THE VARIATION OF ONE MACHINE SCHEDULING PROBLEM

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Abstract

A generalization of one machine maximum lateness minimization problem is considered. There are one machine with controllable speed and n weighting jobs $J_j, j = 1, 2, \cdots, n$ with ambiguous duedates. Introducing fuzzy formulation, a membership function of the duedate associated with each job J_j, which describes the satisfaction level with respect to completion time of J_j. Thus the duedates are not constants as in conventional scheduling problems but decision variables reflecting the fuzzy circumstance of the job completing. We develop the polynomial time algorithm to find an optimal schedule and jobwise machine speeds, and to minimize the total sum of costs associated with jobwise machine speeds and dissatisfaction with respect to completion times of weighting jobs.

Keywords : Fuzzy duedate, Variable Machine Speed, Weighting Jobs, Scheduling, Combinatorial Analysis

1 Introduction

The classical problem of scheduling n jobs with duedates on a one machine has been extensively studied and has various variations. Many variations of the scheduling problem have been proposed from the viewpoint of different optimality criterion. Examples are maximum lateness, maximum tardiness, certain function etc. J.R. Jackson solved a one machine maximum lateness minimization problem by using the earliest duedates first rule in $O(nlogn)$ computational time [1][2].

However, there exist many real situations that any constant, as in the conventional scheduling problems, is not sufficient to completly characterize the duedate. That is, the duedates is not rigid in some situations and some violations may be accepted. And we may weight the jobs accordingly, since some jobs may be considered more important than others. Further machine speed can be changeable jobwise. This variation is practical in many applications. A typical example is a writer's scheduling problem. A famous writer is responsible for manuscrips by closing-dates from publishing companies with different authorities. The writer wishes to establish a writing schedule and speeds of writing manuscrips, subjectively. Usually, The more he evaluates authority of publisher highly, the more he recognizes the closing-date of the publisher severely. And the existance of easygoing gab from the closing-date asked by publisher loosely to the deadline for real publishing may make the closing-date ambigous. As another example, consider the building industry. The duedate of construction is to be ambigous since it is influenced by internal or external factors, that is, materials, worker, climates, war etc.

Thus the viewpoint of above concept of duedates, it is necessary to investigate not rigid constants as in the conventional scheduling problems but decision variables reflecting the fuzzy circumstance of the weighted job completing. As for changeable machine speeds, E. Nowicki & S. Zdralka considered on a two machine folw shop [3].

Considering these cases, this paper investigates the problem of scheduling weighted n jobs with fuzzy duedates on a one machine with controllable speeds. The objective is to find an optimal schedule and jobwise machine speeds, and to minimize the total sum of costs associated with jobwise machine spees and dissatisfaction with respect to completion times of weighting jobs.

Section 2 introduces fuzzy duedates with membership functions describing the degree of satisfaction with respect to job completion times and formulates the problem P. P is first transformed into equivalent problem $\bar{P}$. Then $\bar{P}$ is divided into subproblems $\bar{P}_1, \bar{P}_2, \cdots, \bar{P}_q$. Section 3 proposes an algorithm solving each subproblem $\bar{P}_l$ and gives a polynomial solution procedure for $\bar{P}$. Finally, section 4 summarizes this paper and discusses further research problems.

2 Problem Formulation

This paper makes the following assumptions.

(1) There is one machine with changeable speed for each job.

(2) The n jobs $J_1, J_2, \cdots, J_n$, to be processed on the machine, are weighted by importance.

(3) A degree of satisfaction with respect to the completion time is associated with each job, which is denoted with a certain membership function of a fuzzy set defined on R^+ (Nonnegative part of real number). This reflects the actual situation that earlier completion will give us more satisfaction.

(4) The objective function consists of costs related to minimal satisfaction(maximal dissatisfaction) with respect to completion times of jobs and jobwise speeds. That is, if we speed up the processing of jobs, completions of jobs become earlier and the degree of satisfaction is improved. But it makes the running costs and burden of the machine larger, and furthermore, the quality of products may become worse. So we must balance them.

We seek the optimal schedule and optimal jobwise speeds of the machine minimizing the above mentioned objective function.

