'93 한국자동제어학술회의논문집 (1993. 10. 20~22)

Feedback Control Synthesis for a Class of Controlled Petri Nets with Time Constraints

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Abstract

This paper presents an efficient solution for a class of forbidden state problems by introducing a cyclic timed controlled marked graphs (TCMG's), a special class of timed controlled Petri nets (TCPN's) as a model of a class of discrete event systems (DES's). The state feedback control is synthesized, which is maximally permissive while guaranteeing the forbidden states will be avoided. The practical applications of the theoretical results for an automated guided vehicle (AGV) coordination problem in a flexible manufacturing facility is illustrated.

I. INTRODUCTION

Recently the synthesis of controllers for achieving desired closed-loop behavior has been one of the considerable interesting researches on discrete event systems (DES's) [1]-[4]. Among the specifications of the desired behavior for the controlled DES's, it is the most essential to guarantee that the forbidden conditions (states) will be avoided in the system, which is referred to as the forbidden state problem (specification).

The forbidden state problem for a DES was introduced by Ramadge and Wonham who proposed a state feedback solution in [2]. Krogh considered the forbidden state problem in the context of controlled Petri nets (CPN's) [5]. Concurrent transitions in CPN's model a more general class of DES's for which the uniqueness of the maximally permissive control found by Ramadge and Wonham no longer holds [3]. CMG model which was introduced by Krogh and Holloway permits us to synthesize an efficient feedback control policy from the Petri net model of the uncontrolled system dynamics [3],[4] while other Petri net analysis methods could have been applied to verify properties of controlled systems [6],[7]. This, however, modeled only a logical point of view of the controlled system, and gave no attention to any time constraints. The control policy in [3],[4] was somewhat restrcitive in most forbidden state problems. Moreover on some cases, it might be possible to avoid forbidden states by not allowing the controlled system to operate at all, but this would seldom be an acceptable control policy.

Lately, there have been some papers to extend the results on feedback control policies for untimed (logical) Petri net models to control problems in TPN's [8],[9]. These papers proposed an attractive framework for control problems in which real time specifications are incorporated into the models and control objectives, and introduced an efficient feedback control policy. In these models, however, there are two types of transitions; uncontrolled transitions which have bounds of the firing interval are one and controlled transitions which are forced to fire by the controller are the other, which makes those unrealistic to some extent, and have somewhat restrictive initial states which should meet not only the marking state but also the clock state.

In this paper, we propose a cyclic timed controlled marked graphs (TCMG's) as a model of a class of DES's, which are a special case of the TCPN's that are an extension of CPN's in which time delays attached to the places, and formulate and solve the forbidden state problem for the DES's which can be modeled as TCMG's. Time delays attached to the places make the firing rule of transitions simple, and the feedback control policy concerning all the clock states in that marking makes the initial state less restrictive. Due to the distributed representation of the system state in terms of the net marking and the time delays attached to the places, TCMG's provide a compact, intuitive modeling framework for describing both the state transition dynamics of the DES with time constraints and the forbidden state specifications.

The following section defines TCMG's and introduces notation and results which are used in the remainder of the paper. The specification of forbidden state conditions using the distributed state representation and the set of time delays attached to the places of TCMG's is discussed in Section III.

In Section IV, the adaptive controls, which guarantee that the next state transition will not lead uncontrollably to a forbidden marking, are considered and the maximally permissive state feedback problem is solved. The application of our results to an automated guided vehicle (AGV) coordination problem in a flexible manufacturing system is illustrated in section V.

