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파라메타 불확실성을 갖는 선형시스템에 대한 강한 신뢰 H_∞ 제어

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Robust and Reliable H_∞ Control for Linear Systems with Parameter Uncertainty

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Abstract

In this paper, a robust and reliable H_∞ control problem is considered for linear uncertain systems with time-varying norm-bounded uncertainty in the state matrix, which performs well despite of actuator outages. Using linear static state feedback and the quadratic stabilization with H_∞-norm bound, a robust and reliable H_∞ controller is obtained that stabilizes the plant and guarantees an H_∞-norm bound constraint on disturbance attenuation for all admissible uncertainties and normal state as well as faulty state of actuators. It provides a sufficient condition for robust and reliable stabilization with H_∞-norm bound. Reliability is guaranteed provided actuator outages only occur within a prespecified subset of actuators.

1. Introduction

Robust state feedback control for linear systems with parameter uncertainty has been increasingly studied in recent years[2,4,5,10-13]. The resultant control systems are robust and provide guaranteed stability and satisfactory performance for all admissible uncertainties. However, they may result unsatisfactory performance and even unexpected instability in case of actuator or sensor outages. Since actuator or sensor outages can be occasionally found in real world, they should be taken into account in practice when a control system is designed. This paper develops robust and reliable control methodologies via state feedback for uncertain systems with time-varying norm-bounded uncertainty in the state matrix, which guarantees satisfactory closed-loop behavior despite actuator outages.

Various robust control theories have been developed. Especially, interests have focused on the problem of robust H_∞ control for linear systems with parameter uncertainties. Most of results have utilized the Riccati equation approach in view of the quadratic stabilization of uncertain systems[2,4,5,13]. The objective is to design a controller stabilizing an uncertain system while satisfying an H_∞-norm bound constraint on disturbance attenuation for all admissible uncertainties. For time-invariant linear uncertain systems, robust H_∞ controllers are constructed for uncertainties in the input matrix[10] or for uncertainties in the state matrix[11]. For linear uncertain systems with time-varying norm-bounded parameter uncertainty in both the state and input matrices, necessary and sufficient conditions for quadratic stabilization with an H_∞-norm bound have been derived in [12].

Despite of often finding outages of control component in practice, e.g. sensor outage, actuator outage, etc., only a few reliable control methodologies have been developed with various reliability goals. Reliable control laws using multiple controllers for a single plant have been represented in [1,3,6,7,9]. These approaches generally guarantee normal operation under failures in some of controllers. Very recently, the methodology for the design of reliable centralized and decentralized control systems using observer-based output feedback was introduced[8]. The resultant control system is reliable in that they provide guaranteed stability and satisfy H_∞-norm bound constraint on disturbance attenuation not only when all control components are operational, but also in case of sensor or actuator outages in the centralized case, or in case of control channel outages in the decentralized case within a prespecified subset of control components. However, this controller is restricted to known linear time-invariant systems.

In this paper, using the special result of robust H_∞ control in [12], which is robust H_∞ control for the uncertain linear system with parameter uncertainties in the state matrix only, along with the philosophy of constructing reliable controller in [8], robust and reliable H_∞ control problem for the uncertain systems with parameter uncertainty in the state matrix is solved not only in case of normal operation, but also in case of actuator outages within a prespecified subset of actuators. The result of this paper is distinguished from that in [12], because actuator outage type assumed in here cannot be treated as the uncertainties of input matrix as in [12], and can be regarded as an extension for uncertain systems of centralized reliable control in [8] using state feedback different from observer-based output feedback.

2. System Specification and Definition

Consider a class of uncertain linear systems described by state-space models of the form Then, it follows that

$$\dot{x}(t) = [A + \Delta A(t)]\,x(t) + B_1 w(t) + B_2 u(t) \quad (1a)$$

$$z(t) = C_1 x(t) + D_1 u(t) \quad (1b)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $w(t) \in R^q$ is the disturbance input, $z(t) \in R^p$ is the controlled output, A, B_1, B_2, C_1, and D_1 are real constant matrices of appropriate dimensions describing the nominal system, and $\Delta A(\cdot)$ is real-valued matrix function representing time-varying parameter uncertainties. The parameter uncertainties considered here are norm-bounded as

$$\Delta A(t) = DF(t)E_1 \quad (2)$$

where $D \in R^{n\times i}$ and $E_1 \in R^{j\times n}$ are known constant matrices and $F(t) \in R^{i\times j}$ is an unknown matrix function satisfying $F^T(t)F(t) \leq I$ with the elements of $F(\cdot)$ being Lebesgue measurable. Without loss of generality, we shall make the following assumption for technical simplification[12].

