Spline Method with Application to Ship Classification

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ABSTRACT

The first objective of this study is to derive an automated method that minimizes the number of spline regions and optimizes the locations of the knots to provide and adequate fit of a given nonlinear function. This has been accomplished by the development of the Optimal Spline Method discussed herein. The second objective is to apply the derived automated method to an important application. This objective has been accomplished by the successful application of the Optimal Spline Method to ship classification .

I. INTRODUCTION

A nonlinear function can be chopped into subarcs having small nonlinearities which can be represented by simple spline functions. The domain of a subarc is called a spline region and a point joining one spline region with an adjacent one is called a knot. The piecewise combination of the simple spline functions that fit the subarcs is a spline representation of the nonlinear function. Spline representations can closely approximate the nonlinearities of any function. The goodness of fit depends on the number of spline regions and on the location of the knots.

A major technological area in ocean surveillance is the over-the-horizon detection and classification of surface ships. The high range resolution radar system is a candidate obtaining surface ship signatures (stern-bow profiles) having classification potential. Accordingly, identifying a ship signature is largely a software problem.

The integrated signal returns of a high range resolution radar system provide data from which the radar cross-section per range cell of a ship can be estimated. Radar cross-section is a measure of the amount of a ship's superstructure contained in a range cell. As a result, a graph of the radar cross-sections per range cell drawn over a ship's length(i.e., a stern-bow profile) provides a means for classifying ships. The stern-bow profiles are the signatures for ship classification. The separate superstructure appears as valleys. An automated technique is needed for determining the number of separate superstructure masses, their separation from each other, their locations and their extended widths. These are the independent features of a ship's stern-bow radar cross-section profile. The Optimal Spline Method is an automated technique that fulfills this need.

The problem of identifying a pattern increases exponentially with the number of features. Each set of data contains a fixed number of independent features. If too many spline regions are used the independent feature are distributed into a higher number of spline coefficients. In this case the set of spline coefficients form a dependent set of features, making the classification problem unnecessarily complex. An independent set of features can be accomplished by optimizing on the locations of the spline regions for each fixed number of knots and by minimizing the number of knots. This is precisely what the Optimal Spline Method accomplishes.

2. QUADRATIC SPLINE FUNCTIONS

Consider the following general 2nd order nonlinear differential equation

$$r^{(2)}(t) = f(t,r(t),r^{(1)}(t)) \qquad (2.1)$$

where r and f are scalars and where f is a continuously differentiable function. Integrating both sides of this equation twice with respect to time over the interval $[T_0,t]$ gives

$$r(t) = r^{(1)}(T_0) + r(T_0)(t-T_0) + \int_{T_0}^{t}\int_{T_0}^{\tau} f(s,r(s),r^{(1)}(s))\,ds\,d\tau \qquad (2.2)$$

The double integral can be rewritten as

$$\int_{T_0}^{t}\int_{T_0}^{t} \delta(s,\tau)\, f(s,r(s),r^{(1)}(s))\,ds\,d\tau \qquad (2.3)$$

where

$$\delta(s,\tau) = 1 \quad s<\tau \qquad (2.4)$$
$$= 0 \quad s>\tau$$

Interchanging the order of integration and carrying out the integration, r(t) can be rewritten as

$$r(t) = r^{(1)}(T_0) + r(T_0)(t-T_0) + \int_{T_0}^{t} (t-s)\, f(s,r(s),r^{(1)}(s))\,ds \qquad (2.5)$$

where $r_0 = r(T_0)$ and $r^{(1)}_0 = r^{(1)}(T_0)$

This equation is the integral equivalent of the differential equation. The integral term is evaluated as follows: The total time domain of interest $[T_0, T_f]$ is divided into a number of subregions $[T_{j-1}, T_j]$, j=1,2, ---, m+1 called "splines". The points T_1, T_2, ----T_{m+1} are called knots. Here, $T_{m+1}= T_f$. The integral can be written equivalently as a sum of integrals over each spline region, i.e.,

$$r(t) = r_0 + r^{(1)}(t-T_0) + \sum_{k=1}^{j-1}\int_{T_{k-1}}^{T_k} (t-s)\, f(s,r(s),r^{(1)}(s))\,ds + \int_{T_{j-1}}^{t} (t-s)\, f(s,r(s),r^{(1)}(s))\,ds \qquad (2.6)$$

where $t \varepsilon [T_{j-1}, T_j]$, j=1,2,---, m+1. By choosing sufficiently many splines we can model $f(s, r(s), r^{(1)}(s))$ as a constant $r^{(2)}_j$ over each spline region $[T_{j-1}, T_j]$, j=1,2 ---, m+1. With this approximation r(t) becomes

