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Dynamic Control of Redundant Manipulators Based on Stability Condition

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Abstract

An efficient dynamic control algorithm that outperforms existing local torque optimization techniques for redundant manipulators is presented. The method resolves redundancy at the acceleration level. In this method, a systematic switching technique as a trade-off means between local torque optimization and global stability is proposed based on the stability condition proposed by Maciejewski [1]. Comparative simulations on a three-link planar arm show the effectiveness of the proposed method.

1 Introduction

Kinematically redundant robotic systems are defined as systems which possess more degrees of freedom than are required to perform the specified task. Additional redundant joints can be used to improve the performance of redundant manipulator and to avoid obstacles and singularities. In addition, the redundancy provides room for optimization of certain desired criteria when tracking a specified trajectory. One of these criteria is the utilization of redundancy for joint torque optimization.

When redundancy is resolved at the acceleration level to instantaneously optimize joint torque, the joint acceleration is related to the end-effector acceleration as follows:

$$J\ddot{\theta} = \ddot{x} - \dot{J}\dot{\theta} \tag{1}$$

where $\ddot{x} \in \Re^m$ represents the acceleration of an end-effector, $\dot{\theta}$, $\ddot{\theta} \in \Re^n$ are joint velocities and accelerations, respectively, J is the Jacobian matrix and $\dot{J}$ is its time derivative. For redundant manipulators, Eq. (1) will be underdetermined since $m < n$. If a desired end-effector acceleration vector $\ddot{x}$ is given, and current joint positions and velocities are known, the general solution to Eq. (1) is typically presented in the form

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\dot{\theta}) + (I - J^+J)\ddot{\phi} \tag{2}$$

where $^+$ denotes the pseudoinverse, $(I - J^+J)$ is a projection operator onto the null space of J, and $\ddot{\phi}$ is an arbitrary vector in $\ddot{\theta}$ space.

On the other hand, it is well known that joint torques are expressed as

$$\tau = M\ddot{\theta} + N \tag{3}$$

where M is the inertia matrix, and N is a vector containing terms such as Coriolis, centripetal and gravitational torques. If $\tilde{\tau}$ is used to denote the torque due to the minimum-norm acceleration, that is, the first term of Eq. (2), $\tilde{\tau}$ can be obtained as

$$\tilde{\tau} = MJ^+(\ddot{x} - \dot{J}\dot{\theta}) + N \tag{4}$$

Hollerbach and Suh [2] recast the optimization problem of locally minimizing joint torques as finding the vector $\ddot{\phi}$ to minimize $\|\tau\|$, that is,

$$\left\|M(I - J^+J)\ddot{\phi} + \tilde{\tau}\right\| \tag{5}$$

This is a straightforward least-squares problem which can be solved by the pseudoinverse with the solution given by

$$\ddot{\phi} = -[M(I - J^+J)]^+\tilde{\tau}. \tag{6}$$

Thus the solution of this torque optimization problem yields joint accelerations as

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\dot{\theta}) - [M(I - J^+J)]^+\tilde{\tau} \tag{7}$$

which was simplified using the following identity [3]

$$B[CB]^+ = [CB], \tag{8}$$

where B is Hermitian and idempotent. They applied this local torque optimization technique to the motion of three-link planar manipulator. Although joint torques are successfully minimized for short trajectories, some *instabilities* which may require physically unrealizable torques have been observed for long trajectories for this method.

Kazerounian and Nedungadi [4] proposed a solution for the torque-minimization least-squares problem as follows:

> *Minimize $\tau^T\tau$ subject to Eq. (1) where τ is given by Eq. (3).*

Using Lagrangian multipliers, they arrived at a solution that incorporates a generalized inverse, weighted by the squared inertia matrix:

$$\ddot{\theta} = J_B^+(\ddot{x} - \dot{J}\dot{\theta}) - (I - J_B^+J)M^{-1}N, \tag{9}$$

where

$$J_B^+ = B^{-1}J^T(JB^{-1}J^T)^{-1}, \qquad B = M^TM \tag{10}$$

From a computational point of view, the solution Eq. (9) seems to have some advantages compared to Eq. (7); in the latter solution only one pseudoinverse is to be calculated, whereas Eq. (7) requires the calculation of two pseudoinverses. They concluded that a trade-off method between local torque optimization and global stability is necessary, and suggested the following switching technique as:

$$\ddot{\theta} = \begin{cases} J_B^+(\ddot{x} - \dot{J}\,\dot{\theta}) - (I - J_B^+ J) M^{-1} N & \text{if } \|\ddot{\theta}\| < \alpha \\ J_B^+(\ddot{x} - \dot{J}\,\dot{\theta}) & \text{if } \|\ddot{\theta}\| \geq \alpha \end{cases} \tag{11}$$

In this technique, the norm of joint accelerations $\|\ddot{\theta}\|$ is monitored and when this norm is higher than a given threshold α, they suggested that only the pseudoinverse-based particular solution can be used to increase stability and to minimize joint torques to some extent. Unfortunately, the value of the threshold is often critical for the performance, which has not been discussed in [4].

In order to avoid switching during the motion, Ma, Hirose, and Nenchev [5] proposed two types of damped least-squares. One is the damped squared-torque optimization which has the same structure as Eq. (9) obtained by Kazerounian and Nedungadi:

$$\ddot{\theta} = J_W^+(\ddot{x} - \dot{J}\,\dot{\theta}) - \beta (I - J_W^+ J) M^{-1} N \tag{12}$$

where

$$J_W^+ = W^{-1} J^T (J W^{-1} J^T)^{-1} \tag{13}$$
$$W = \left[(1-\beta) I + \beta M^T M\right], \tag{14}$$

and β, $0 \leq \beta \leq 1$ is a continuous function. The other is the damped null-space torque optimization which is a slight modification of Hollerbach and Suh's method:

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\,\dot{\theta}) - \gamma \left[M(I - J^+ J)\right]^+ \tilde{\tau}, \tag{15}$$

where γ, $0 \leq \gamma \leq 1$ is a weighting factor to balance the particular and the homogeneous solution. Unfortunately, it is not possible to derive analytical expressions for the balancing factors, β and γ, in order to guarantee a reasonable real-time implementation.

In order to identify regions of stability and instability for a local torque optimization scheme, Maciejewski [1] presented the stability condition given by

$$\dot{\theta}_h \cdot \ddot{\theta}_h > 0, \tag{16}$$

where $\dot{\theta}_h$ and $\ddot{\theta}_h$ represent homogeneous joint velocity and homogeneous joint acceleration vectors, respectively. Although the above condition is not exact in determining the stability of joint torques, it can be served as a general guideline to determine stability. When Eq. (16) is true, the homogeneous acceleration term will increase the magnitude of the homogeneous joint velocity and will subsequently increase torque requirements. This, in effect, amounts to a positive feedback system and results in the instability of local torque optimization as noted in [2]. However, he did not present a real-time control law using this condition in order to overcome the instability, while he focused on proving that the condition is solely a function of a manipulator's configuration.

In this paper, a new dynamic control algorithm which is based on the formulation of Kazerounian and Nedungadi's method is proposed. The proposed algorithm provides systematic switching between global stability and local torque optimization by means of stability condition, and thus improves the drawback of Kazerounian and Nedungadi's method. The reasons for generating physically unrealizable torques for conventional local torque optimization schemes especially for long trajectories are analyzed using the concept of *aspect* [6] and the stability condition is also analyzed in this respect. Comparative simulations with a planar three-link arm show the good performance of the proposed method.

2 A New Dynamic Control Method

As mentioned in the previous section, Kazerounian and Nedungadi proposed the explicit form of Eq. (9), for the computation of joint accelerations as a solution to the torque-minimization least-squares problem. They suggested that the (unweighted) pseudoinverse J^+ improves the stability of Eq. (9) when it is used instead of J_B^+ as:

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\,\dot{\theta}) - (I - J^+ J) M^{-1} N. \tag{17}$$

Thus the proposed algorithm to be formulated is based on this equation.