First we list up notations used in this paper.

w_j ; weight of job J_j,

s'_j ; speed when machine processes job J_j,

$\bar{p}_j$; processing amount of job J_j (processing time at unit speed of the machine),

C_j ; the completion time of job J_j,

$m_j(C_j)$; the membership function denoting degree of satisfaction with respect to C_j, which is defined as follows:

$$m_j(C_j) = \begin{cases} 1 & (C_j \leq d_j) \\ 1 - \dfrac{(C_j - d_j)}{e_j} & (d_j < C_j < d_j + e_j) \\ 0 & (d_j + e_j \leq C_j) \end{cases}$$

where e_j, d_j are nonnegative constants.

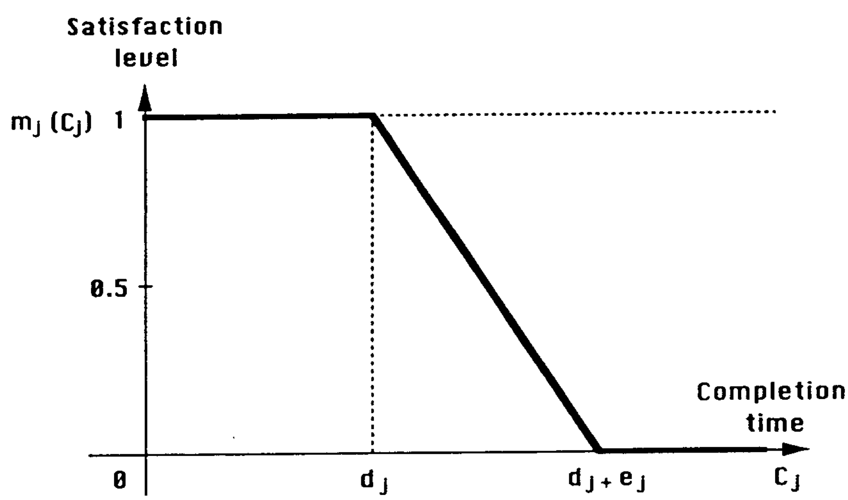

Figure 1. The membership function denoting degree of satisfaction with respect to C_j.

Further we use the notation $s_j \cong 1/s'_j$ for a convenience sake.

Under the above setting, we consider the following problem P.

$$P \; : \; \textit{Minimize} \quad -a_0 \min_j \{w_j \cdot m_j(C_j)\} + \sum_{j=1}^{n} a_j s'_j$$
$$\textit{subject to} \quad s'_j > 0, \; j = 1, 2, \cdots, n,$$

where w_j, a_j, $j = 0, 1, \cdots, n$, are positive constants.

Now let

$$g_j = \min_j\{w_j \cdot m_j(C_j) \mid j = 1, 2, \cdots, n\}, \ t = \min_j\{g_j\}$$

then inequalities

$$C_j \leq d_j + (1 - \frac{t}{w_j})e_j, \ j = 1, 2, \cdots, n \tag{1}$$

must hold. Note that the right hand side of each inequality corresponds to the ordinary duedate of job J_j.

Let π(k) denote the k-th job index of a schedule π. Then for π,

$$C_{\pi(k)} = \sum_{j=1}^{k} \bar{p}_{\pi(j)} s_{\pi(j)}, \ k = 1, 2, \cdots, n. \tag{2}$$

After variable transformation, $p_j = \bar{p}_j s_j, \ j = 1, 2, \cdots, n$, we set $\bar{a}_j = a_j \bar{p}_j, \ j = 1, 2, \cdots, n$ and $\bar{t} = w_j - t$. Then P is equivalent to the following problem $\bar{P}$.

$$\begin{aligned} \bar{P} \ : \ & Minimize \ \ a_0\bar{t} + \sum_{j=1}^{n} (\bar{a}_j / p_j) \\ & subject\ to \ \ \sum_{j=1}^{k} p_{\pi(j)} \leq d_{\pi(k)} + \frac{\bar{t}}{w_{\pi(k)}} e_{\pi(k)}, \ k = 1, 2, \cdots, n, \\ & \qquad p_j > 0, \ j = 1, 2, \cdots, n, \ 0 \leq \bar{t} \leq w_j, \ \pi \in \Pi, \end{aligned}$$

