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An Iterative Learning Approach to Error Compensation of Position Sensors for Servo Motors

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Abstract — In this paper, we present an iterative learning method of compensating for position sensor error. The previously known compensation algorithms need a special perfect position sensor or a priori information about error sources, while ours does not. To our best knowledge, any iterative learning approach has not been taken for sensor error compensation. Furthermore, our iterative learning algorithm does not have the drawbacks of the existing iterative learning control theories. To be more specific, our algorithm learns a uncertain function itself rather than its special time-trajectory and does not request the derivatives of measurement signals. Moreover, it does not require the learning system to start with the same initial condition for all iterations. To illuminate the generality and practical use of our algorithm, we give the rigorous proof for its convergence and some experimental results.

I. Introduction

The control accuracy of servo systems driven by electric motors depends highly on the accuracy of position sensors. Unfortunately, the output of a practical position sensor is distorted to some extent by its nonideal dynamic characteristics. Therefore, the outputs of practical position sensors need be compensated for in order to meet the high accuracy specifications of servo systems.

In the prior literature, not much attention has been paid to the compensation method for position sensor error. Hung and Hung [8] measured directly and compensated digitally for the position sensor error. To do this, they needed a special ideal sensor. On the other hand, Hanselman [5] analyzed the various sources that could give rise to the position error. Based on this error analysis, he proposed in [6] a method of reducing or eliminating the position sensor error. The error analysis in [5], however, considered only the individul effect of each error source on position error.

In this paper, we present a learning algorithm which compensates for the deterministic error of a position sensor by estimating the inverse of its input-output mapping iteratively. Our compensation method can correct directly the position sensing error resulting from a combination of all error sources. Recently, considerable research effort has been devoted to high performance control of AC servo motors. The torque control algorithms in [4, 7, 13] can force AC servo motors to behave like DC servo motors, provided that the true values of rotor position are available. The key idea of our learning algorithm for position sensor error compensation lies in that inaccurate information of rotor position causes torque ripple. Our compensation method does not require a priori information of torque controller, position sensor model, or motor parameters, but is based only on the steady-state responses of the servo system with a nonideal position sensor. Hence, it does not need a special ideal sensor. To our best knowledge, any iterative learning approach has not been taken for sensor error compensation. Furthermore, our iterative learning algorithm for position sensor error does not have the drawbacks of the existing iterative learning control theories [1, 2, 3, 9, 12] in the following respects.

Our algorithm can learn the inverse of the input-output mapping of the nonideal position sensor, but not just its special time-history. In our sensor compensation problem, only the output signal of the nonideal position sensor is accessible. Therefore, its differentiation can produce significant noise and hence is not desirable. For this reason, our algorithm does not use the derivatives of the sensor output signal. In addition, we show that the sensor output signal tends to be a stable limit cycle at each iteration.

As the result, our algorithm does not require an initial position reset mechanism in order to assure that the learning system starts with the same initial conditions for all iterations.

The paper is organized as follows. In Section II, we state precisely our problem of sensor error compensation and discuss the typical servo motor drive system with a nonideal position sensor. In Section III, we present our learning argorithm to compensate for the position sensor error with the rigorous proof for its uniform convergence. We introduce two lemmas to describe the dynamic behavior of the torque control system with a nonideal position sensor. In Section IV, we show that a wide class of servo motors satisfies the convergence condition of our learning argolithm. In order to illuminate further the practical significance of our learning algorithm, we perform some experiments through use of an NSK VR type DD motor with a VR type resolver as the position sensor. Finally, Section V contains our concluding remarks.

II. Preliminaries

The dynamics of the electric actuators such as DC motors, Brushless DC motors (BLDCM), and step motors can be described by

$$J\ddot{\theta} + B\dot{\theta} + Csgn(\dot{\theta}) = \tau_e - \tau_L. \tag{2.1}$$