In Eq. (17), the first term denotes the minimum-norm of joint accelerations $\|\ddot{\theta}\|$ as "minimum-norm acceleration." For the accurate tracking of the Cartesian trajectory $x_d(\cdot)$, the usual error correcting term $K_p e + K_v \dot{e}$ is added to $\ddot{x}_d$ in place of $\ddot{x}$ where $e \triangleq x_d - x$ is the tracking error, K_p and K_v are constant position and velocity feedback gain matrices. Then the minimum-norm acceleration $\ddot{\theta}_m$ is given by

$$\ddot{\theta}_m = J^+(\ddot{x}_d + K_v \dot{e} + K_p e - \dot{J}\,\dot{\theta}). \tag{18}$$

The second term of Eq. (17) can be shown to be the homogeneous (or null-space) acceleration denoted by $\ddot{\theta}_h$, by projecting $\ddot{\theta}$ onto the null-space of the Jacobian matrix by means of the projection operator $(I - J^+ J)$ as follows:

$$\begin{aligned} \ddot{\theta}_h &= (I - J^+ J)\ddot{\theta} \\ &= (J^+ - J^+ J J^+)(\ddot{x} - \dot{J}\,\dot{\theta}) - (I - J^+ J)^2 M^{-1} N \\ &= -(I - J^+ J) M^{-1} N \end{aligned} \tag{19}$$

where the property of pseudoinverse, $J^+ J J^+ = J^+$, and the idempotency of $(I - J^+ J)$, $(I - J^+ J)^2 = (I - J^+ J)$, are used.

In order to increase the performance of the proposed method in the sense of global stability, we need a systematic switching criterion to determine whether $\ddot{\theta}_h$ is added to $\ddot{\theta}_m$ or not. As mentioned earlier, the stability condition proposed by Maciejewski is adopted as a switching criterion for the proposed method. To be more specific, if Eq. (16) is true, the homogeneous acceleration must not be used in order to guarantee the global stability of the proposed method. Otherwise, $\ddot{\theta}_h$ is added to $\ddot{\theta}_m$ so that the local torque optimization is achieved in the stable region of operation.

According to the above strategy, the command acceleration $\ddot{\theta}_d$ is generated as follows:

$$\ddot{\theta}_d = \begin{cases} \ddot{\theta}_m + \ddot{\theta}_h & \text{if } \dot{\theta}_h \cdot \ddot{\theta}_h \leq 0 \\ \ddot{\theta}_m & \text{if } \dot{\theta}_h \cdot \ddot{\theta}_h > 0 \end{cases} \tag{20}$$

where $\dot{\theta}_h$ is given by

$$\dot{\theta}_h = (I - J^+ J)\,\dot{\theta}\,. \tag{21}$$

The actual values of $\dot{\theta}$ in Eq. (21) can be measured from a redundant robot system. In the case of computer simulation, the values of $\dot{\theta}$ are obtained by solving the forward dynamics of a manipulator through the fourth-order Runge-Kutta algorithm. Consequently, the stability condition is incorporated in actively avoiding the instability region of operation *online* rather than passively identifying the regions of stability and instability *offline*. As shown in Eq. (20), the change of the command acceleration according to the stability condition generates the discontinuity on joint torque, which is inherent for the switching process. However, the discontinuity does not degrade the proposed method because the step-like torque can easily be induced by normal electric actuators in the stable region of operation. This will be illustrated by computer simulation in the following section.