where Π is the set of all permutation schedules. Note that p_j is the actual processing time of J_j. When $\bar{t}$ is fixed, $\bar{P}$ seeks an optimal schedule and optimal processing time of each job J_j minimizing $\sum_{j=1}^{n}(\bar{a}_j/p_j)$ under duedates $d_j + \frac{\bar{t}}{w_j} e_j, \ j = 1, 2, \cdots, n$. In this case, an optimal schedule is obtained by processing jobs in a nondecreasing order of $d_j + \frac{\bar{t}}{w_j} e_j, \ j = 1, 2, \cdots, n$ [2]. Defining,

$$\bar{t}_{jk} = (d_j - d_k)/(\frac{e_k}{w_k} - \frac{e_j}{w_j})$$

for $j \neq k$, and sorting different $\bar{t}_{jk}$, let

$$0 = \bar{t}_0 < \bar{t}_1 < \cdots < \bar{t}_q = \min_j \{w_j\}.$$

By dividing the above interval into subintervals $T_i = [\bar{t}_{i-1}, \bar{t}_i], \ i = 1, 2, \cdots, q$, we introduce the following subproblems $\bar{P}_l, \ l = 1, 2, \cdots, q$, since a certain schedule π_l is always optimal on $T_l, \ l = 1, 2, \cdots, q$.

$$\begin{aligned} \bar{P}_l \ : \ & Minimize \ \ a_0\bar{t} + \sum_{j=1}^{n} (\bar{a}_j / p_j) \\ & subject\ to \ \ \sum_{j=1}^{k} p_{\pi_l(j)} \leq d_{\pi_l(k)} + \frac{\bar{t}}{w_{\pi_l(k)}} e_{\pi_l(k)}, \ k = 1, 2, \cdots, n, \\ & \qquad 0 < \bar{t} \leq w_j, \ \bar{t} \in T_l, \ p_j > 0, \ j = 1, 2, \cdots, n. \end{aligned}$$

Thus the best solution among optimal solutions of $\bar{P}_1, \bar{P}_2, \cdots, \bar{P}_q$ is an optimal solution of $\bar{P}$ and the corresponding schedule is an optimal schedule.

3 Solution Procedure of $\bar{\mathrm{P}}$

First we propose a solution procedure of subproblem $\bar{P}_l$. For convenience, we suppress subscript l in order that any confusion does not occur. Further let its optimal schedule be $\pi_l(j) = j$ for easiness of explanation by changing the job number if necessary.

Now let define functions $f(\bar{t}), h(\bar{t})$ as follows:

$$\begin{aligned} f(\bar{t}) &= Min\,\{\sum_{j=1}^{n}(\bar{a}_j/p_j) \mid \sum_{j=1}^{k} p_j \le d_k + \frac{\bar{t}}{w_k}e_k,\ p_k > 0,\ k = 1, 2, \cdots, n\}, \\ h(\bar{t}) &= a_0\bar{t} + f(\bar{t}). \end{aligned}$$

Theorem 1 *$f(\bar{t})$ is nonincreasing and a convex function of $\bar{t} \in T_l$.*

Proof : Nonincreasingness of $f(\bar{t})$ is clear and so we show its convexity only.

For $\bar{t}_\alpha, \bar{t}_\beta \in T_l$, let

$$f(\bar{t}_\alpha) = \sum_{j=1}^{n}(\bar{a}_j/p_j^\alpha), \qquad f(\bar{t}_\beta) = \sum_{j=1}^{n}(\bar{a}_j/p_j^\beta)$$

respectively. And for $\bar{t}_\lambda = \lambda\bar{t}_\alpha + \bar{\lambda}\bar{t}_\beta$ $(0 \le \lambda \le 1,\ \bar{\lambda} = 1 - \lambda)$, let

$$f(\bar{t}_\lambda) = \sum_{j=1}^{n}(\bar{a}_j/p_j^\lambda).$$

Then since

$$\sum_{j=1}^{k}(\lambda p_j^\alpha + \bar{\lambda}p_j^\beta) = \lambda\sum_{j=1}^{k} p_j^\alpha + \bar{\lambda}\sum_{j=1}^{k} p_j^\beta \le \lambda(d_k + \frac{\bar{t}_\alpha}{w_k}e_k) + \bar{\lambda}(d_k + \frac{\bar{t}_\beta}{w_k}e_k) = d_k + \frac{\bar{t}_\lambda}{w_k}e_k,$$

$p_j^\gamma = \lambda p_j^\alpha + \bar{\lambda}p_j^\beta$ is feasible for $\bar{t} = \bar{t}_\lambda$.