Here, the constants J, B, and C are, respectively, moment of inertia, viscous friction coefficient, and Coulomb friction coefficient. The variables θ and τ_L represent rotor position and load torque, respectively. On the other hand, τ_e represents motor torque and is a continuous function of rotor position θ and phase current $i \in R^q$, where q denotes the number of phases. Let $i^* \in R^q$ be the reference phase current. If the phase current is directly controlled [4, 10], we can assume that $i = i^*$ and hence that the motor torque is a function of θ and i^*. That is,

$$\tau_e = T(\theta, i^*). \tag{2.2}$$

It is a natural property of rotating machines that the above function $T : R \times R^q \mapsto R$ is periodic such that

$$T(\theta + 2\pi, i^*) = T(\theta, i^*), \quad \forall\, \theta \in R,\ i^* \in R^q. \tag{2.3}$$

Now, suppose that we have a function $I : R \times R \mapsto R^q$ satisfing

$$T(\theta, I(\theta, \tau^*)) = \tau^*, \quad \forall\, \theta \in R,\ \tau^* \in R. \tag{2.4}$$

Then, choose the reference phase current i^* by

$$i^* = I(\theta, \tau^*), \tag{2.5}$$

where $\tau^* \in R$ represents the reference torque. Then,

$$\tau_e = T(\theta, i^*) = T(\theta, I(\theta, \tau^*)) = \tau^*, \quad \forall\, \theta \in R,\ \tau^* \in R. \tag{2.6}$$

Hence, the torque controller in (2.5) linearizes the nonlinear dynamic of an electric motor in (2.1) and (2.2) as follows.

$$J\ddot{\theta} + B\dot{\theta} + Csgn(\dot{\theta}) = \tau^* - \tau_L \tag{2.7}$$

On the other hand, it follows from (2.3) and (2.4) that the inverse of T is also periodic, i.e.,

$$I(\theta + 2\pi, \tau^*) = I(\theta, \tau^*), \quad \forall\, \theta \in R,\ \tau^* \in R. \tag{2.8}$$

The above feedback linearizing approach is currently popular in the area of motor control [4, 7, 13].

Unfortunately, the output of a practical position sensor is distorted to some extent due to its nonideal dynamic characteristics and hence the error-free information of rotor position is not available to the torque controller I. Since the dynamics of a rotor position sensor is fast enough to be neglected, the input-output dynamic characteristics of a nonideal position sensor can be modeled by a continuous nonlinear function g. That is,

$$\hat{\theta} = g(\theta). \tag{2.9}$$

We define the error function n of a nonideal position sensor by

$$n(\theta) \triangleq g(\theta) - \theta. \tag{2.10}$$

Then, the block diagram representing the function of a nonideal position sensor can be depicted as in Fig.2.1.(a). It is obvious that the error function n is periodic and is upper bounded.

$$n(\theta + 2\pi) = n(\theta) \tag{2.11}$$

$$|n(\theta)| \quad \leq \quad \mu \triangleq \sup_{\theta \in [0, 2\pi]} |n(\theta)| \tag{2.12}$$

In general, the reference point of rotor position does not coincide with that of the position sensor. This reference offset corresponds to $g(0)$ or $n(0)$ here. Fortunately, it is practically simple to detect $g(0)$. In the case of VR type DD motors, for example, we turn on one phase and turn off the other phases. Then, one of stator teeth is aligned with one of rotor teeth. We choose this rotor position as the reference point of rotor position. Then, $g(0)$ is just the output of the nonideal position sensor at this rotor position. From now, we assume that the reference offset is removed, i.e.,

$$g(0) = n(0) = 0. \tag{2.13}$$

As can be seen from (2.9), the true value θ of rotor position can be recovered from the output $\hat{\theta}$ of a nonideal position sensor, if g^{-1} is known. The function f_k in Fig.2.1.(b) stands for the k-th estimate of g^{-1}. In this paper, we attemp to find a learning algorithm which can make the sequence $\{f_k\}_{k=1}^{\infty}$ converge uniformly to g^{-1}. Throughout this paper, the subscript k denotes the iteration number.