II. Timed Controlled Marked Graphs

Timed controlled Petri nets (TCPN's) are an extension of controlled Petri nets (CPN's) [3]-[5] in which time constraints can be attached to the state places, and are defined as a 6-tuple $\mathcal{G} = \{\mathcal{P}, \mathcal{T}, \mathcal{E}, \mathcal{C}, \mathcal{B}, d\}$. Here $\mathcal{P}$ is the finite set of *state places*, $\mathcal{T}$ is the finite set of *transitions*, $\mathcal{E} = (\mathcal{P} \times \mathcal{T}) \cup (\mathcal{T} \times \mathcal{P})$ is a set of directed arcs associating state places and transitions, $\mathcal{C}$ is the finite set of *control places*, $\mathcal{B} = (\mathcal{C} \times \mathcal{T})$ is a set of directed arcs associating control places with transitions, and $d : \mathcal{P} \rightarrow \{[a, b] \subseteq \mathcal{R}^+ \cup \{0\}\}$ assigns a closed *availing time interval* (defined below) to each state place $p \in \mathcal{P}$ where $\mathcal{R}^+$ is the set of positive real numbers. The lower and upper bounds of the availing time interval $d(p)$ for state place $p \in \mathcal{P}$, will be denoted by $d_{\min}(p)$ and $d_{\max}(p)$, that is, $d(p) = [d_{\min}(p), d_{\max}(p)]$. For all $p \in \mathcal{P}$, $0 \leq d_{\min}(p) \leq d_{\max}(p) < \infty$. A TCPN with $d_{\min}(p) = 0$ and $d_{\max}(p) = \infty$ for all $p \in \mathcal{P}$ reduces to a CPN. We assume that there is at most one arc in $\mathcal{B}$ and $\mathcal{E}$ between two nodes. Readers are considered to be familiar to CPN notations, and notations to be used in this paper will be shortly explained (Refer to [3],[4]).

The state of a TCPN is given by its current *marking* $m : \mathcal{P} \rightarrow \mathcal{N}$, where $\mathcal{N}$ is the set of nonnegative integers. The marking indicates the current distribution of *tokens* in the state places. In a TCPN, when a token is put in a state place $p \in \mathcal{P}$ it remains *unavailable* for an amount of time $d(p)$, which we call *availing time delay* for the state place p, such that $d_{\min}(p) \leq d(p) \leq d_{\max}(p)$, after which it becomes and keeps *available* as long as the token exists in the place. This indicates that state places represent conditions which are true for an amount of time, and that for a state place p, the amount of time which it takes for the condition to be true is between $d_{\min}(p)$ and $d_{\max}(p)$ when the system normally operates. In this paper, the given system is assumed to be in normal operation. A transition $t \in \mathcal{T}$ is said to be *state-enabled* under a marking m if for all $p \in {}^{(p)}t$, $m(p) \geq 1$ and the token in the place p is available. A *control* $u : \mathcal{C} \rightarrow \{0, 1\}$ assigns a binary token count to each control place. A transition $t \in \mathcal{T}$ is said to be *control-enabled* under a control u if $u(c) = 1$ for all $c \in {}^{(c)}t$, with the convention that any transition $t \notin \mathcal{T}_c$ is always control-enabled. A set of transitions $T \subseteq \mathcal{T}$ in a TCPN $\mathcal{G}$ is just said to be *enabled* for a given marking m and control u if all transitions $t \in T$ are both state-enabled and control-enabled. The set of enabled transitions T *fires*, and state transitions occur (i.e., the TCPN marking changes). The time when a transition $t \in \mathcal{T}$ fires, which is called the *firing time* of the transition t and denoted by $\tau(t)$, is determined by the latter between the time when the transition t is state-enabled and the time when the transition t is control-enabled. Note that the control u does not influence a trnasition to be state-enabled. When the enabled transition set T fires under the given marking m and control u, the state of the TCPN changes from marking m to a new marking m' defined by the *state transition equation* $m'(p) = m(p) - |p^{(t)} \cap T| + |{}^{(t)}p \cap T|$, where $|\cdot|$ indicates the cardinality of the set argument. The control u can be updated or not in response to state changes in the system. In this section, we regard the control u as one fixed in any number of state transitions.

$\mathcal{R}_{INF}(u, m)$ and $\mathcal{R}_{ONE}(u, m)$ denote the set of markings *reachable* under valid transition firing sequences of any length and the *immediately reachable* marking set under the firing of a single transition set, respectively, under the control u in a TCMG.

We now restrict our attention to a cyclic TCMG model that for any place $p \in \mathcal{P}$, $|p^{(t)}| = 1$ and $|{}^{(t)}p| = 1$, and that every place $p \in \mathcal{P}$ is contained in a cycle [3],[6]. Figure 1 illustrates a cyclic TCMG where circles represent state places, squares represent control places, bars represent transitions, and $p[a, b]$ represent time constraints attached to state places such that a and b denote the minimum and the maximum availing time delay, respectively. We note that in TCMG's, there is no conflict among transitions since each place is the input to only one transition. Thus, any set of transitions which are both state- and control-enabled is enabled. Moreover, it can be shown that the cyclic structure of TCMG's guarantees the net is live under control u_{one}, provided the initial marking is chosen from the initial marking set $\mathcal{M}$ such that every cycle in the given TCMG contains at least one marked place, and that every place $p \in \mathcal{P}$ is contained within some cycle which has exactly one marked place [3],[4]. Figure 1 shows an example of the initial marking for the given TCMG.