Assumption 1. $D_1^T[C_1, D_1] = [0, I]$

Let us consider the following uncertain system with parameter uncertainty $\Delta A_c(t)$ in the state matrix:

$$\dot{x}(t) = [A_c + \Delta A_c(t)]\,x(t) + B_c w(t) \quad (3a)$$

$$z(t) = C_c x(t) \quad (3b)$$

In this paper, design method for a linear state feedback controller will be based on the quadratic stability with disturbance attenuation of the closed-loop system[2,4,5,13]. In order to use the technique for quadratic stability, we will first recall the following definition[12].

Definition 1. Given a scalar $\gamma > 0$, the system (3) is said to be ***quadratically stable with disturbance attenuation*** γ if there exists a symmetric positive definite matrix P such that for all admissible uncertainty $\Delta A_c(\cdot)$

$$[A_c + \Delta A_c(t)]^T P + P[A_c + \Delta A_c(t)] + \gamma^{-2} P B_c B_c^T P + C_c^T C_c < 0. \quad (4)$$

Similarly, given a scalar $\gamma > 0$, the uncertain system (1) is said to be ***quadratically stabilizable with disturbance attenuation*** γ ***via linear state feedback*** if there exists a linear state feedback control $u(t) = Kx(t)$ such that the closed-loop system is quadratically stable with disturbance attenuation γ.

Remark 1. Note that the quadratic stability with disturbance attenuation γ implies uniformly asymptotic stability for all admissible $\Delta A_c(\cdot)$ and that with zero-initial condition for $x(t)$, $\|z\|_2 < \gamma\|w\|_2$ for all admissible $\Delta A_c(\cdot)$ and all nonzero $w \in L_2[0,\infty)$, where $\|\cdot\|_2$ denotes the usual $L_2[0,\infty)$-norm.

We conclude this section by clarifying the terminology used in following sections. The terms of fault, outage, and reliability used without describing words have the meanings of actuator fault, actuator outage, and reliability for actuator faults, respectively.

3. Main Results

In this section, Robust and reliable controller design problem is solved to find a linear static state feedback control law $u(t) = Kx(t)$ for the system (1) with parameter uncertainty in the state matrix such that the closed-loop system is quadratically stable with disturbance attenuation γ for all admissible parameter uncertainties despite actuator outages within a prespecified subset of actuators. It is assumed that perfect information of the states of the plant is available for feedback.

In case of an actuator outage, we can consider that the control input value for a faulty actuator is zero, i.e. the faulty actuator is excluded in control loop. We can find this phenomenon in the digital equipments in practice even if this doesn't completely describe generally faulty situations. Let $\Omega \subseteq \{1, 2, \cdots, m\}$ correspond to a selected subset of actuators susceptible to outages. Introduce a decomposition

$$B_2 = B_{2\Omega} + B_{2\bar{\Omega}} \tag{5}$$

where $B_{2\Omega}$ and $B_{2\Omega}$ are formed from B_2 by zeroing out columns corresponding to susceptible actuator set Ω and to actuator set except Ω, respectively. Let $\omega \subseteq \Omega$ correspond to a particular subset of susceptible actuators that actually experience outages. Again we can decompose B_2 for ω analogous to (5) as

$$B_2 = B_{2\omega} + B_{2\bar{\omega}}. \tag{6}$$

Since $\omega \subseteq \Omega$, it follows that $B_{2\omega}B_{2\omega}^T \leq B_{2\Omega}B_{2\Omega}^T$.

Theorem 1. For actuator outages corresponding to any $\omega \subseteq \Omega$, the uncertain system (1) is quadratically stabilizable with an H_∞-norm bound $\gamma > 0$ if for a sufficiently small $\delta > 0$, there exists a constant $\epsilon > 0$ such that a symmetric positive definite matrix $Q > 0$ satisfies the following Riccati equation:

$$\begin{aligned} A^TQ + QA &+ \gamma^{-2}QB_1B_1^TQ + \epsilon QDD^TQ - QB_{2\Omega}B_{2\Omega}^TQ \\ &+ \frac{1}{\epsilon}E_1^TE_1 + C_1^TC_1 + \delta I = 0. \end{aligned} \tag{7}$$

Moreover, a suitable feedback control law is given by

$$u(t) = Kx(t); \qquad K = -B_2^TQ. \tag{8}$$

Proof. The proof is accomplished to show that Theorem 1 is true both when all actuators are operational, i.e. $\omega = \emptyset$, and in case of actuator outages, i.e. $\omega \neq \emptyset$.

Suppose that there exists a constant $\epsilon > 0$ such that the Riccati equation (7) has a solution $Q > 0$.