Thus

$$\begin{aligned} f(\bar{t}_\lambda) &= \sum_{j=1}^{n}(\bar{a}_j/p_j^\lambda) \le \sum_{j=1}^{n}(\bar{a}_j/p_j^\gamma) \\ &\le \lambda\sum_{j=1}^{n}(\bar{a}_j/p_j^\alpha) + \bar{\lambda}\sum_{j=1}^{n}(\bar{a}_j/p_j^\beta) = \lambda f(\bar{t}_\alpha) + \bar{\lambda}f(\bar{t}_\beta), \end{aligned} \tag{3}$$

where the second inequality holds from the convexity of the function $1/x$.

(3) shows convexity of $f(\bar{t})$. ∎

Theorem 1 implies $h(\bar{t})$ is convex. Now we propose a calculation method of $f(\bar{t}_c)$ for each $\bar{t}_c \in T_l$. First we introduce the following Lagrange function $L(\cdot)$.

$$L(u_1, \cdots, u_n, p_1, \cdots, p_n) = \sum_{j=1}^{n}(\bar{a}_j/p_j) + \sum_{k=1}^{n} u_k(\sum_{j=1}^{k} p_j - d_k - \frac{\bar{t}_c}{w_k}e_k), \tag{4}$$

where $u_1, u_2, \cdots, u_n$ are Lagrange multipliers and restricted to be nonnegative. Let us define

$$L(u_1,u_2,\cdots,u_n)=\min\{L(u_1,u_2,\cdots,u_n,p_1,p_2,\cdots,p_n)\mid p_1,p_2,\cdots,p_n>0\}.$$

Then

$$f(\bar{t}_c)=\max\{L(u_1,u_2,\cdots,u_n)\mid u_1,u_2,\cdots,u_n\geq 0\}$$

by the theory of convex programming. Since it holds that

$$\begin{aligned}
L(u_1,\cdots,u_n,p_1,\cdots,p_n) &= \sum_{j=1}^{n}(\bar{a}_j/p_j)+\sum_{k=1}^{n}u_k(\sum_{j=1}^{k}p_k-d_k-\frac{\bar{t}_c}{w_k}e_k)\\
&= \sum_{j=1}^{n}\{(\bar{a}_j/p_j)+p_j\sum_{k=j}^{n}u_k\}-\sum_{k=1}^{n}(d_k+\frac{\bar{t}_c}{w_k}e_k)u_k\\
&\geq \sum_{j=1}^{n}2\sqrt{\bar{a}_j}\sqrt{\sum_{k=j}^{n}u_k}-\sum_{k=1}^{n}(d_k+\frac{\bar{t}_c}{w_k}e_k)u_k \qquad (5)
\end{aligned}$$

by the relation between arithmetic and geometric means,

$$L(u_1,\cdots,u_n)=2\sum_{j=1}^{n}\sqrt{\bar{a}_j}\sqrt{\sum_{k=j}^{n}u_k}-\sum_{k=1}^{n}u_k(d_k+\frac{\bar{t}_c}{w_k}e_k) \qquad (6)$$