From now on, we restrict our attention to TCMG's with initial markings in $\mathcal{M}$, which have properties such that if $m_0 \in \mathcal{M}$, then m_0 is binary, the net is live under the control u_{one}, and $\mathcal{R}_{INF}(u_{one}, m_0) \subseteq \mathcal{M}$ [3],[6].

We use the notation $\mathcal{P}^*$ to indicate the set of all finite sequences of elements of $\mathcal{P}$, including the empty string ϵ. In a TCMG, a sequence of places $\pi = p_0 \cdots p_N \in \mathcal{P}^*$ such that $p_{i-1}^{(t)} = {}^{(t)}p_i$ for $0 < i \leq N$ is said to be a *path*. For a path π, we let $\pi(j, N) = p_{N-j+1} \cdots p_N, 0 < j \leq N$ and $\pi(0, N) = \epsilon$ denote the sequence of places consisting of the last j places of the path, which is said to be a *suffix* of the path. The set

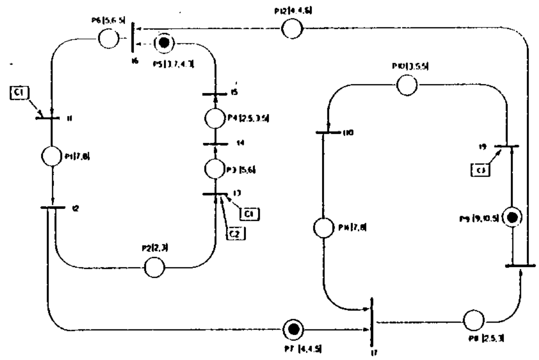

Figure 1: A cyclic timed controlled marked graph

of all suffixes of a path π is denoted by $sf(\pi)$.

In a TCMG, the firing time of the transition t, denoted by $\tau(t)$, is recursively represented in terms of the availing time delays, $d(p)$, for the inputs p to the transition t and the firing times of the transitions t' for which the place p is output, that is, $\tau(t) = \max_{p\in^{(s)}t}(d(p) + \tau(t'))$ if $u(c) = 1$ for all $c \in {}^{(c)}t$, and $\tau(t) = \infty$ if $u(c) = 0$ for some $c \in {}^{(c)}t$, since the transition t can not be control-enabled. The forbidden state problem in a TCMG is related to the time when a token is put in and released from the individual state places in the forbidden marking set. The former is the firing time of the transition for which the state place is output and the latter is the firing time of the transition for which the state place is input. These firing times are computed along the particular paths, called *precedence paths*, which end to the interesting state places in a TCMG. We define a precedence path for a place $p \in \mathcal{P}$ as follows.

Definition 1 : For a place $p \in \mathcal{P}$ in a TCMG, a *precedence path* is defined as a path $\pi_p = p_0 p_1 \cdots p_n$ such that

(1) $p_n = p$,
(2) $p_i \neq p_j$, $0 \leq i \neq j \leq n$,
(3) ${}^{(t)}p_i \neq p_j^{(t)}$, $0 < i \leq j \leq n$, and
(4) ${}^{(t)}p_0 = p_i^{(t)}$, $0 \leq i \leq n$.

In Figure 1, a path $p_6p_1p_2p_3p_4p_5$ is the precedence path for p_5, and the precedence paths for p_8 are paths $p_9p_{10}p_{11}p_8$, $p_{12}p_6p_1p_7p_8$, and $p_2p_3p_4p_5p_6p_1p_7p_8$. For a place $p \in \mathcal{P}$, we denote the set of all precedence paths, the set of all controlled transitions for which a place in a $\pi_p \in \Pi_p$ is an output, and the set of all suffixes of all precedence paths by Π_p, T_{Π_p}, and $sf(\Pi_p)$, respectively. Note that every precedence path for a place $p \in \mathcal{P}$ includes a cycle and contains at least one marked place for all initial markings in $\mathcal{M}$, and that given an intial marking $m \in \mathcal{M}$ and control u_{one}, the place p will be continually marked at irregular intervals which have bounds as stated in the following lemma. The interval time is called the *remarking time* for the place p and denoted by $\tau_R(p)$. To get the remarking time, we define particular places to be input to the transition for which the place p is input as follows.