From Fig.2.1.(b), we can see that the compensated value ϕ of the sensor output $\hat{\theta}$ can be represented by

$$\phi = f_k \circ g(\theta) \qquad (2.14)$$

and that the k-th compensation error e_k can be defined by

$$e_k(\theta) \triangleq \phi - \theta = f_k \circ g(\theta) - \theta. \qquad (2.15)$$

More specifically speaking, our objective is to find an update rule for f_k , $k = 1, 2, 3, \ldots$ which can make $\bar{e}_k \longrightarrow 0$ as $k \longrightarrow \infty$, where $\bar{e}_k$ is defined as follows.

$$\bar{e}_k \triangleq \sup_{\theta \in (-\infty,\infty)} |e_k(\theta)| \qquad (2.16)$$

Let us investigate the effect of the position sensor error on torque response. Suppose that we have a torque controller I which satisfies (2.4). Suppose further that the position sensor is nonideal but is compensated as is shown in Fig.2.1.(b). Instead of (2.5) and (2.6), we then have

$$i^* = I(\phi, \tau^*) \qquad (2.5')$$

and

$$\tau_e = T(\theta, I(\phi, \tau^*)). \qquad (2.6')$$

As the result, (2.6) will not necessarily hold, since $f_k \neq g^{-1}$ in general. For each $\tau^* \in R$ and $\mu \in [0, \infty)$, however, there exists a constant α such that

$$|\frac{\partial}{\partial \phi} T(\theta, I(\phi, \tau^*))| \leq \alpha, \quad \text{if } |\phi - \theta| < \mu. \qquad (2.17)$$

This is due to continuity and periodicity of T and I.

The block diagram representation of the torque control system given by (2.1), (2.2), and (2.5′) is given in Fig.2.2. And its dynamic equations can be written as follows.

$$J\ddot{\theta}_k + B\dot{\theta}_k + Csgn(\dot{\theta}_k) = T(\theta_k, I(\phi_k, \tau^*)) - \tau_L, \qquad (2.7')$$
$$\phi_k = f_k(g(\theta_k))$$

Here and in what follows, we insert the subscript k into all variables that are affected by the k-th estimate f_k of g^{-1}. In the next section, we present a learning algorithm for g^{-1} which does not depend on motor parameters(J, B, C), torque model($T : R \times R^q \mapsto R$), or torque control strategy($I : R \times R \mapsto R^q$).

III. Main Results

Before presenting our learning rule to update f_k, $k = 1, 2, \ldots$ iteratively, we need to make some assumptions on the system in Fig.2.2. First, we assume that

(C 1) $\quad |\frac{d}{d\theta} n(\theta)| < 1, \quad \forall\, \theta \in R.$

This assures the existence of g^{-1}. In turn, this implies that the true value of rotor position can be recovered from the output of nonideal position sensor, if g^{-1} is known. Most of good industrial position sensors are expected to satisfy *(C 1)*.

We also make an assumption on the initial conditions of the system in Fig.2.2 as follows.

(C 2) $\quad |\dot{\theta}_k(0)| < \tau_M / B, \qquad \text{for } k = 1, 2, \ldots.$

Here, τ_M denotes the maximum allowable torque of the electric motor on which a nonideal position sensor is mounted. The quantity τ_M/B in *(C 2)* corresponds to the rotor speed that the electric motor driven by the maximum allowable torque τ_M achieves in the steady state. Note that *(C 2)* is much less restrictive than the conditions that the existing iterative learning control theories [1, 2, 3, 9, 12] impose on initial states.

Finally, we assume that

(C 3) $\quad \tau_L$ is constant during learning process.

In practice, the load torque τ_L is not necessarily constant throughout learning process. However, it is reasonable to assume that τ_L is constant in the case when learning period is short.

(C 4) $\quad \tau^*$ is set constant during learning process such that

$$\tau^* - |\tau_L| - C > 6\mu\alpha, \quad \tau^* + |\tau_L| + C \leq \tau_M - \mu\alpha.$$

As will be shown later, the condition *(C 4)* guarantees that the rotating direction of the electric motor is determined only by the sign of τ^* but is not affected by other sources such as position sensing error, Coulomb friction torque, and load torque.

Now, we describe the dynamic behaviors of the system in Fig.2.2 under the prescribed assumptions. Speaking specifically, we show that the time functions θ_k, $\hat{\theta}_k$, $k = 1, 2, \ldots$ are invertible and periodic.

Lemma 3.1 : Suppose that

(i) f_k is chosen so that

$$f_k(x + 2\pi) = f_k(x) + 2\pi, \quad \forall\, x \in R. \qquad (3.1)$$

(ii) $\bar{e}_k$ is sufficiently small so that

$$\bar{e}_k \leq \mu. \qquad (3.2)$$

Then, the system in Fig.2.2 has the following properties in the steady state.