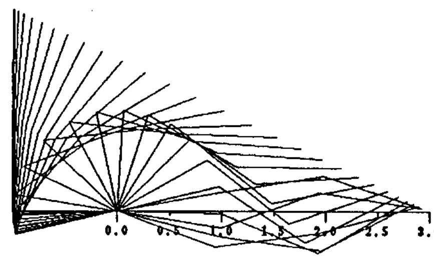
3. Computer Simulation

The comparative evaluation of the proposed method against the unweighted pseudoinverse method and Hollerbach and Suh's method were conducted by computer simulation. Link lengths, masses, and the type of trajectory were chosen similar to those in Hollerbach and Suh's work in order to compare with their results. The simulated model is a planar three-link manipulator model with revolute joints. Links have unit length, mass of 10.0 kg, and are modeled as uniform thin rods. Gravity is neglected. Relative joint variables are adopted and the torques are defined accordingly. Actuators are assumed to be located at the base of the manipulator. The joints are labelled 1, 2, 3 from the base. The kinematic task is specified as a Cartesian path for the end-effector, with zero initial and final velocity. Joints are commanded to start at rest from a given initial arm configuration. End-effector acceleration is of the bang-bang type, with equal acceleration and deceleration in the first and the last half, respectively.

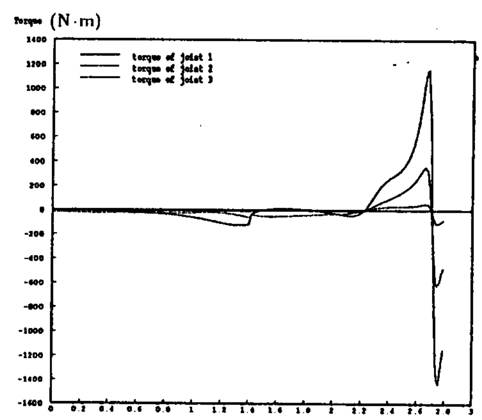
The command torque τ, which is adopted as a control input, is obtained from Eq. (3) using the command acceleration given by Eqs. (18), (7), and (20) for the three methods mentioned above, respectively. The fourth-order Runge-Kutta integration algorithm is used to obtain joint positions and velocities from the nonlinear ordinary differential equations of forward dynamics. An integration step of 2 ms is accurate enough to avoid the use of closed-loop correction. The joint control system is simulated with position and velocity feedback gain matrix K_p = diag(256, 256, 256) rad/(m · s^2) and K_v = diag(32, 32, 32) rad/(m · s), respectively. In this simulation, 1.96 m and 0.98 m long path in x and y directions, respectively, is considered. The arm starts from θ_0 = (180°, −90°, 0°), with accelerations $\ddot{x}_d$ = (2, −1) m/s^2 and $\ddot{x}_d$ = (−2, 1) m/s^2 for the first and the last half of the path respectively.

Arm motion and torque profiles for the unweighted pseudoinverse method are reported in Fig. 1 (a) and (b), respectively. As shown in Fig. 1 (b), the method resulted in remarkably large torque, about 1400 N · m, at 2.6 s to 2.8 s. As pointed out in [2], the method fails to rule out the stability problem. Figure 2 (a)–(c) show the plots of arm motion, joint torques, and $\dot{\theta}_h \cdot \ddot{\theta}_h$ for Hollerbach and Suh's method. The method is simulated utilizing the resolved acceleration control [7] with position and velocity feedback matrices K_p and K_v (that is, $\ddot{x}_d + K_v \dot{e} + K_p e$), instead of $\ddot{x}$ in Eq. (7). In Fig. 2 (b), physically unachievable joint torques, about 85,000 N · m, are observed at t = 2.5 s, which means that the method has unrealistic characteristic of joint torque. This instability is verified in Fig. 2 (c) where the stability condition $\dot{\theta}_h \cdot \ddot{\theta}_h$ has a large positive value of 10^4 order at the same instant of peak torque. As noted in [1], this amounts to an unstable system with large positive feedback gains.

The reason of torque instability can also be analyzed in terms of the concept of *aspect* [6]. The joint space of a robot can be decomposed into volumes corresponding to the various classes of configurations, called "aspects." When joint limits are taken into account, the reachable domain in joint space is divided into ${}_nC_m$ aspects for m task variables and n joint variables. One of the separating surfaces between aspects is the locus of joint coordinates at which one of the m-order minors (namely, *full row-rank minors*) of the manipulator Jacobian $J \in \Re^{m \times n}$ is equal to zero. Mathematically , an aspect is defined in [6].



