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A Method of Dynamic Error Reduction for a Sensor with First Order Lag Using a Digital Convolution Integrator

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Abstract

This paper describes a new method of dynamic error compensation, using a digital convolution integrator and two digital low pass filters. In this method, the process of compensation consists of three steps. First, sampling and digitizing of input signal, second, removing the noise in sampled data by the low pass filter and third, making a convolution integral using the output data of low pass filters. This method showed a good experimental result of reducing dynamic error even if there was a slight noise in the input signal. As a result, the detecting time constant of resistance thermo-bulb was improved to about 1/10th.

1 Introduction

Dynamic error in a measurement has been a big problem in instrumentation and control engineering[1]. Recently practical research has been made in the fast estimation of the stationary value using data which is in a transient state. One example of this is the weight estimation method in a measurement using a spring balance and a Kalman filter[2]. Another example is the application of the weight estimation ideas of Ikeda and his "dynamic systems theory" in a measurement using a load cell[3].

On the other hand, many researches have been made on the dynamic error reduction of temperature sensors. The first idea ever for solving this problem was to use an inverse transfer-function as a compensating method for the dynamic error, i.e. a pole-zero canceler construction using a sensor and a compensator[4]. However, this method didn't have natural causality, and a connection between the input and the output in a state diagram[5]. We developed an analog dynamic error compensator using the approximate inverse transfer-function and the detecting time constant of a resistance thermo-bulb was improved by about 1/10th[6]. This method satisfied natural causality and the necessary condition of observability and controllability[7]. However, this method could not become popular because the analog compensator was found to be susceptible to noise. And the analog compensator was not only too bulky and expensive, but also unstable.

Therefore we attempted to reduce the dynamic error by using a digital convolution integrator[8]. But the bare digital compensator is susceptible to noise in input singnal. So we adopted the method in which the smoothing process is inserted before the compensating process. Therefore, new dynamic error compensator is constructed of five elements, an A-D converter, two low pass filters ,a digital compensator and a D-A converter.

2 Theory and Experiment

Fig 1 shows a general system for compensating the dynamic error of a sensor. $c_i(t)$ is the true values, in other words the physical quantity that is to be mesured. $e_d(t)$ is the output of the sensor. This involves dynamic error and is the input of the compensator, $e_o(t)$ is the output of the compensator, and $g_d(t), g_c(t)$ are the impulse response of the sensor and compensator respectively. $E_i(z), E_d(z), E_o(z), G_d(z)$ and $G_c(z)$ are the z transform of $e_i(t), e_d(t), e_o(t), g_d(t)$ and $g_c(t)$ respectively.

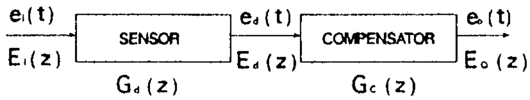

Fig.1 A block diagram of a general compensating system for an instrumentation with dynamic error.

The relationship between the input and the output of the sensor and compensator in Fig.1 are shown in equation(1) and (2).

$$e_d(t) = \int_0^t g_d(\tau) e_i(t-\tau)\, d\tau \qquad (1)$$

$$e_o(t) = \int_0^t g_c(\tau) e_d(t-\tau)\, d\tau \qquad (2)$$

These are the forms of the convolution integral. After this, we will treat this convolution intgral in the discrete time. The values of $e_i(t), e_d(t)$ and $e_o(t)$ are sampled at equal intervals of $\Delta\tau$. Therefore this convolution integral can be rewritten by z transform as follows,

$$\begin{aligned} E_d(z) &= G_d(z) \cdot E_i(z) \\ E_o(z) &= G_c(z) \cdot E_d(z) = G_c(z) \cdot G_d(z) \cdot E_i(z) \end{aligned} \qquad (3)$$

In this paper we are investigating the dynamic error compensation of a sensor that has the transfer characteristics of a first order lag type(T_d is a time constant) only.
When the input of sensor: $e_i(t)$ is a unit-step function, $e_i(t) = E_i$ for $t \geq 0$, then the output of sensor: $e_d(t)$ is $E_i(1 - exp(-t/T_d))$.

In the discrete time,

$$e_d(n) = E_i\{1 - \exp(-\frac{\Delta\tau}{T_d} n)\} \qquad (4)$$

By taking the z transform of equation(4) and evaluating the pulse transfer function for sensor.

$$\begin{aligned} E_d(z) &= \sum_{n=0}^{\infty} e_d(n) z^{-n} \\ &= \frac{E_i}{1 - z^{-1}} - \frac{E_i}{1 - \exp(-\Delta\tau/T_d) z^{-1}} \\ &= \frac{E_i}{1 - z^{-1}} \cdot \frac{1 - \exp(\Delta\tau/T_d)}{z^{-1} - \exp(\Delta\tau/T_d) \cdot z^{-1}} \cdot z^{-1} \end{aligned} \qquad (5)$$