(a) θ_k and $\hat{\theta}_k$ are strictly increasing in t.

(b) There exists a positive constant T_k such that

$$\begin{aligned} \theta_k(t + T_k) &= \theta_k(t) + 2\pi, \\ \hat{\theta}_k(t + T_k) &= \hat{\theta}_k(t) + 2\pi, \\ \phi_k(t + T_k) &= \phi_k(t) + 2\pi. \end{aligned}$$

Proof : The proof is quite elaborate and lengthy. In this paper, it is omitted due to limited space. □

Suppose that the hypotheses (i), (ii) of Lemma 3.1 are satisfied and that the system in Fig.2.2 with f_k as the k-th estimate of g^{-1} reaches the steady state. Then, the properties (a), (b) in Lemma 3.1 justify the following procedure to define the function $f_{k+1} : R \mapsto R$ on the basis of the responses of the system in Fig.2.2 with the function f_k as the k-th estimate of g^{-1}.

Step 1 : Pick up t_k and t'_k such that $\hat{\theta}_k(t_k) = 2\pi m_k$ and $\hat{\theta}_k(t'_k) = 2\pi(m_k + 1)$ for a positive integer m_k.

Step 2 : Compute $T_k \triangleq t'_k - t_k$.

Step 3 : Determine the time function $\psi_k : [t_k, t_k + T_k] \mapsto [2\pi m_k, 2\pi(m_k + 1)]$ by

$$\psi_k(t) \triangleq \frac{2\pi}{T_k}(t - t_k) + 2\pi m_k, \quad t_k \le t \le t_k + T_k. \quad (3.3)$$

Step 4 : Define $f^0_{k+1} : [2\pi m_k, 2\pi(m_k + 1)] \mapsto [2\pi m_k, 2\pi (m_k + 1)]$ by

$$f^0_{k+1}(x) \triangleq \psi_k \circ \hat{\theta}_k^{-1}(x), \quad \forall\, x \in [2\pi m_k, 2\pi(m_k + 1)]. \quad (3.4)$$

Step 5 : Define $f_{k+1} : R \mapsto R$ by periodically expanding f^0_{k+1} as follows. For all $j = 0, \pm 1, \pm 2, \ldots$ and $x \in [2\pi m_k, 2\pi(m_k + 1)]$,

$$f_{k+1}(x + 2\pi j) \triangleq f^0_{k+1}(x) + 2\pi j. \quad (3.5)$$

It should be clear from the definition of ψ_k in (3.3) and f^0_{k+1} in (3.4) with the properties (a), (b) in Lemma 3.1 that (3.5) determines the function f_{k+1} uniquely reqardless of different choices of m_k. On the other hand, (2.9)-(2.11), (2.13), and *(C1)* imply that, for $j = 0, \pm 1, \pm 2, \ldots$,

$$\hat{\theta}_k = 2\pi j \quad \text{iff} \quad \theta_k = 2\pi j. \quad (3.6)$$

Therefore, we see that the function ψ_k defined by (3.3) is just the "time-average signal" of θ_k.

Using the function ψ_k, we now define the position ripple ϵ_k and its upper bound $\bar{\epsilon}_k$ as follows.

$$\epsilon_k(t) \triangleq \psi_k(t) - \theta_k(t), \quad \forall\, t \in [t_k, t_k + T_k] \quad (3.7)$$

$$\bar{\epsilon}_k \triangleq \sup_{t \in [t_k, t_k + T_k]} |\epsilon_k(t)| \quad (3.8)$$

The following Lemma 3.2 clarifies the relationship between the upper bound $\bar{e}_k$ of compensation error and the upper bound $\bar{\epsilon}_k$ of position ripple.

Lemma 3.2 : Suppose that all the hypotheses of Lemma 3.1 are satisfied. Then,

$$\bar{\epsilon}_k \le \eta \bar{e}_k, \quad (3.9)$$

where

$$\eta \triangleq \frac{8\pi\alpha}{\tau^* - \tau_L - C - 2\mu\alpha}. \quad (3.10)$$

Proof : The proof is omitted due to lack of space. □

Now, we are ready to present our learning rule that can update f_k, $k = 1, 2, \ldots$ iteratively so that the sequence $\{f_k\}_{k=1}^{\infty}$ converges uniformly to g^{-1}.