and (6) is given by

$$p_j=(\bar{a}_j/\sum_{k=j}^{n}u_k)^{\frac{1}{2}},\ \ j=1,2,\cdots,n.$$

Hence let

$$y_k=\sqrt{\sum_{j=k}^{n}u_j},\ \ k=1,2,\cdots,n$$

then

$$u_j=y_j^2-y_{j+1}^2,\ \ j=1,2,\cdots,n-1,\ \ u_n=y_n^2 \qquad (7)$$

and

$$y_1\geq y_2\geq\cdots\geq y_n\geq 0 \qquad (8)$$

by the nonnegativity of u_j, $j=1,2,\cdots,n$. Substituting (7) into (6), then

$$\begin{aligned}
&L(u_1,\cdots,u_n)\\
&= 2\sum_{j=1}^{n}\sqrt{\bar{a}_j}y_j-\sum_{j=1}^{n-1}(y_j^2-y_{j+1}^2)(\frac{\bar{t}_c}{w_j}e_j+d_j)\\
&= -\{y_1^2(\frac{\bar{t}_c}{w_1}e_1+d_1)-2\sqrt{\bar{a}_1}y_1\}\\
&\quad -\sum_{j=2}^{n}[y_j^2\{\frac{\bar{t}_c}{w_j}(e_j-e_{j-1})+(d_j-d_{j-1})\}-2\sqrt{\bar{a}_j}y_j]\\
&= -\sum_{j=2}^{n}\{\frac{\bar{t}_c}{w_j}(e_j-e_{j-1})+d_j-d_{j-1}\}[y_j-\sqrt{\bar{a}_j}/\{\frac{\bar{t}_c}{w_j}(e_j-e_{j-1})+d_j-d_{j-1}\}]^2\\
&\quad -(\frac{\bar{t}_c}{w_1}e_1+d_1)\{y_1-\sqrt{\bar{a}_1}/(\frac{\bar{t}_c}{w_1}e_1+d_1)\}^2+\bar{a}_1/(\frac{\bar{t}_c}{w_1}e_1+d_1)\\
&\quad +\sum_{j=2}^{n}\bar{a}_j/\{\frac{\bar{t}_c}{w_j}(e_j-e_{j-1})+d_j-d_{j-1}\}\\
&\cong g(y_1,y_2,\cdots,y_n). \qquad (9)
\end{aligned}$$

Now we define α_j, β_j and γ_j, $j = 1, 2, \cdots, n$ as follows;

$$\begin{aligned}\gamma_j &= \sqrt{\bar{a}_j},\ j = 1, 2, \cdots, n,\\ \beta_1 &= \frac{\bar{t}_c}{w_1}e_1 + d_1,\ \beta_j = \frac{\bar{t}_c}{w_j}(e_j - e_{j-1}) + d_j - d_{j-1},\ j = 2, 3, \cdots, n,\\ \alpha_j &= (\gamma_j/\beta_j),\ j = 1, 2, \cdots, n.\end{aligned}$$

Using these notations,

$$g(y_1, y_2, \cdots, y_n) = -\sum_{j=1}^{n} \beta_j (y_j - \alpha_j)^2 + \sum_{j=1}^{n} \gamma_j^2/\beta_j.$$

Theorem 2 *(i) If $\alpha_j \geq \alpha_{j+1}$, $1, 2, \cdots, n-1$, then*

$$f(\bar{t}_c) = \sum_{j=1}^{n} \gamma_j^2/\beta_j$$

by setting $y_j = \alpha_j$, $j = 1, 2, \cdots, n$.
(ii) If for a certain i,

$$\alpha_i < \alpha_{i+1},$$

then a solution giving $f(\bar{t}_c)$ satisfies

$$y_i = y_{i+1}.$$

Proof : The former part (i) is clear. For the latter part (ii), it is sufficient to show $\bar{y}$ with $\bar{y}_i > \bar{y}_{i+1}$ never maximizes $g(y_1, y_2, \cdots, y_n)$.

First note that y must satisfy $y_i \geq y_{j+1}$, $j = 1, 2, \cdots, n-1$ from (8). So we can not set $y_i = \alpha_i$, $y_{i+1} = \alpha_{i+1}$. We show (ii) by checking following cases.

Case (1) ; $\bar{y}_{i+1} \geq \alpha_i$.

When $y_i \to \alpha_i$, $g(\cdot)$ increases, i.e., setting $y_i = y_{i+1}$ produces better solution than $\bar{y}$.

Case (2) ; $\bar{y}_i > \alpha_i \geq \bar{y}_{i+1}$.

As $y_i \downarrow \alpha_i$ and $y_{i+1} \uparrow \alpha_{i+1}$, $g(\cdot)$ increases, i.e., by decreasing y_i and increasing y_{i+1}, the value of $g(\cdot)$ is improved. But due to the constraint $y_i \geq y_{i+1}$, the above change produces the solution $y_i = y_{i+1}$, and it is better than $\bar{y}$.

Case (3) ; $\alpha_i \geq \bar{y}_i$.