In the equation(5), the first term $E_i/(1 - z^{-1})$ is the z transform of the input $e_i(t)$, then we can say that the second term is the inpulse transfer function of the sensor. We obtain

$$\begin{aligned} G_d(z) &= \frac{E_d(z)}{E_i(z)} \\ &= \frac{1 - \exp(\Delta\tau/T_d)}{z^{-1} - \exp(\Delta\tau/T_d) \cdot z^{-1}} \cdot z^{-1} \end{aligned} \qquad (6)$$

In case we choose the pulse transfer function of the compensator as follows,

$$G_c(z) = \frac{z^{-1} - \exp(\Delta\tau/T_d)}{1 - \exp(\Delta\tau/T_d)} \qquad (7)$$

By substituting $G_c(z)$ and $G_d(z)$ into equation(3), we could obtain the output of compensator as follows,

$$E_o(z) = E_i(z) \cdot z^{-1} \qquad (8)$$

Equation(8) shows that the output of the compensator is the same as $E_i(z)$ except a time lag for one sampling period. Therefore, the compensator with the pulse transfer function that expressed by equation(7), enables one to reduce the dynamic error of a sensor with first order lag. Fig.2 shows a basic dynamic error compensating system to which equation(7) is applied.

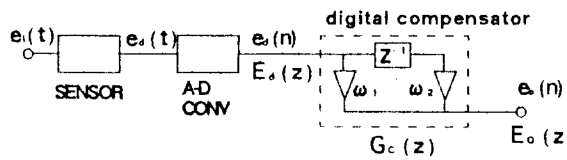

Fig.2 A basic compensating system for dynamic error.

The pulse transfer function of this basic digital compensator is as follows:

$$G_c(z) = w_1 + w_2 \cdot z^{-1} \qquad (9)$$

where,

$$w_1 = \frac{-\exp(\Delta\tau/T_d)}{1 - \exp(\Delta\tau/T_d)} \qquad w_2 = \frac{1}{1 - \exp(\Delta\tau/T_d)} \qquad (10)$$

But this bare digital compensating system has a serious weak point. It is susceptible to noise in the input signals. Therefore compensating process should be done after reducing the noise. So we adopted the method in which the smoothing process is inserted before the compensating process.

A new dynamic error compensating system using this method is shown in Fig.3. In this system, two low pass filters are added to the basic compensating system as shown in Fig.2. The distance between two low pass filters is m sampling periods. These m and α have a important role for total compensating characteristics. The output of this low pass filter is as follows:

$$\overline{e_d(n)} = \sum_{j=0}^{N-1} H_j e_d(n-j) \tag{11}$$

where, N is the length and H_j is the coefficient of digital low pass filter respectively.

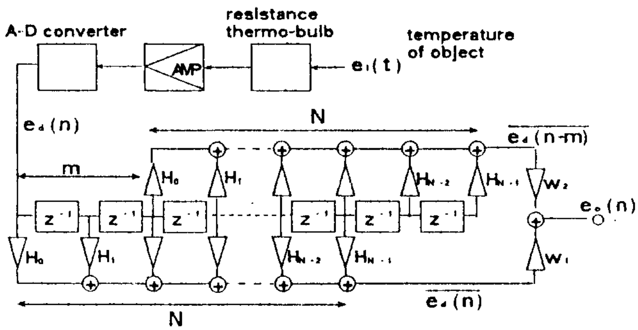

Fig.3 An overall diagram of new dynamic error compensating system.

In this new system, the input of digital compensator, $\overline{e_d(n)}$ and $\overline{e_d(n-m)}$ have been smoothed by low pass filters. Thus the effect of noise in $e_d(n)$ has been reduced.

The output of compensator, when the input of sensor: $e_i(t)$ is unit-step function, is as follows:

$$\begin{aligned} e_o(n) &= W_1 \cdot \overline{e_d(n)} + W_2 \cdot \overline{e_d(n-m)} \\ &= W_1 \sum_{j=0}^{N-1} H_j e_d(n-j) + W_2 \sum_{j=0}^{N-1} H_j e_d(n-m-j) \\ &= W_1 \sum_{j=0}^{N-1} H_j E_i\{1-\exp(-\frac{\Delta\tau(n-j)}{T_d})\} \\ &+ W_2 \sum_{j=0}^{N-1} H_j E_i\{1-\exp(-\frac{\Delta\tau(n-m-j)}{T_d})\} \\ &= E_i\{\sum_{j=0}^{N-1} H_j + F(N,m,\alpha)\exp(-n\frac{\Delta\tau}{T_d})\} \end{aligned} \tag{12}$$

where,

$F(N,m,\alpha)$

$$= \sum_{j=0}^{N-1} H_j \exp(\frac{\Delta\tau}{T_d}j) \cdot \frac{\exp(\alpha \cdot m\frac{\Delta\tau}{T_d}) - \exp(m\frac{\Delta\tau}{T_d})}{1-\exp(m\frac{\Delta\tau}{T_d})} \tag{13}$$

$$W_1 = \frac{-\exp(\alpha m\frac{\Delta\tau}{T_d})}{1-\exp(\alpha m\frac{\Delta\tau}{T_d})} \qquad W_2 = \frac{1}{1-\exp(\alpha m\frac{\Delta\tau}{T_d})} \tag{14}$$

where, α is a factor to decide a characteristic of the compensator.