(a)



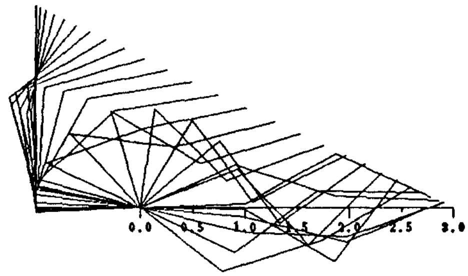
(b)

Fig. 1. Simulation results for the unweighted pseudoinverse method: (a) arm motion; (b) torque profiles.

The relation between aspects and joint torques are analyzed as follows. Chang [8] proved that the manipulability measure [9] can be rewritten in terms of the full row-rank minors:

$$w = \sqrt{\det(JJ^T)} = \left(\sum_{i=1}^{p} \delta_i^2\right)^{\frac{1}{2}} \tag{22}$$

where δ_i's, $i = 1, 2, \ldots, p$, with $p = {}_nC_m$, are the full row-rank minors of $J \in \Re^{m\times n}$ with $m < n$. The pseudoinverse of J with full rank is given by



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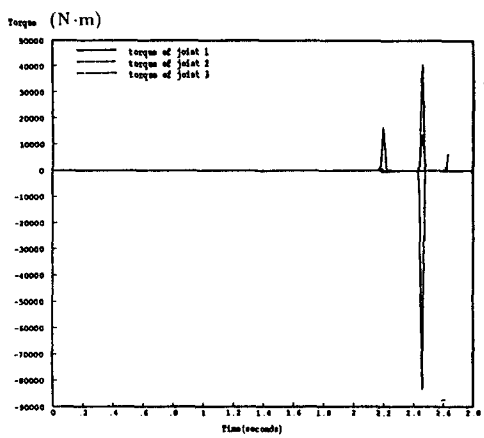

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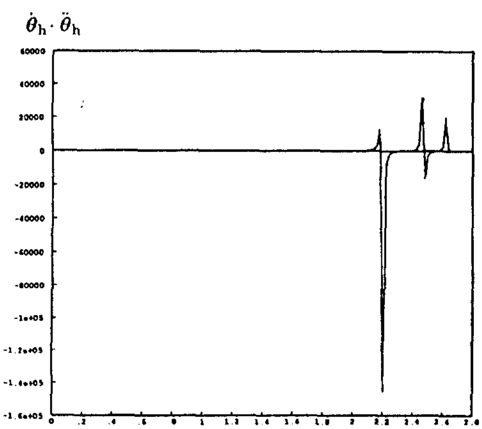

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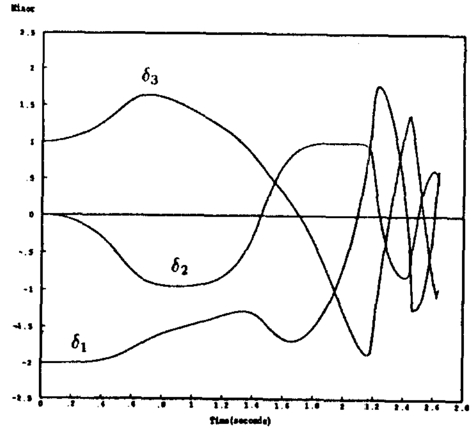


(d)

Fig. 2. Simulation results for the Hollerbach and Suh's method: (a) arm motion; (b) torque profiles; plot of $\dot{\theta}_h \cdot \ddot{\theta}_h$; (d) minor profiles.

$$J^+ = J^T(JJ^T)^{-1}. \tag{23}$$

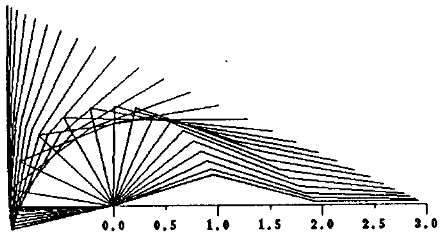
The situation that the full row-rank minors have zero values frequently means that aspects are frequently switched. So, reviewing Eqs. (22) and (23), we can state that when the minors δ_i's have smaller values in the neighborhood of frequent switching of aspects, they will induce the larger values of the elements of J^+. Accordingly, the command accelerations given by Eqs. (18) and (7) have large values, which in turn induces large torque requirements. Therefore, it can be said that the frequent switching of aspects induces the instability problem.