Case(3) implies

$$\alpha_{i+1} > \alpha_i \geq \bar{y}_i > \bar{y}_{i+1}. \tag{10}$$

Again, as $y_i \uparrow \alpha_i$ and $y_{i+1} \uparrow \alpha_{i+1}$, $g(\cdot)$ increases. By (10) and the fact that y_{i+1} can not increase above y_i, $y_{i+1} = y_i$ is necessarily reached by the process.
■

When case (ii) of Theorem 2 occurs, i.e., there exists i such that $\alpha_i < \alpha_{i+1}$, we can update y_j, α_j, β_j, γ_j as follows.

$$\begin{aligned}y'_j &= y_j,\ \beta'_j = \beta_j,\ \gamma'_j = \gamma_j \quad (\alpha'_j = \alpha_j),\ j = 1, 2, \cdots, i-1,\\ y'_i &= y_i,\ \beta'_i = \beta_i + \beta_{i+1},\ \gamma'_i = \gamma_i + \gamma_{i+1} \quad (\alpha'_i = (\gamma_i + \gamma_{i+1})/(\beta_i + \beta_{i+1}))\end{aligned}$$

and (11)

$$y'_j = y_{j+1},\ \beta'_j = \beta_{j+1},\ \gamma'_j = \gamma_{j+1} \quad (\alpha'_j = \alpha_{j+1}),\ j = i+1, \cdots, n-1.$$

Above updating of (11) induces the updating of $g(\cdot)$, and results are as follows.

$$g(y'_1, \cdots, y'_{n-1}) = -\sum_{j=1}^{n-1} \beta'_j (y'_j - \alpha'_j)^2 + \sum_{j=1}^{n-1} \gamma'^2_j / \beta'_j \tag{12}$$

For the revised $g(\cdot)$, we check the condition of Theorem 2 again. Note that $g(y'_1, \cdots, y'_{n-1})$ is the very same type of $g(y_1, y_2, \cdots, y_n)$. After each check, either the number of variables is decreased by one or the condition (i) of Theorem 2 is reached. Therefore after at most $(n-1)$ updatings of $g(\cdot)$, the condition (i) of Theorem 2 occurs, and $f(\bar{t}_c)$ and corresponding (y_j) are found. The type of $g(\cdot)$ is not changed independent of the value $\bar{t}_c$. Of course, α_j, is the function of $\bar{t}$ and so we describe α_j with $\alpha_j(\bar{t})$. According to the value of $\bar{t}$, the ordering of $\alpha_j(\bar{t})$ is changed, i.e., which condition (i) or (ii) of Theorem 2 occurs is changed.

Here we denote optimal value of $\bar{t}$ with $\bar{t}^*$ and its corresponding optimal solution (y_j^*). Now we are ready to give an algorithm for $\bar{P}_l$, $l = 1, 2, \cdots, q$, which is based on the same principle as Megiddo's [4].

Algorithm for $\bar{P}_l$

Step 0 : Set

$$\begin{aligned}
\alpha_1(\bar{t}) &= \sqrt{\bar{a}_1}/(\frac{\bar{t}}{w_1} e_1 + d_1) \ \ (= \gamma_1(\bar{t})/\beta_1(\bar{t})), \\
\alpha_j(\bar{t}) &= \sqrt{\bar{a}_j}/\{\frac{\bar{t}}{w_j}(e_j - e_{j-1}) + d_j - d_{j-1}\} \ \ (= \gamma_j(\bar{t})/\beta_j(\bar{t})), \ j = 2, 3, \cdots, n,
\end{aligned}$$

and $s = 1$.

Step 1 : Find $\bar{t}^s$ satisfying $\alpha_s(\bar{t}) = \alpha_{s+1}(\bar{t})$.

Step 2 : If $\bar{t}^s \notin T_l$, then go to Step 4. Otherwise, go to Step 3.

Step 3 : Calculate $f(\bar{t}^s)$ and corresponding y^s. Next calculate its subdifferential $\partial f(\bar{t}^s)$. If $\partial f(\bar{t}^s) = -a_0$, then set $\bar{t}^* = \bar{t}^s$ and calculate (p_j^*) from (y_j^s), and terminate. If $\min\{\rho \mid \rho \in \partial f(\bar{t}^s)\} > -a_0$, then set $T_l = T_l \cap [t_{l-1}, \bar{t}^s]$ and go to Step 4. If $\max\{\rho \mid \rho \in \partial f(\bar{t}^s)\} < -a_0$, then set $T_l = T_l \cap [\bar{t}^s, \bar{t}_l]$ and go to Step 4.