Definition 2 : For a place $p \in \mathcal{P}$ in a TCMG, an ***associate place*** $\hat{p}_p$ is defined as a state place such that $p^{(t)} = \hat{p}_p^{(t)}$.

For a place p, we let $\hat{P}_p$ denote the place set which consists of the place p and all associate places for the place p, and $T_{\Pi_{\hat{P}_p}}$ denote the set of all controlled transitions for which a place in some π_{p_r}, for some $p_r \in \hat{P}_p$ is an output. The set of all cycles such that exist within a precedence path for a place p and that include the place p is denoted by C_{Π_p}, and the element cycle of the set is denoted by C_{π_p}.

Lemma 1 : Given a place p in a TCMG and an initial marking $m_0 \in \mathcal{M}$. If $u(c) = 1$ for all $c \in {}^{(c)}t$ for each $t \in T_{\Pi_{\hat{P}_p}}$ then

$$\max_{C_{\pi_p}\in C_{\Pi_p}} \Big(\sum_{p_i \in C_{\pi_p}} d_{\min}(p_i) \Big) \leq \tau_R(p) \leq \sum_{p_m \in P_{\Pi_{\hat{P}_p}}} d_{\max}(p_m),$$

where $P_{\Pi_{\hat{P}_p}} = \{p_m \in \mathcal{P} | p_m \in \pi_{\hat{p}_p} \text{ and } \pi_{\hat{p}_p} \in \Pi_{\hat{P}_p}\}$.

We will let $\tau_{R_{\min}}(p) = \max_{C_{\pi_{p_r}} \in C_{\Pi_{\hat{P}_p}}} (\sum_{p_i \in C_{\pi_{p_r}}} d_{\min}(p_i))$, and $\tau_{R_{\max}}(p) = \sum_{p_m \in P_{\Pi_{\hat{P}_p}}} d_{\max}(p_m)$.

Our solution for the forbidden state problem is based on the structure of the TCMG, particularly among the suffixes of the precedence paths for the individual state places in the forbidden state, the paths which influence the state place markings. These are the paths beginning with state places which are marked or outputs of controlled transitions. We call these paths *marking paths* or *control paths*, which are defined as follows.

Definition 3 : For a place $p \in \mathcal{P}$ in a TCMG, a *control path* $\pi_{p\_c} \in sf(\Pi_p)$ is defined as a suffix of some precedence paths $\pi_p \in \Pi_p$ such that

(1) $p_n = p$,
(2) ${}^{(t)}p_i \notin T_c, 0 < i \leq n$(when $n > 0$), and
(3) ${}^{(t)}p_0 \in T_c$.

Definition 4 : For a place $p \in \mathcal{P}$ and the given marking m in a TCMG, a *marking path* $\pi_p(m) \in sf(\Pi_p)$ is defined as a suffix of some precedence paths $\pi_p \in \Pi_p$ such that

(1) $p_n = p$,
(2) $m(p_i) = 0, 0 < i \leq n$(when $n > 0$), and
(3) $m(p_0) \geq 1$.

In Figure 1, the paths p_7p_8 and $p_9p_{10}p_{11}p_8$ are the marking paths for p_8, and the control paths for p_8 are $p_{10}p_{11}p_8$ and $p_1p_7p_8$. The set of all control paths for a place $p \in \mathcal{P}$ is denoted by Π_{p_c}, and the set of all marking paths for a place $p \in \mathcal{P}$ and the given marking m is denoted by $\Pi_p(m)$. Given a place $p \in \mathcal{P}$ and a marking m in a TCMG, for a marking (respectively, control) path, the transition for which the first place in $\pi_p(m)(respectively, \pi_{p\_c})$ is an output is denoted by

$t_{\pi_p(m)}$(*respectively*, $t_{\pi_{p\_c}}$), and the set of all controlled transitions for which a place in some $\pi_p(m) \in \Pi_p(m)$, not the first place, is an output is denoted by $T_{\Pi_p(m)}$, and the set of all suffixes of all marking paths excluding marking path itself and ϵ is denoted by $sf^+(\Pi_p(m))$.