$$r(t) = r_0 + r^{(1)}(t-T_0) + \sum_{k=1}^{j-1}(T_k- T_{k-1})\,(t-\tfrac{1}{2}((T_k- T_{k-1})\, r^{(2)}_k + \tfrac{1}{2}(t- T_{j-1})^2\, r^{(2)}_j \qquad (2.7)$$

where $t \varepsilon [T_{j-1}, T_j]$, j=1,2,---, m+1. This is the quadratic spline function. In vector form the quadratic spline function can be written as

$$r(t) = a^T(t;\underline{T})\cdot \underline{p} \qquad (2.8)$$

where the knot vector $\underline{T}$ is given by

$$\underline{T} = (T_0, T_1, ----, T_m),$$
$$\underline{p} = (r^{(2)}_1, r^{(2)}_2, ---, r^{(2)}_m, r_0, r^{(1)}_1)$$
$$a^T(t;\underline{T}) = (a_1(t;\underline{T}), a_2(t;\underline{T}), ---a_{m+3}(t;\underline{T}))$$
$$a_{m+2}(t;\underline{T}) = 1$$
$$a_{m+3}(t;\underline{T}) = t - T_0 \qquad (2.9)$$

and where, for j=1,2, ---, m+1,

$$\begin{array}{ll} 0 & T_0 \le t < T_{j-1} \\ \tfrac{1}{2}(t- T_{j-1})^2 & T_{j-1} \le t < T_j \\ (T_j- T_{j-1})\,(t-\tfrac{1}{2}(T_j- T_{j-1})) & T_j \le t \end{array} \qquad (2.10)$$

In an application to ship classification the function r represents the radar cross-section per unit range resolution (i.e., density) along the length of the ship from the stern and at the end of the bow the constraints

$$r_0 = 0, \quad r(T_{m+1}) = 0 \qquad (2.11)$$

must necessarily be invoked. The coefficients r_j, j=1, 2, ---, m+1 provide a composite picture of the ship's superstructure.

Let d(t) be a measured time history of r(t):

$$d(t) = r(t) + \eta \qquad (2.12)$$

where η is a stationary Gaussian process with variance σ^2. A general time function optimization problem has the error criterion

$$E(p,T) = \int_{T_0}^{T_f} [d(t)-a^T(t;T)\cdot p)^2]\, dt \qquad (2.13)$$

and where the (m+1) dimensional vector T denote the set of parameters on which the vector $a^T(t;T)$ depend. In the case of quadratic splines the vector T represents the set of knots.

Since the argument p occurs linearly inside the square brackets the error criterion $E(p,T)$ can be minimized with respect to p as a function of T. Minimizing $E(p,T)$ with respect to q gives in matrix form

$$B(p,T)\, q\text{^}(T) = \gamma(T) \qquad (2.14)$$

where $q\text{^}(T)$ denotes the optimum q, the column vector $\gamma(T)$ has components

$$\gamma_i(T) = \int_{T_0}^{T_f} d(t)\cdot a_j(t;T)dt,\ j=1, 2, --, N \quad (2.15)$$

and the matrix $B(p,T)$ has elements

$$b_{ij}(T) = \int_{T_0}^{T_f} a_i(t;T)\cdot a_j(t;T)dt,\ \ i, j=1, 2, ---, N \qquad (2.16)$$

Using the definition of $\gamma(T)$ we can rewrite the matrix $E(p,T)$ as

$$E(q\text{^}(T),T) = \int_{T_0}^{T_f} d^2(t) - \gamma^T(T)\cdot q\text{^}(T) \qquad (2.17)$$

Since the first term is independent of T it will have no bearing on the generation of the optimum $T\text{^}$. Consequently, we redefine the error criterion E as

$$E(T) = \gamma^T(T)\cdot B^{-1}(T)\cdot \gamma(T) \qquad (2.18)$$

which is the last term of (2.17). Our objective is to maximize with respect to the knot vector T in the presence of the constraints

$$T_{j-1} \leq T_j,\ j=1, 2, ---, m+1 \qquad (2.19)$$

where $T_{m+1} = T_f$.