First, consider that all actuators are operational. The closed-loop system of (1) with the control law (8) is given by the state-space equations

$$\dot{x}(t) = A_c(t)x(t) + B_1w(t) \tag{9a}$$

$$z(t) = C_cx(t) \tag{9b}$$

where

$$\begin{aligned} A_c(t) &= A + DF(t)E_1 - B_2B_2^TQ \\ C_c &= C_1 - D_1B_2^TQ. \end{aligned}$$

Then, it follows that

$$\begin{aligned} A_c^T(t)Q &+ QA_c(t) = A^TQ + QA - 2QB_2B_2^TQ \\ &+ E_1^TF^T(t)D^TQ + QDF(t)E_1. \end{aligned} \tag{10}$$

From the facts that

$$\left[\sqrt{\epsilon}QD - \frac{1}{\sqrt{\epsilon}}E_1^TF^T(t)\right]\left[\sqrt{\epsilon}D^TQ - \frac{1}{\sqrt{\epsilon}}F(t)E_1\right] \geq 0$$

and $F^T(t)F(t) \leq I$, we have

$$E_1^TF^T(t)D^TQ + QDF(t)E_1 \leq \epsilon QDD^TQ + \frac{1}{\epsilon}E_1^TE_1. \tag{11}$$

By considering Assumption 1 and $C_c = C_1 + D_1K$, we obtain

$$C_c^T C_c = C_1^T C_1 + K^T K. \tag{12}$$

Now, considering (9) and substituting (8) into (12), we obtain

$$\begin{aligned} A_c^T(t)Q + QA_c(t) + \gamma^{-2}QB_1B_1^TQ + C_c^TC_c \\ \le A^TQ + QA + \gamma^{-2}QB_1B_1^TQ + \epsilon QDD^TQ \\ -QB_2B_2^TQ + \frac{1}{\epsilon}E_1^TE_1 + C_1^TC_1. \end{aligned} \tag{13}$$

Using (7), it follows that

$$\begin{aligned} A_c^T(t)Q + QA_c(t) + \gamma^{-2}QB_1B_1^TQ + C_c^TC_c \\ \le -\delta I - QB_{2\Omega}B_{2\Omega}^TQ < 0. \end{aligned}$$

Hence, with all actuators operational, Theorem 1 is true from Definition 1.

Next, consider for actuator outages corresponding to $\omega \subseteq \Omega$ with $\omega \neq \emptyset$. Due to actuator outages, B_2 and K respectively correspond to $B_{2\bar{\omega}}$ and $K_{\bar{\omega}}$ in the closed-loop system. So, the closed-loop system is given by the state-space equations

$$\dot{x}(t) = A_{c\bar{\omega}}(t)x(t) + B_1w(t) \tag{14a}$$

$$z(t) = C_{c\bar{\omega}}x(t) \tag{14b}$$

where

$$\begin{aligned} A_{c\bar{\omega}}(t) &= A + DF(t)E_1 - B_{2\bar{\omega}}B_{2\bar{\omega}}^TQ \\ C_{c\bar{\omega}} &= C_1 - D_1B_{2\bar{\omega}}^TQ. \end{aligned}$$

Then, it follows that (10) and (12) become, respectively,

$$\begin{aligned} A_c^T(t)Q + QA_c(t) = A^TQ + QA - 2QB_{2\bar{\omega}}B_{2\bar{\omega}}^TQ \\ +E_1^TF^T(t)D^TQ + QDF(t)E_1 \end{aligned} \tag{15}$$

and

$$C_c^TC_c = C_1^TC_1 + K_{\bar{\omega}}^TK_{\bar{\omega}} \tag{16}$$

where $K_{\bar{\omega}} = -B_{2\bar{\omega}}^TQ$.

Using (15), (11), and (16), we obtain

$$\begin{aligned} A_c^T(t)Q + QA_c(t) + \gamma^{-2}QB_1B_1^TQ + C_c^TC_c \\ \le A^TQ + QA + \gamma^{-2}QB_1B_1^TQ + \epsilon QDD^TQ \\ -QB_{2\bar{\omega}}B_{2\bar{\omega}}^TQ + \frac{1}{\epsilon}E_1^TE_1 + C_1^TC_1. \end{aligned} \tag{17}$$

Finally, using (7) and the fact that $B_{2\bar{\Omega}}B_{2\bar{\Omega}}^T \le B_{2\bar{\omega}}B_{2\bar{\omega}}^T$ since $\bar{\omega} \supseteq \bar{\Omega}$,

$$\begin{aligned} A_c^T(t)Q + QA_c(t) + \gamma^{-2}QB_1B_1^TQ + C_c^TC_c \\ \le -\delta I - QB_{2\bar{\omega}-\bar{\Omega}}B_{2\bar{\omega}-\bar{\Omega}}^TQ < 0 \end{aligned}$$

where $B_{2\bar{\omega}-\bar{\Omega}}$ is formed from B_2 by zeroing out columns corresponding $\bar{\omega} - \bar{\Omega}$.