For a place $p \in \mathcal{P}$ in a TCMG, the firing of the transition ${}^{(t)}p$ under the given marking $m \in \mathcal{M}$ and control u can exist within the duration, $\tau_p(u,m)$, which is represented in terms of availing time delays for the places in the marking paths for the place p as stated in the following lemma. We call the duration *possible firing time* of the transition ${}^{(t)}p$ under the given marking $m \in \mathcal{M}$ and control u. In order to simply denote the duration, we let

$$d_{\min}(\Pi_p(m)) = \max_{\pi_p(m)\in\Pi_p(m)}\left(\sum_{i=1}^{n-1} d_{\min}(p_i)\right)$$

and

$$d_{\max}(\Pi_p(m)) = \max_{\pi_p(m)\in\Pi_p(m)}\left(\sum_{i=0}^{n-1} d_{\max}(p_i)\right).$$

Lemma 2 : Given a place p in a TCMG and a marking $m \in \mathcal{M}$ for which $m(p) = 0$.

(1) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for every $t \in T_{\Pi_{\hat{P}_p}}$ then

$$\begin{aligned} d_{\min}(\Pi_p(m)) + k \cdot \tau_{R_{\min}}(p) &\le \tau_p(u,m), \quad k = 0,1,2,\cdots \\ &\le d_{\max}(\Pi_p(m)) + k \cdot \tau_{R_{\max}}(p). \end{aligned}$$

(2) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for every $t \in T_{\Pi_p}$, $t = p^{(t)}$, and every $t \in T_{\Pi_{p_r}(m)}$ where $p_r \in \hat{P}_p$ and $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_{\hat{P}_p}}$ then

$$\begin{aligned} d_{\min}(\Pi_p(m)) + k \cdot \tau_{R_{\min}}(p) &\le \tau_p(u,m), \quad k = 0,1 \\ &\le d_{\max}(\Pi_p(m)) + k \cdot \tau_{R_{\max}}(p). \end{aligned}$$

(3) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for each $t \in T_{\Pi_p(m)}$ and $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_{\hat{P}_p}}$ then

$$d_{\min}(\Pi_p(m)) \le \tau_p(u,m) \le d_{\max}(\Pi_p(m)).$$

(4) If $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_p(m)}$ then $\tau_p(u,m) = \infty$.

The above lemma shows that the possible firing time of the transition for which the given place is an output under the given marking m and control u in a TCMG can be computed in terms of the availing time delays of the individual places. From the above lemma, we can get the time, denoted by $\bar{\tau}_p(u,m)$, which a token can exist in the place $p \in \mathcal{P}$ as stated in the following corollary. The time is called *possible marking time* of the place p.

Corollary 1 : Given a place p in a TCMG and a marking $m \in \mathcal{M}$ for which $m(p) = 0$.

(1) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for every $t \in T_{\Pi_{\hat{P}_p}}$ then
$d_{\min}(\Pi_p(m)) + k \cdot \tau_{R_{\min}}(p) \le \bar{\tau}_p(u,m) \le \max_{p_r\in\hat{P}_p}(d_{\max}(\Pi_{p_r}(m)) + d_{\max}(p_r)) + k \cdot \tau_{R_{\max}}(p), k = 0,1,2,\cdots.$

(2) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for every $t \in T_{\Pi_p}$, $t = p^{(t)}$, and every $t \in T_{\Pi_{p_r}(m)}$ where $p_r \in \hat{P}_p$ and $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_{\hat{P}_p}}$ then

$$d_{\min}(\Pi_p(m)) \le \bar{\tau}_p(u,m) \le \max_{p_r\in\hat{P}_p}(d_{\max}(\Pi_{p_r}(m)) + d_{\max}(p_r))$$

and

$$d_{\min}(\Pi_p(m)) + \tau_{R_{\min}}(p) \le \bar{\tau}_p(u,m).$$

(3) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for every $t \in T_{\Pi_{P_p}(m)}$ and $t = p^{(t)}$ and $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_{\hat{P}_p}}$ then

$$d_{\min}(\Pi_p(m)) \le \bar{\tau}_p(u,m) \le \max_{p_r\in\hat{P}_p}(d_{\max}(\Pi_{p_r}(m)) + d_{\max}(p_r)).$$