Step 4 : If $\alpha_s \geq \alpha_{s+1}$, then go to Step 5. Otherwise, set

$$\begin{aligned}
y_j &= y_j, \ \beta_j = \beta_j, \ \gamma_j = \gamma_j \quad (\alpha_j = \alpha_j), \ j = 1, 2, \cdots, s-1, \\
y_s &= y_{s+1}, \ \beta_s = \beta_s + \beta_{s+1}, \ \gamma_s = \gamma_s + \gamma_{s+1} \quad (\alpha_s = (\gamma_s + \gamma_{s+1})/(\beta_s + \beta_{s+1})) \\
y_j &= y_{j+1}, \ \beta_j = \beta_{j+1}, \ \gamma_j = \gamma_{j+1} \quad (\alpha_j = \alpha_{j+1}), \ j = s+1, \cdots, n-1,
\end{aligned}$$

and $n = n - 1$. Go to Step 6.

Step 5 : Set $s = s + 1$. If $s = n$, go to Step 7. Otherwise, return to Step 1.

Step 6 : If $s = n$, go to Step 7. If $s = 1$, return to Step 1. Otherwise set $s = s - 1$ and return to Step 1.

Step 7 : Based on all informations about y_j, α_j, $j = 1, 2, \cdots, n$, seek for $\bar{t}^o$ satisfying $\frac{d}{d\bar{t}} f(\bar{t}) = -a_0$. If there exists such $\bar{t}^o$, then set $\bar{t}^* = \bar{t}^o$, calculate (p_j^*) from $\bar{t}^*$ and terminate. Otherwise, set

$$\bar{t}^* = \begin{cases} \bar{t}_{l-1} & (if \frac{d}{dt} f(\bar{t}) > -a_0 \text{ on the current } T_l) \\ \bar{t}_l & (if \frac{d}{dt} f(\bar{t}) < -a_0 \text{ on the current } T_l) \end{cases}$$

then calculate (p_j^*) from $\bar{t}^*$ and terminate.

Solution Procedure for $\bar{P}$

(I) Solve all $\bar{P}_l$ according to the above algorithm.

(II) Choose the best solution among optimal solutions of $\bar{P}_l$'s and calculate an optimal schedule and optimal jobwise machine speeds from it.

Theorem 3 *If nonlinear equation*

$$\sum_{i=1}^{n} F_i/(D_i + E_i x)^2 = A$$

is solved in $O(n^2)$ computational times, our procedure solves $\bar{P}$ in $O(n^4)$ computational times.

Proof : Validity is clear from the above discussions. For the complexity, first we show the complexity of our algorithm for $\bar{P}_l$. the computational time of each step is as follows.

(a) $O(n)$ to calculate $f(\bar{t}^s)$ in Step 3.

(b) It takes $O(n)$ from Step 1 to Step 6.

(c) From (a) and (b), $O(n^2)$ till Step 7.

(d) By assumption, it takes $O(n^2)$ in Step 7.

In total, $O(n^2)$ for each $\bar{P}_l$.

Since the number of $\bar{P}_l$ is at most $O(n^2)$, $O(n^4)$ computational time is enough to solve $\bar{P}$ by our procedure. ■

4 Discussion

This paper considered one machine weighting jobs scheduling problem with fuzzy duedates. Though a linear membership function is not enough to describe an actual situation of flexible duedates, we think it is a first attempt to generalize an ordinary scheduling problem into a more flexible model by fuzzy duedates. In this sense, other factors such as processing times[6], precedence relations [9]etc. may be also fuzzified, and generalization of membership function as variable, non-linear form may be investigated. But our solution procedure does not utilize this structure of fuzziness enough. So refinement to our model may be the first effort to be done.

Whether investigations of a so called "fuzzy scheduling model" are successful or not, depends greatly on the degree of difficulty of original model. In summary, we think this generalized area of scheduling theory may be fruitful and interesting, and many issues are left remained without any study. Especially our model has a single objective and it may not reflect actual situations and so a multi-objective model of scheduling problems [7][8][9]are to be fuzzified and solved.

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