Theorem 3.1 : Define the update rule for f_k, by (3.3)-(3.5), where f_1 is chosen as the identity mapping, i.e.,

$$f_1(x) \triangleq x, \quad \forall\, x \in R. \quad (3.11)$$

Then, the update rule is well defined for all $k = 1, \ldots$, if

$$\eta < 1. \quad (3.12)$$

Futhermore, it holds that

$$\bar{e}_k \le \eta^{k-1}\mu, \quad \text{for} \quad k = 1, 2, \ldots \quad (3.13)$$

and hence that

$$\bar{e}_k \longrightarrow 0 \text{ as } k \longrightarrow \infty. \quad (3.14)$$

Proof : We show by induction that our assertion is true. Clearly, f_1 in (3.11) is well defined and satisfies (3.1). On the other hand, we can see from (2.10), (2.12), and (2.15) that (3.13) holds with $k = 1$.

Next, suppose that f_r is well-defined and

$$f_r(x + 2\pi) = f_r(x) + 2\pi, \quad \forall\; x \in R \quad (3.15)$$

$$\bar{e}_r \le \eta^{r-1}\mu. \quad (3.16)$$

By (3.12), (3.15), and (3.16), Lemma 3.1 holds for $k = r$. Consequently, f_{r+1} via *Step 1–Step 5* is well defined and

$$f_{r+1}(x + 2\pi) = f_{r+1}(x) + 2\pi, \quad \forall\; x \in R. \quad (3.17)$$

By (3.17) with (2.10), (2.11), and (2.15),

$$e_{r+1}(x + 2\pi) = e_{r+1}(x), \quad \forall\, x \in R. \quad (3.18)$$

By (2.15), *Step 1–Step 5*, (3.6), and (3.18),

$$\begin{aligned} \bar{e}_{r+1} &= \sup_{x \in [2\pi m_r, 2\pi(m_r+1)]} |f_{r+1}(g(x)) - x| \\ &= \sup_{t \in [t_r, t_r + T_r]} |f_{r+1}(g(\theta_r(t))) - \theta_r(t)| \\ &= \sup_{t \in [t_r, t_r + T_r]} |f_{r+1}(\hat{\theta}_r(t)) - \theta_r(t)| \\ &= \sup_{t \in [t_r, t_r + T_r]} |\psi_r(t) - \theta_r(t)| \\ &= \bar{\epsilon}_r. \end{aligned}$$

By Lemma 3.2, this and (3.16) imply that (3.13) holds with $k = r + 1$.

Hence, by induction, we can conclude that the update rule is well defined for all $k = 1, 2, \ldots$ and (3.13) holds for all $k = 1, 2, \ldots$. Finally, (3.14) is the immediate consequence of (3.13) under the condition in (3.12). And this complets the proof. □

In our update rule, we have chosen f_1 as the identity mapping. This means that, at the first iteration, we feedback the output of the nonideal position sensor directly into the torque controller without compensation.

The direct application of the existing iterative learning control theories [1, 2, 3, 9, 12] to our compensation problem does not seem possible since the true information of system states (here, θ and $\dot{\theta}$) is not available. Moreover, our iterative learning approach differs from the prior approaches in the following respects. First, ours uses the time-averaged estimate ψ_k but not the true data of rotor position, as can be seen from (3.3). Second, ours uses only the available information $\hat{\theta}$ but not its derivatives, as can be seen from (3.4) and (3.5). Third, ours estimates the function g^{-1} rather than its special time-histories. Finally, the convergence of our update rule does not impose any restriction on the initial conditions $\theta_k(0)$, $\dot{\theta}_k(0)$.

In our iterative learning approach, the reference torque τ^* is chosen large enough to ensure that θ_k is strictly increasing for all iterations. The asymptotic periodicity of θ_k shown in Lemma 3.1 is guaranteed purely by the natural properties of a rotating machine. After the torque control system in Fig.2.2 reaches the steady state at each iteration, our learning scheme updates the estimate of g^{-1} based on the waveform of $\hat{\theta}$ for one period only.