Hence, in case of actuator outages, Theorem 1 is also true from Definition 1. ▽▽▽

Theorem 1 provides a sufficient condition for quadratic stabilization with an H_∞-norm bound γ for uncertain system (1) despite actuator outages. It includes the meaning that if we allow all actuator outages, the system (1) must be open-loop quadratically stable with disturbance attenuation γ. If a subset Ω of actuators susceptible to outages is empty set, It becomes a special result in case of only considering parameter uncertainties in the state matrix in [12].

In the wide sense, actuator outages can be considered to special type of parameter uncertainties in the input matrix because actuator outages assumed here is represented to replacing column vectors of input matrix corresponding the outages with zero vectors. But we can show that this uncertainty is not described to parameter uncertainties in the input matrix in [12]. Therefore we know that Theorem 1 is a new result that can not solved to the results in [12].

4. Example

Consider the following linear system with time-varying uncertainty

$$\begin{aligned} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -4 & 2+cos(2t) \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} w(t) \\ + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} u(t) \end{aligned}$$

$$z(t) = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} u(t) \qquad (18)$$

where the parameter uncertainty $\Delta A(t)$ can be represented as follows:

$$\Delta A(t) = DF(t)E_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} cos(2t) \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The nominal open-loop system is unstable, since all its poles are right half plane. If we let $\gamma = 10$, $\epsilon = 0.2$ and $\delta = 0.5$, and assume $\Omega = \{2\}$, i.e. the actuator corresponding to u_2 is susceptible to outage, then we obtain the positive definite solution of (7) as follows:

$$Q = \begin{bmatrix} 33.0282 & 1.2827 \\ 1.2827 & 6.7372 \end{bmatrix}. \qquad (19)$$

With (19), we compose the controller under the control law in (8). Fig. 1 shows the transient of the states, the controlled outputs, and the control inputs of the system (18) with initial state $x(0) = [3, -2]^T$, when actuator outage occurs at 5 second and the disturbance of $w = 2$ is inputed from 3 to 6 second. It shows that the control system is robust and reliable for the parameter uncertainty and actuator outage.

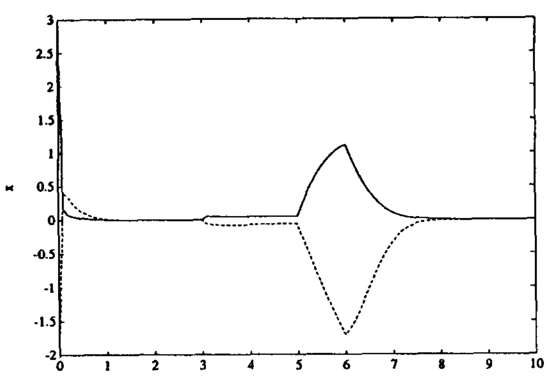

Fig. 1a States for linear system in Example (solid-line: x_1, dashed-line: x_2)

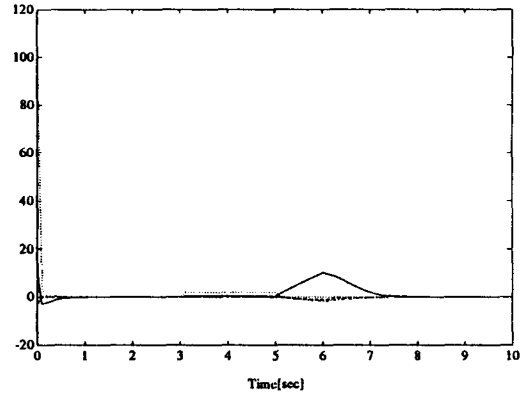

Fig. 1b Controlled outputs for linear system in Example (solid-line: z_1, dashed-line: z_2, dotted-line: z_3, dotdash-line: z_4)

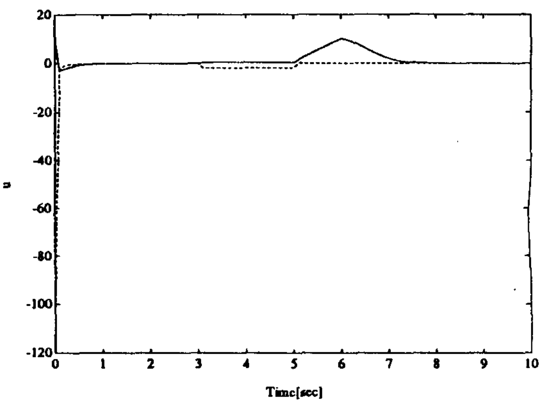

Fig. 1c Control inputs for linear system in Example (solid-line: u_1, dashed-line: u_2)

5. Conclusions

For linear systems with time-varying parameter uncertainties in the state matrix, we has developed a state feedback H_∞ control technique despite of actuator outages within a prespecified subset of actuators. The scheme utilizes linear static state feedback based on the quadratic stability with disturbance attenuation. The result has solved the problem of robust H_∞ control, quadratic stability, and reliable control at one time.

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