Along this line of reasoning mentioned above, we can find the behavior of the full row-rank minors given by

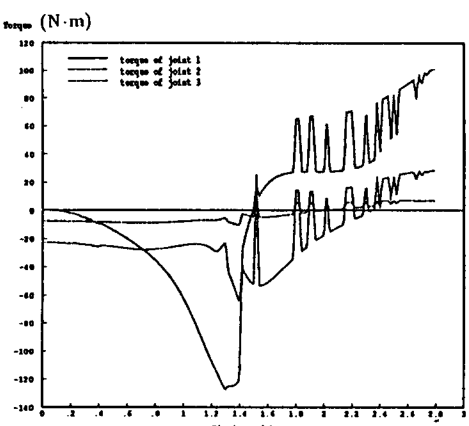
$$\delta_1 = \det[J^1 \;\; J^2] = \ell_1\ell_2 s_2 + \ell_1\ell_3 s_{23} \tag{24}$$
$$\delta_2 = \det[J^2 \;\; J^3] = \ell_2\ell_3 s_3 \tag{25}$$
$$\delta_3 = \det[J^3 \;\; J^1] = -\ell_2\ell_3 s_3 - \ell_1\ell_3 s_{23} \tag{26}$$

in Fig. 2(b) where J^i is the i-th column vector of the Jacobian J; $s_2 = \sin\theta_2$, $s_3 = \sin\theta_3$, and $s_{23} = \sin(\theta_2 + \theta_3)$, behave especially when extremely large torques are required. From this comparison, we can understand that the peak torques occur during the last period of 0.4 s where the aspects are switched as frequently as six times since each δ_i, i = 1, 2, 3 becomes zero as shown in this figure.

As expected, the proposed method with the systematic switching between local torque optimization and globl stability can hold down the peak torque at reasonable low level, about 130 N · m, as shown in Fig. 3(b). The plot of $\dot{\theta}_h \cdot \ddot{\theta}_h$ is presented in Fig. 3(c). Although the values of $\dot{\theta}_h \cdot \ddot{\theta}_h$ are positive during almost entire movement, they are kept at reasonably low positive values of 10^{-1} order when compared with those of Hollerbach and Suh's method. This implies that the stability of joint torque depends not on whether the sign of $\dot{\theta}_h \cdot \ddot{\theta}_h$ is positive or negative but on how large the positive peak value of $\dot{\theta}_h \cdot \ddot{\theta}_h$ is. Therefore the proposed method is able to minimize joint torques at a great extent and to maintain global stability by guiding the homogeneous acceleration according to the stability condition.



(a)



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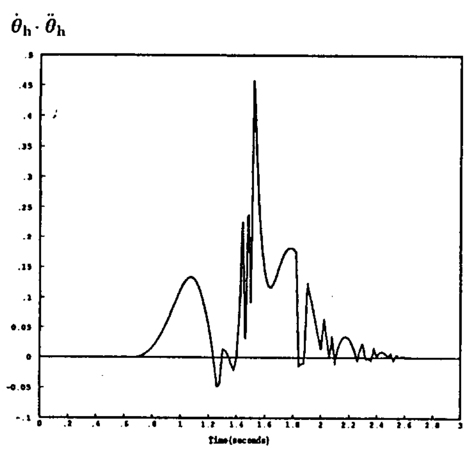