(4) If $u(c) = 1$ for all $c \in {}^{(c)}t$ for each $t \in T_{\Pi_p(m)}$ and $u(c) = 0$ for some $c \in {}^{(c)}t$ for either $t = p^{(t)}$ or some $t \in T_{\Pi_{\hat{P}_p}(m)}$ then

$$d_{\min}(\Pi_p(m)) \le \bar{\tau}_p(u,m).$$

(5) If $u(c) = 0$ for some $c \in {}^{(c)}t$ for some $t \in T_{\Pi_p(m)}$ then $\bar{\tau}_p(u,m) = \infty$.

This corollary indicates that the possible marking time for the given place p can be computed under the given marking m and the control u in a TCMG. For a marking m such that $m(p) = 1$, the possible marking time for the place p is extended as $0 \le \bar{\tau}_p(u,m) \le \max(\tau_{p_{next}}(u,m))$. For a given place p, a marking m, and a control u, if $\bar{\tau}_p(u,m) < \infty$, then the place p is said to be *markable* from m under the control u, otherwise the place p is said to be *unmarkable* from m.

Control paths permit us to identify which controls can regulate the marking of places in a TCMG as stated in the following lemma.

Lemma 3 : Given a place p in a TCMG and an initial marking $m_0 \in \mathcal{M}$ for which $m_0(p) = 0$. If $\pi_{p\_c} \in sf^+(\Pi_p(m_0))$ for some $\pi_{p\_c} \in \Pi_{p\_c}$, and $u(c) = 0$ for some $c \in {}^{(c)}t_{\pi_{p\_c}}$, then $m(p) = 0$ for all $m \in \mathcal{R}_{INF}(u, m_0)$.

From the above lemma, it follows that the marking of the given place may be regulated by external controls which are inputs of the control path for the place in the sense that if the path is empty, a control exists which will prevent the place from ever being marked.

III. Forbidden States

The forbidden state problem for DES's involves the control syntheses which guarantee the system never enters a specified set of forbidden states. In a TCMG a set of forbidden states is expressed by a set of markings $M \subseteq \mathcal{M}$. Following [3],[4], the specification of a set of forbidden markings in a TCMG is given a class of *set conditions*, so called *class condition*, $\mathcal{F} \subseteq 2^{\mathcal{P}}$, which defines a set of forbidden markings, $M_{\mathcal{F}} = \{m \in \mathcal{M} | \exists F \in \mathcal{F}, \forall p \in F \;\; m(p) = 1\}$. For a particular set $F \in \mathcal{F}$, M_F denotes the set of markings with tokens in all state places in $F \subset \mathcal{P}$, and for a state place $p \in \mathcal{P}$, M_p denotes the set of markings with a token in state place p.

In general, given a set $M_{\mathcal{F}}$) of forbidden markings, a larger set of markings must be avoided, due to the possibility of uncontrollable firing sequences, which is said to be the *weakly forbidden markings*, $W(M_{\mathcal{F}})$, with respect to $M_{\mathcal{F}}$ defined as $W(M_{\mathcal{F}}) = \{m \in \mathcal{M} | \mathcal{R}_{INF}(u_{zero}, m) \cap M_{\mathcal{F}} \neq \emptyset\}$ [6]. In words, $W(M_{\mathcal{F}})$ is the set of markings from which a marking in the forbidden marking set $M_{\mathcal{F}}$ can be reached uncontrollably, that is, the set of markings such that $\exists F \in \mathcal{F}, \bigcap_{p \in F} \vec{\tau}_p(u_{zero}, m) \neq \emptyset$. By definition, $M_{\mathcal{F}} \subseteq W(M_{\mathcal{F}})$. Given a set $M_{\mathcal{F}} \subseteq \mathcal{M}$ for a TCMG of forbidden markings, the set $\tilde{M}_{\mathcal{F}} \equiv \mathcal{M} - W(M_{\mathcal{F}})$ is said to be the set of *admissible markings* with respect to $M_{\mathcal{F}}$, and $\tilde{M}_{\mathcal{F}} \cap W(M_{\mathcal{F}}) = \emptyset$.