IV. PRACTICAL EXAMPLES

Our learning algorithm for position sensor error compensation presented in Section III can be applied to a wide class of electric motors. In this section, we show that AC syncro motors and VR type DD motors satisfy the conditions for the convergence of our learning algorithm. Especially, for the case of VR type DD motors, we present some experimental results in order to illuminate further the practical significance of our compensation method.

Example 4.1 : (AC Syncro motor)
The torque model of a three-phase AC syncro motor takes the following form.

$$T(\theta, i^*) = k_T[i_1^* \sin(\theta) + i_2^* \sin(\theta - \frac{2}{3}\pi) + i_3^* \sin(\theta - \frac{4}{3}\pi)] \qquad (4.1)$$

A choice of the torque controller I satisfying (2.4) is

$$I(\theta, \tau^*) = (\frac{2\tau^*}{3k_T}) \begin{bmatrix} \sin(\theta) \\ \sin(\theta - \frac{2}{3}\pi) \\ \sin(\theta - \frac{4}{3}\pi) \end{bmatrix}. \qquad (4.2)$$

Then, simple calculation yields the following fact.

$$T(\theta, I(\phi, \tau^*)) = \tau^* \cos(\phi - \theta), \quad \forall \ \theta, \phi \in R \qquad (4.3)$$

By (4.3), we have the following inequality.

$$|\frac{\partial}{\partial \phi} T(\theta, I(\phi, \tau^*))| = |\tau^* \sin(\phi - \theta)| \leq |\tau^*||\phi - \theta| \quad (4.4)$$

This shows that (2.17) can be satisfied by

$$\alpha = |\tau^*|\mu. \qquad (4.5)$$

This choice gives

$$\eta = \frac{8\pi|\tau^*|\mu}{\tau^* - \tau_L - C - 2\mu^2|\tau^*|}. \qquad (4.6)$$

In the particular case of $\tau_L = C = 0.1\tau^*$ and $\mu = 0.02$, we have $\eta = 0.629$. Note that $\mu = 0.02$ corresponds to about 1.146°. This large sensor error can be compensated for at the fast convergence rate of $\eta = 0.629$. □

Example 4.2 : (VR type DD Motor)
The torque model of a VR type DD motor with 3 phases can be written as

$$T(\theta, i^*) = T_0(\theta, i_1^*) + T_0(\theta - \frac{2}{3}\pi, i_2^*) + T_0(\theta - \frac{4}{3}\pi, i_3^*), \quad (4.7)$$

where the function $T_0 : R \times R \mapsto R$ has the properties :

$$T_0(\theta, y) = T_0(\theta, -y), \ T_0(-\theta, y) = -T_0(\theta, y), \ \ \forall \ \theta, y \in R. \qquad (4.8)$$

In order to maximize torque/mass ratio, VR type DD motors usually operate with magnetic saturation. As the result, T_0 is a highly nonlinear function and hence is hard to be described in explicit form.

On the other hand, Wallace and Taylor [13] proposed recently an inverse function technique, which corresponds here to finding the torque controller I which satisfies (2.4). The torque controller I for VR type DD motors can be described by

$$I(\theta, \tau^*) = (I_0(\theta, \tau^*), I_0(\theta - \frac{2}{3}\pi, \tau^*), I_0(\theta - \frac{4}{3}\pi, \tau^*))^T, \quad (4.9)$$

where the function $I_0 : R \times R \mapsto R$ has the property.

$$I_0(\theta, \tau^*) = 0, \quad \forall \ \theta \in [0, \pi], \ \tau^* \in R^+ \qquad (4.10)$$

For our experimental study, we used a VR type DD motor (Ref. No. RS0608FN001) with 150 poles, maximum stator current 6 [A], maximum speed 1 [rps], and $\tau_M = 25$ [Nm], which is manufactured by NSK Co., Japan. The position sensor mounted on the NSK VR type DD motor is a VR type resolver. VR type resolvers are cheap and rugged, but are not accurate enough for high-precision control. Therefore, we have to compensate for the nonideal characteristics of VR type resolvers if we want to use them for high-precision control. The functions T_0, I_0 of the NSK VR type DD motor we obtained in look-up table form by using a method similar to those in [11, 13] are depicted graphically in Fig.4.1.(a) and Fig.4.1.(b), respectively.