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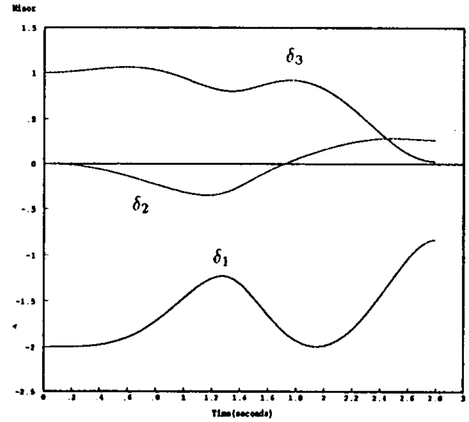

(d)

Fig. 3. Simulation results for the proposed method: (a) arm motion; (b) torque profiles; (c) plot of $\dot{\theta}_h \cdot \ddot{\theta}_h$; (d) minor profiles.

As shown in Fig. 3(d), the switching of aspects occurs only one time at about 1.7 s. Owing to the capability of the proposed method to suppress the switching of aspects, the peak torque of the proposed method can be drastically held down at low values in comparison with those of the unweighted pseudoinverse method and Hollerbach & Suh's method. This implies that the stability condition is closely related to suppressing the switching of aspects. There is one important observation concerning the stability condition and the aspects; that is, both are solely functions of manipulator's configuration.

4 Conclusion

A new dynamic control of redundant manipulators for guaranteeing globally stable behavior of joint torque was proposed. The proposed method incorporates the systematic switching technique between local torque optimization and global stability by applying the homogeneous acceleration to the minimum-norm acceleration according to the stability condition. The method is efficient and feasible for on-line implementation, requiring only one pseudoinverse operation. Good performance was verified through computer simulation in comparison with other existing local torque optimization schemes. It was shown that the unstable behavior of joint torque depends not on whether the sign of $\dot{\theta}_h \cdot \ddot{\theta}_h$ is positive or negative but on how large the positive peak value of $\dot{\theta}_h \cdot \ddot{\theta}_h$ is. It was also pointed out that the stability condition ultimately aims at suppressing the switching of aspects, which should be proved rigorously later.

References

[1] A. A. Maciejewski, "Kinematic limitations on the use of redundancy in robotic manipulators," *IEEE Trans. on Robotics and Automation*, vol. 7, no. 2, pp. 205–210, 1991.

[2] J. M. Hollerbach and K. C. Suh, "Redundancy resolution of manipulators through torque optimization," *IEEE J. Robotics and Automation*, vol. RA-3, no. 4, pp. 308–316, 1987.

[3] A. A. Maciejewski and C. A. Klein, "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environment," *Int. J. Robotics Res.*, vol. 1, no. 3, pp. 109–117, 1985.

[4] K. Kazerounian and A. Nedugadi, "Redundancy resolution of robotic manipulators at the acceleration level," *Proc. 7th IFToMM World Congress* (Sevilla, Spain), Sept. 1987, pp. 1207–1211.

[5] S. Ma, S. Hirose, and D. Nenchev, "Improving local torque optimization techniques for redundant robotic mechanisms," *J. Robotic Systems*, vol. 8, no. 1, pp. 75–91, 1991.

[6] P. Borrel and A. Liègeois, "A study of multiple manipulator inverse kinematic applications to trajectory planning and workspace determination," *Proc. IEEE Int. Conf. Robotics and Automation* (San Francisco, CA), April 1986, pp. 1180–1185.

[7] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, "Resolved-acceleration control of mechanical manipulators," *IEEE Trans. Automatic Control*, vol. AC-24, pp. 468–474, 1980.

[8] P. H. Chang, "Development of a dexterity measure for kinematically redundant manipulators," *Proc. American Control Conf.* (Pittsburgh, PA), June 1989, pp. 496–506.

[9] T. Yoshikawa, "Analysis and control of robot manipulators with redundancy," *Proc. Robotics Research: 1st Int. Symp.*, M. Brady and R. Paul, Eds., Cambridge, MA: MIT press, pp. 439–446, 1984.