A control objective is to prevent any forbidden markings $M_{\mathcal{F}}$ from being reached. To carry out the control objective, we will use the control policy to prevent any forbidden markings $M_{\mathcal{F}}$ from being reached while the control u is updated upon each state transition using state feedback, which will be referred to as *adaptive control* which can be used to achieve the maximally permissive closed-loop behavior [3],[5].

IV. Adaptive Control

If the system state can be measured and the control can be updated immediately when state transitions occur, it is only necessary to assure that the set of next possible markings are admissible. Such a state feedback policy is the motivation for the adaptive control problem considered in this section.

Given a set of forbidden markings $M_{\mathcal{F}}$, and an admissible marking $m \in \tilde{M}_{\mathcal{F}}$, the objective of adaptive control is to choose a control u such that any marking reachable from m in one state transntion is also within $\tilde{M}_{\mathcal{F}}$ while permitting a maximal number of state transitions from the marking m. It is shown in [6] that for the maximally permissive adaptive control problem, it is necessary to solve the problem only when the marking is on the *boundary* of the admissible marking set $\tilde{M}_{\mathcal{F}}$, where the notion of boundary markings is defined as follows.

Definition 5 : [3] Given a marking set M for a TCMG, a marking $m \in M$ is a *boundary marking* in M if there exists some $m' \in \mathcal{R}_{ONE}(u_{onc}, m)$, such that $m' \notin M$.

For a set of markings M, the set of boundary markings is denoted by δM. It is also shown in [3] that for admissible markings which are not boundary markings, the unique maximally permissive control is the most permissive control, u_{onc}. In this section, we consider the identification of boundary markings for sets of admissible markings, and characterization of the set of maximally permissive controls for these markings.

We now consider the adaptive control problem for a class condition satisfied the cycle condition defined as follows.

Definition 6 : A class condition $\mathcal{F}$ for a TCMG is said to satisfy the *cycle condition (cc)* if, for every $F \in \mathcal{F}$, no place $p \in F$ lies within any cycle including another place $p' \in F, p' \neq p$.

A class condition satisfying the above cycle condition will be referred to as *cc class condition*. The following theorem identifies the set of maximally permissive controls for a given cc class condition. Without loss in generality, we assume that $\exists p \in F$, $\Pi_{p\_c} \neq \epsilon$ for every $F \in \mathcal{F}$.

Theorem 1 : Given a TCMG with a cc class condition $\mathcal{F}$ and an admissible marking $m \in \tilde{M}_{\mathcal{F}}$, a control u is maximally permissive with respective to the forbidden markings $M_{\mathcal{F}}$ for a marking m if and only if the following hold.

(1) For each $F \in \mathcal{F}$ such that $m(p) = 1$ for all $p \in {}^{(p)}t$ for each $t \in T_{\Pi_F(m)} \neq \emptyset$ and $\bigcap_{p_i \in F} \vec{\tau}_{p_i}(u_{zero}, m_{T_{\Pi_F(m)}}) \neq \emptyset$, there exists $c \in {}^{(c)}t$ for one $t \in T_{\Pi_F(m)}$ such that $u(c) = 0$.

(2) For each $c \in C$ such that $u(c) = 0$, there exists an $F \in \mathcal{F}$ for which $m(p) = 1$ for all $p \in {}^{(p)}t$ for each $t \in T_{\Pi_F(m)} \neq \emptyset$ and $\bigcap_{p_i \in F} \vec{\tau}_{p_i}(u_{zero}, m_{T_{\Pi_F(m)}}) \neq \emptyset$, such that $c \in {}^{(c)}t$ for one $t \in T_{\Pi_F(m)}$ and $u(c') = 1$ for all $c' \neq c \in {}^{(c)}t$ for each $t \in T_{\Pi_F(m)}$.

V. Example

In this section we apply the theory from the previous section, to an example to get the maximally permissive controls for them. The maximally permissive control is computed in two stages. It is assumed that the uncontrolled system is fully observed. First stage is off-line-computation to find the weakly forbidden marking set and the boundary marking set for the forbidden marking specifications, and to generate masks for the control places which are inputs to the input transition for each control path for the places in the forbidden specification. This off-line-computation is based entirely on the structure of the TCMG, the time delays attached to the state places, and the given class condition. Second stage is on-line-computation to generate a maximally permissive control set for the given marking, followed by the selection of the maximally permissive controls using the control masks.