In order that this system work as a compensator for first order lag type sensor, it must satisfy the following conditions:

$$\begin{aligned} \sum_{j=0}^{N-1} H_j &= 1 \\ \alpha &= 1.0 \end{aligned}$$

Fig.4 shows that the indicial response in the sensor with characteristic of ideal first order lag, and compensated ones using the new compensator for three values of α. In $\alpha > 1$, compensated result is characterized by an undershoot, and in $\alpha < 1$, compensated result has an overshoot.

On the other hand, the filter length N decides the immunity against noise and the rising time of the compensation. The larger the N, the better for the noise cancelling, but in this case we obtain a larger rising time. Therefore, we should choose the value of α, m and N, which gives the most desirable compensated result.

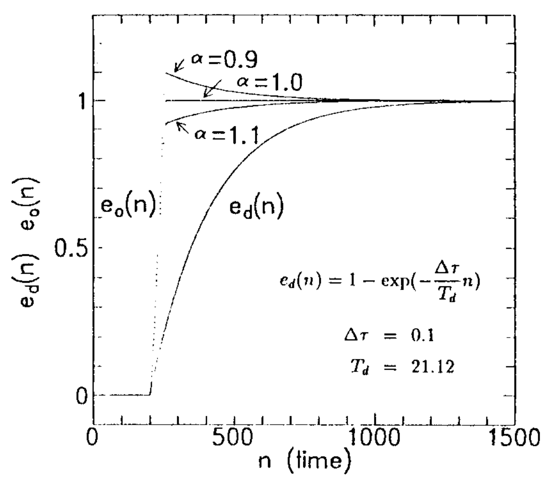

Fig.4 The compensation characteristics of new compensation method for ideal first order lag data. $N = 50, m = 10, H_j = 1/N = 0.02$

Fig.5 shows an overall diagram of the experimental system. The sensor is the resistance thermo-bulb(50 Ω at 0 ℃, Pt) with protective tube, and its time constant is 21.1 sec. The A-D converter's input voltage range is from 0 ~ 10v, and the output has 12bits digital value with sampling period $\Delta\tau = 0.1$ sec. In this experiment, we adopted the digital low pass filter which has same coefficient $(1/N)$, this makes simple moving average value. An abrupt change in temperature $(e_i(t))$ was made by quickly moving the resistance thermo-bulb from water(20 ℃) to water(80 ℃).

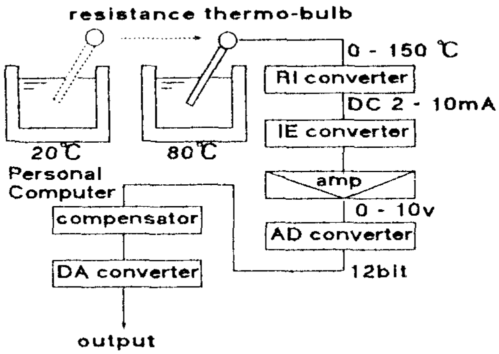

Fig.5 An overall diagram of the experimental system

The results, the measured $e_d(n)$ and compensated $e_o(n)$ are shown in Fig.6 for $N = 50$, $m = 10$, $\alpha = 0.9, 1.0$ and 1.1 respectively. This shows that the most desirable compensated result shoud be obtained in $\alpha = 1.1$.

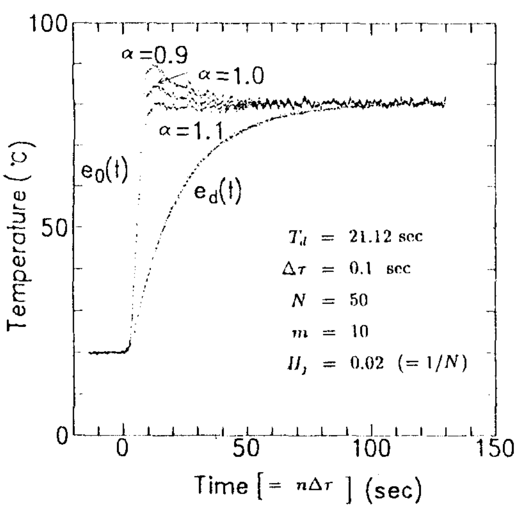

Fig.6 A result of the experiment.
($N = 50$, $m = 10$, $H_j = 0.02$).

3 Conclusion

A satisfactory dyanmic error reduction was obtained with the new method using a digital compensator and two low pass filters. By compensating, the time constant of the sensor was improved by about 1/10th even if there was a slight noise in signal. we believe this method is effective for dynamic error reduction.

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