In our experiment, the torque command τ^* was set to 1.25 [Nm], which corresponds to 5% of τ_M. The experimental results are summarized in Fig.4.2.(a) (c) and

Fig.4.3. The time-histories of ϕ_k for $k = 1,3,5$ in the steady state are depicted in Fig.4.2. Fig.4.3 presents the graphic plots of $n_k(x) \triangleq f_k^{-1}(x) - x$, $k = 1, \ldots, 5$. Just in four iterations, the sequence $\{n_k\}$ converges and velocity ripple is reduced one-tenth or less, as can be seen from Fig.4.3 and Fig.4.2. Our experimentation firmly demonstrates the practical use of our learning algorithm. □

V. Conclusion

In this paper, we have presented a learning algorithm which can compensate automatically for position sensing error without using any other special sensors. In order to justify the generality and practical signigicance of our learning algorithm, we have provided the rigorous proof for its uniform convergence under reasonable assumptions. And, we have shown that typical electric motors satisfy the conditions for the convergence of our learning algorithm. Also, we have demonstrated its practicality through some experiments using a VR type DD motor with a VR type resolver as the position sensor. To our best knowledge, this is the first result to consider an iterative learning approach for sensor error compensation. Moreover, our result suggests that the concept of iterative learning can be used to learn not only the special time-history of an uncertain function but also the uncertain function itself.

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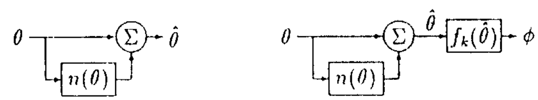


(a) Nonideal Sensor (b) Compensation of Nonideal Sensor

Figure 2.1. Sensor Signal Error and its Compensation

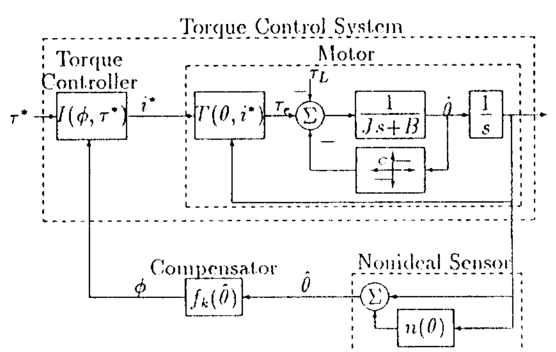
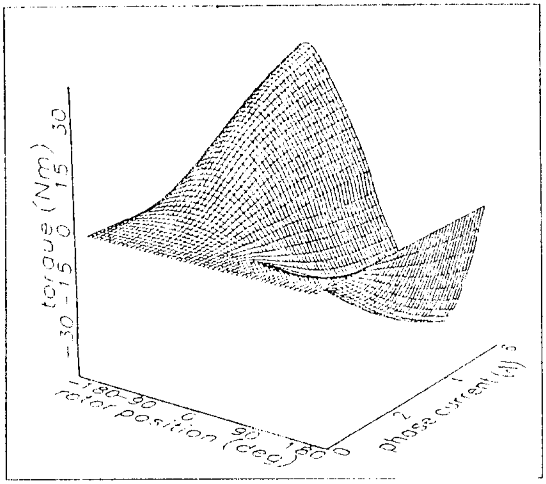
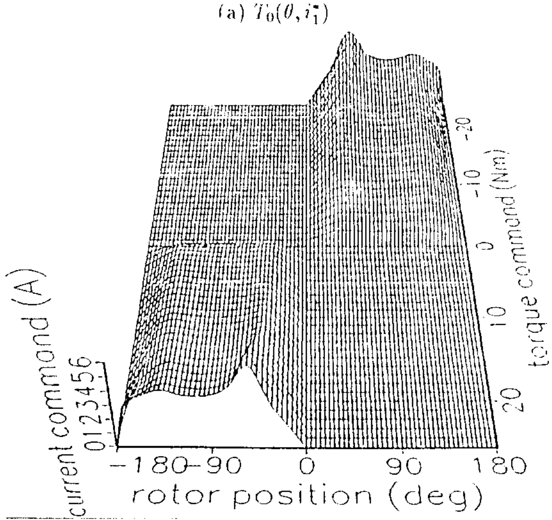