A flexible manufacturing cell, modeled by the CMG in Figure 2 [3], is considered here, in which there are three workstations, two part-receiving stations, and one completed parts station. Five automated guided vehicles (AGV's) transport material between pairs of stations, passing through zones shared by other AGV's. In this example, the control objective is to coordinate the departure of AGV's from stations so that no two AGV's will occupy the same zone simultaneously. CMG provides good solution [3], but is somewhat restrict. To illustrate this restriction, we make the following observations for the making m shown in Figure **2**. For convinient illustration, we assume all the state places in AGV's paths have the time constraint [2,3].

- zone 1 : $m \in \tilde{M}_{zone1}$ and $m \notin \delta\tilde{M}_{zone1}$. Therefore, no control needs to be disabled to prevent a collision in zone 1.
- zone 2 : Since $\vec{\tau}_{p_{21}}(u_{zero}, m_{t_{21}}) \cap \vec{\tau}_{p_{22}}(u_{zero}, m_{t_{21}}) = \emptyset$, $m \notin \delta\tilde{M}_{zone2}$. Therefore, control c_4 does not need to be disabled to prevent a collision in zone 2. In CMG, however, $m \in \delta\tilde{M}_{zone2}$ [3], and control c_4 need to be disabled to prevent a collision in zone 2.
- zone 3 : Since $\vec{\tau}_{p_{31}}(u_{zero}, m_T) \cap \vec{\tau}_{p_{32}}(u_{zero}, m_T) = [0, 6]$ where $T = \{t_{21}, t_{31}\}$, $m \in \delta\tilde{M}_{zone3}$. Therefore, disabling either control c_4 or c_7 is needed to prevent a collision in zone 3.
- zone 4 : Since $\vec{\tau}_{p_{41}}(u_{zero}, m_{T'}) \cap \vec{\tau}_{p_{42}}(u_{zero}, m_{T'}) = [4, 9]$ where $T' = \{t_{31}, t_{41}\}$, $m \in \delta\tilde{M}_{zone4}$. Therefore, disabling either control c_7 or c_{10} is needed to prevent a collision in zone 4.

From Theorem 1, the set of maximally permissive controls for this marking is the controls which $u(c) = 0$ for $c \in \{c_7\}$ and $u(c) = 1$ for $c \notin \{c_7\}$, or which $u(c) = 0$ for $c \in \{c_4, c_{10}\}$ and $u(c) = 1$ for $c \notin \{c_4, c_{10}\}$.

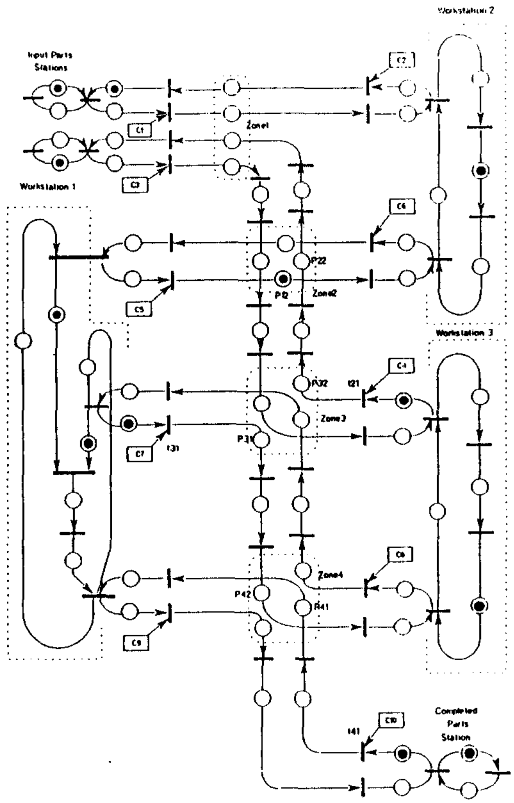

Figure 2: AGV System

VI. Conclusion

In this paper we present a class of discrete event systems (DES's) which can be modeled as cyclic timed controlled marked graphs (TCMG's), a special class of timed controlled Petri nets (TCPN's), and an efficient method for synthesizing feedback control which permits the maximally permissive closed-loop behavior while preventing any forbidden markings M from being reached.

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