The problem is to maximize $E(T)$ of (2.18) with respect to the knot vector T. Several gradient methods are available for carrying out the optimization: steepest descent, Newton-Raphson, Newton, Gauss-Newton, Fetcher-Powell and Davidon. Newton's method is appealing because of its quadratic convergence properties. It can be implemented since the Hessian matrix is analytically computable. If $E(T)$ has a maximum at $T = T^b$ then T^b can be approximated from a previous estimate T^a by

$$T^b = T^a - [\, \delta^2 E(T^a)/\delta T^2]^{-1}\ \delta E(T^a)/\delta T \qquad (2.20)$$

3. RESULTS OF AN APPLICATION OF THE OPTIMAL SPLINE METHOD TO SHIP CLASSIFICATION

The simulated stern-bow profiles are generated by the Image Reconstruction From Projections(IRFP) Method which reconstructs stern-bow images using high range resolution radar data. The horizontal axis of each plot denotes the number of range cells (50 feet per range cell) measured from the stern to the bow along the projection. The vertical axis denotes the radar cross-section of the ship's portion contained in the range cell. The left end of the profile represents the stern and the right end the bow. The IRFP profile has been normalized to a total cross-section.

The results of the application of the Optimal Spline Method to the IRFP stern-bow profiles of Figure 1 and 2 are contained in Figures 3 and 4, respectively. In this application the radar cross-section is normalized to unity. The coefficient of the optimal spline function for each spline region is symbolized by the notation "PBAR". Optimally placed knots are denoted by the notation "OPTIMAL". For example, in Figure 3 the value of $r^{(2)}_1$, (i.e., the first component of the p vector Equation (2.8)) for the first spline region [0, 0.72] is equal to 0.19; the first knot is optimally located at 0.72. The second knot is optimally placed at 2.75. Since each range cell represents 50 feet along the ship's length and since the stern starts at zero it follows that the first knot falls at 0.72 x 50 feet = 36 feet from the stern, the second at 138 feet, etc.

The number of knots, an output of the Optimal Spline Method, is five for the ship #1 - Figure 3. Note that eight knots were calculated for the ship #2 - Figure 3. The knot location and the optimal spline coefficient values are tabulated in Tables 3-1.

Analyzing the two columns of Table 3-1 and the columns of Table 3-2 we see that it is easy to distinguish between the ship #1 and the ship #2 because (1) there are five knots for the ship #1 and seven for the ship #2, (2) their knot locations differ considerably and (3) their optimal spline coefficients differ greatly in magnitude for some of the spline regions.

Optimally locating the knots has two advantages. The first is the extraction of the independent features for classification purposes. the second is the reduction of the least-squares error. In table 3-3 is presented a comparison of the least-squares error between optimally located knots and equally spaced knots. The least-squares error of the ship #1 for optimally located knots is .01 and the error of the same ship for equally spaced knots is 0.87. The ratio of the error associated with equally spaced knots to the error associated with optimally located knots is 87. In all cases this ratio is greater than 10.

Table 3-1. Comparison of Optimal Spline Knot Locations

Knots Index	Ship #1	Ship #2
#1	0.72	0.81
#2	2.75	3.79
#3	4.03	5.03
#4	4.98	5.97
#5	6.13	7.14
#6	*	9.66
#7	*	11.79
#8	*	*
#9	*	*

4. CONCLUSIONS

An automated technique called the Optimal Spline Method has been developed for application to problems having high nonlinearities. The Optimal Spline Method in ship classification applications determines the number of separate superstructure masses, their separation from each other, their locations and their extended widths. These independent features correspond directly to optimal spline coefficients of the method. The results contained in Section 3 show that ships of different type exhibit large differences in their optimal spline coefficients.

References:

(1) Larry L. Schumaker, Spline Functions: Basic theory, John Wiley & Sons, New York. 1981

(2) Richard H. Bartels, John C. Beatty and Brian A. Barsky, An Introduction to Splines for use in Computer Graphics and Geometric Modeling: Morgan Kaufmann Publishers, Los Altos, CA 1987

Table 3-2 Optimal Coefficient Values

Spline Region Index	Ship #1	Ship #2
#1	-0.19	-1.29
#2	0.25	0.13
#3	-0.81	-0.33
#4	1.95	1.83
#5	-1.73	-1.99
#6	0.32	0.47
#7	*	-0.19
#8	*	0.07
#9	*	*

Table 3-3 Comparison of Least-Squares Between Optimally Located Knots and Equally Spaced Knots

	Number of Knots	Least-Squares Error	
		Optimally Located Knots	Equally Spaced Knots
Ship #1	5	0.01	0.87
Ship #2	7	0.06	1.45

Fig 1. Simulated IRFP Data
Ship #1 Stern-Bow Profile [Sq. M.]