Figure 2.2: Torque Control System with Nonideal Position Sensor

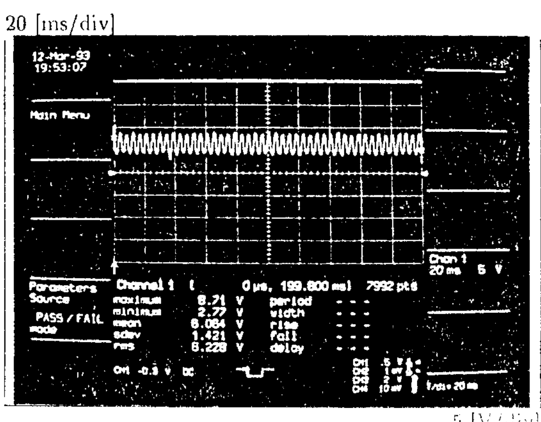


(a) $T_0(\theta, i_1^*)$

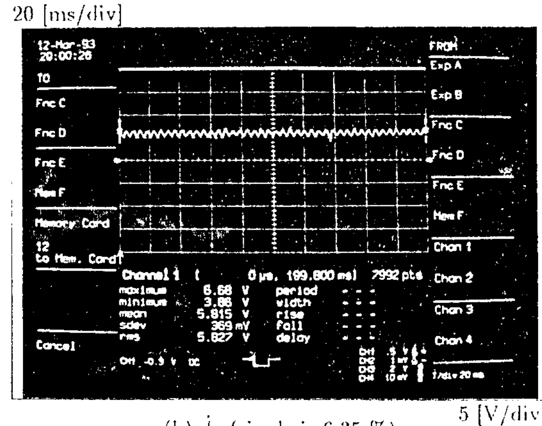


(b) $I_0(\theta, \tau^*)$

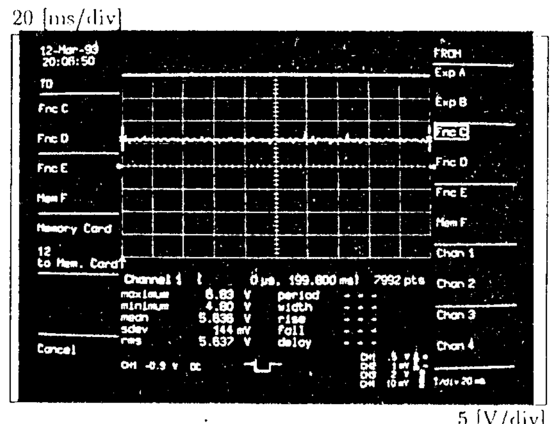
Figure 4.1: Graphic Representation of T_0 and I_0 for NSK VR Type DD Motor



(a) $\dot{\phi}_1$ (ripple is 23.4 %)



(b) $\dot{\phi}_3$ (ripple is 6.35 %)



(c) $\dot{\phi}_5$ (ripple is 2.56 %)

Figure 4.2: Time-History of $\dot{\phi}_k$, $k = 1, 3, 5$ in the Steady State

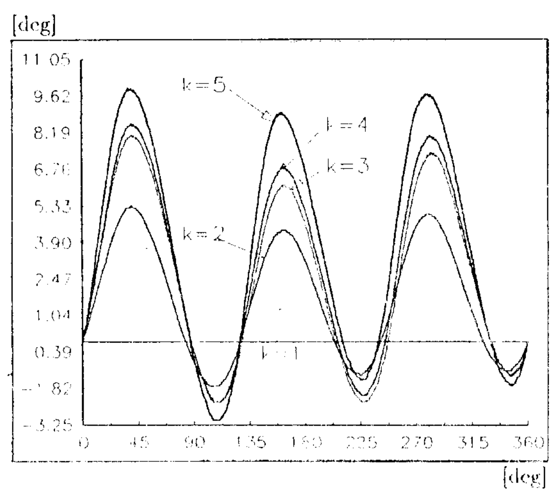


Figure 4.3. Graphic Plot of n_k, $k = 1, 2, 3, 4, 5$