weight means that we have no information about the individual experts' ability or credibility.

Let s is an ideal point as aggregated opinion. If s is the arithmetic mean of r_{ij}^k, the all of w_k should be $1/n$. But, as an aggregated value, the all of experts have not the same opinion as the mean. For example, consider the subjective numbers 0.1, 0.2, and 0.9. The combined point of an individual expert is defined as $0.1(\mu_1) + 0.2(\mu_2) + 0.9(\mu_3)$, where μ_i are membership functions of interval [0,1]. Most experts may respond as $\mu_2 \geq \mu_1 \geq \mu_3$ rather than $\mu_1 = \mu_2 = \mu_3$.

We let s_k be the subjective assertion of expert k as a combined value about a set. The aim is to find out the optimal weight which minimizes the difference the final aggregated value r_{ij} (which is not known as yet) and the experts' opinions s_k. Thus we obtain the following model:

$$\text{Minimize } z = \sum_k \left| \sum_k r_{ij}^k w_k - s_k \right|^m \tag{2a}$$

subject to

$$\sum_k w_k = 1 \tag{2b}$$

$$w_k \geq 0 \tag{2c}$$

Then the objective equation, *i.e.*, distance function is considered as absolute distance (if m = 1) or square distance model (if m = 2). If m is 1, the system can be solved as a LP (Linear Problem) form. The case that m equals 2, the model is a quadratic programming problem [5]. Hence the both cases can be easily obtained the optimal solutions, respectively.

VI. Conclusions

In recent years, much attention is directed toward the problems of strategic decision and decision making in dynamic and crisis. Cognitive map is used for structuring and analyzing complex and ill–structured decision domains, and it is a representation of the subjective perception of individual decision maker.

In practice, constructing cognitive map would be a burden process requiring much time and efforts. This paper has considered the construction of cognitive map models under the participation of multiple persons.

This paper is not a completed academic one in that more intensive research on cognitive mapping, especially testing method and pooling method of subjective expert assertions, should be done. The complete results of studies will be presented in the forthcoming papers of Park and Kim [10, 11].

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