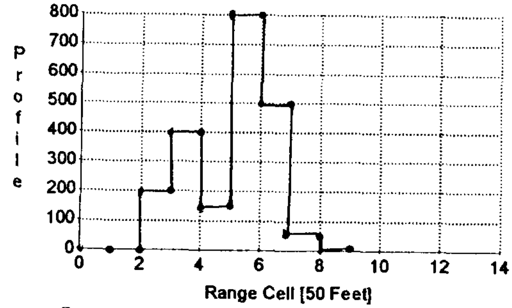
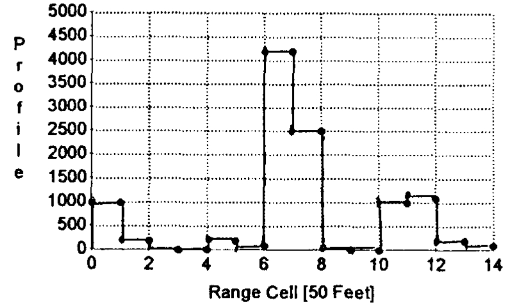


Fig. 2 Simuated IRFP Data
Ship #2 Stern-Bow Profile [Sq. M.]



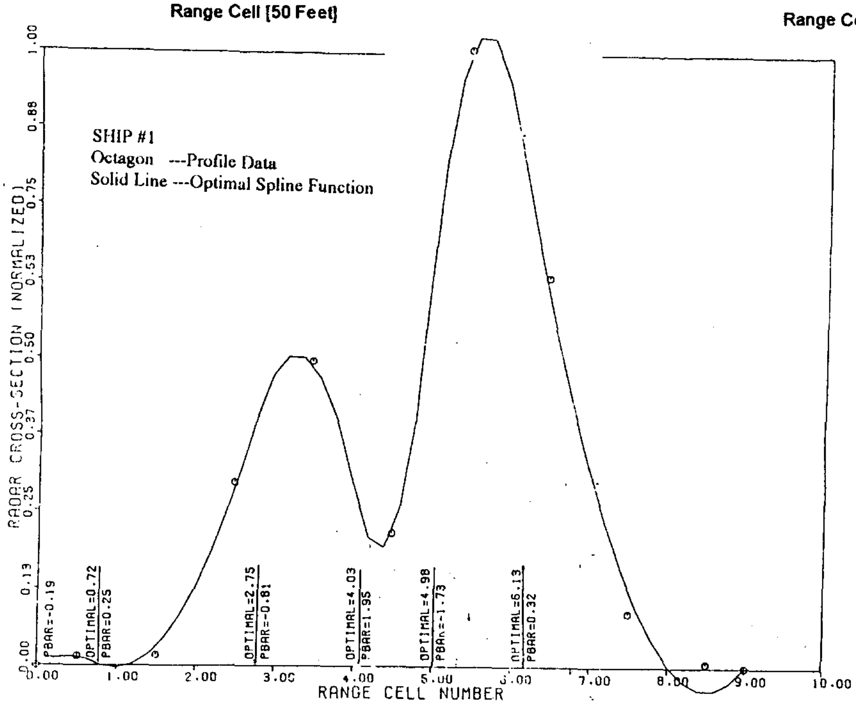


Figure 3. Optimal Spline Method Results of Ship #1

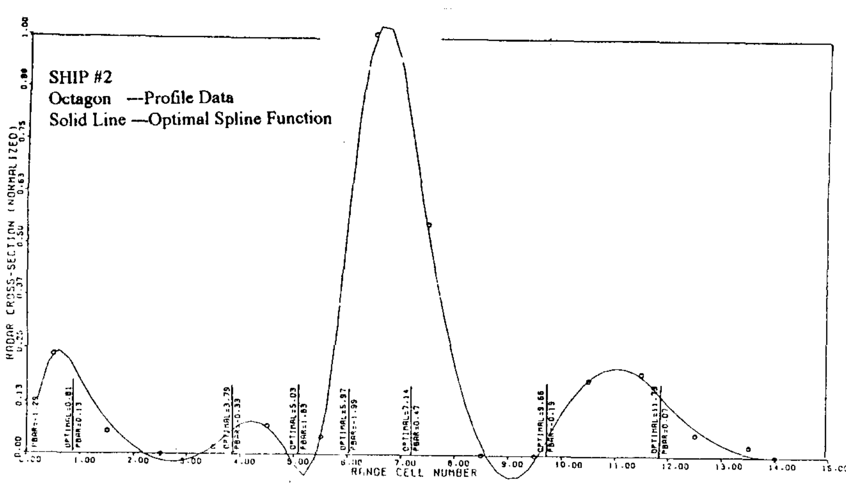


Figure 4. Optimal Spline Method Results of Ship #2