System Model Reduction by Weighted Component Cost Analysis

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ABSTRACT

Component Cost Analysis considers any given system driven by a white noise process as an interconnection of different components, and assigns a metric called "component cost" to each component. These component costs measure the contribution of each component to a predefined quadratic cost function. One possible use of component costs is for model reduction by deleting those components that have the smallest component costs. The theory of Component Cost Analysis is extended to include finite-bandwidth colored noises. The results also apply when actuators have dynamics of their own. When the dynamics of this input are added to the plant, which is to be reduced by CCA, the algorithm for model reduction process will be called Weighted Component Cost Analysis (WCCA). Closed-form analytical expressions of component costs for continuous time case, are also derived for a mechanical system described by its modal data. This is very useful to compute the modal costs of very high order systems beyond Lyapunov solvable dimension. A numerical example for NASA's MINIMAST system is presented.

1. Introduction

There exist numerous schemes for model reduction. However, due to the requirement of many of these methods to solve Lyapunov equations these schemes are not applicable to the model reduction of large flexible space structures due to the large dimension of these models. Modal Cost Analysis (MCA) is one method which has been developed especially for such large scale systems. The MCA is a special case of Component Cost Analysis (CCA) [1,2]. CCA considers any given system driven by a white noise process as an interconnection of different components. The definition of these components is up to analyst ; they may have physical significance, or they may be defined for mathematical convenience. For example, in a multibody system, each body may be considered as a component and each component body may have several subcomponents. For any choice of components CCA assigns a metric called "component cost" to each component. These component costs measure the contribution of each component to a predefined quadratic cost function. A reduced-order model of the given system may be obtained by deleting those components that have the smallest component costs, although only special coordinates can offer any guarantees by this reduction.

In the theory of CCA, the input is assumed to be a white noise process. However, such infinite-bandwidth white noise processes do not exist in the real world. In fact, any real actuator and sensor devices can only have finite bandwidth. Furthermore, the drawbacks of the infinite-bandwidth assumption for white noise processes are evident in infinite dimensional systems, since the standard quadratic cost function is not finite in all cases (such as torque inputs and velocity outputs) [6]. To cope with this unrealistic situation we propose the practical approach of considering the dynamics of finite-bandwidth inputs. When the dynamics of this input are added to the plant, which is to be reduced by CCA, the algorithm for model reduction process will be called Weighted Component Cost Analysis (WCCA).

The purpose of this paper is to extend the theory of Component Cost Analysis when a linear system to be reduced is subjected to a finite-bandwidth colored noise which is modeled by linear dynamics. When a mechanical system is described by its modal data, each mode is considered as a component and analytical expressions of component costs (modal costs) will be derived for continuous time case. This analytical expression is very useful to compute the modal costs of very high-order systems since Lyapunov equations need not be computed.

This paper is organized as follows: section 2 reviews the theory of CCA and section 3 provides analytical expressions of modal costs when a mechanical system is driven by white noises. Section 4 develops the theory of Weighted Modal Cost Analysis (WMCA) for the system subjected to finite-bandwidth noises. A numerical example for NASA's MINIMAST system is presented in section 5.

2. Theory of Component Cost Analysis

Let a state space realization of a linear time-invariant system driven by zero mean white noise w with intensity W, be given as

$$\left.\begin{aligned} \dot{x} &= Ax + Dw \ , \quad x \in R^{n} \ , \quad w \in R^{n_w} \ , \\ y &= Cx \ , \quad y \in R^{n_y} \end{aligned}\right\} \tag{2.1}$$

where x and y are, respectively, state and output vectors. The component form may be written as follows:

$$\left.\begin{aligned} \dot{x}_i &= \sum_{j=1}^{N} A_{ij}x_j + D_i w \ , \\ y &= \sum_{i=1}^{N} C_i x_i \ , \quad \sum_{i=1}^{N} n_i = n \\ x_i &\in R^{n_i} \ , \quad i = 1, 2, \cdots, N \end{aligned}\right\} \tag{2.2}$$

where N is the number of components and the state vector x_i define the i-th component. Given the system (2.1) a simple quadratic cost function is defined by

$$V \triangleq E_\infty V(t) \quad , \quad V(t) \triangleq y(t)^T Q\, y(t) \tag{2.3}$$

where $E_\infty \triangleq \lim_{t\to\infty} E$ is the expectation operator and Q is a positive semi-definite output weighting matrix. Then, the component cost V_i associated with each component x_i is defined by

$$V_i \triangleq \frac{1}{2} E_\infty\left(\frac{\partial V(t)}{\partial x_i} x_i \right) \quad , \quad i = 1, 2, \cdots, N . \tag{2.4}$$

It can be shown [4] that V_i is calculated by the following formula :

$$V_i = \mathrm{tr}\, [XC^T QC]_{ii} \quad , \quad i = 1, 2 \cdots, N \tag{2.5a}$$

where tr is the matrix trace operator and the steady state covariance of the states X satisfies the Lyapunov equation :

$$0 = AX + XA^T + DWD^T . \tag{2.5b}$$

Clearly, since $V = \mathrm{tr}\,[XC^T QC]$, the component costs V_i satisfy the cost decomposition property :

$$V = \sum_{i=1}^{N} V_i . \tag{2.6}$$

Because of the property (2.6) component costs V_i in (2.5a) may be normalized as

$$\hat{V}_i = \frac{V_i}{V} \quad , \quad i = 1, 2, \cdots, N . \tag{2.7}$$

Then a reduced-order model of the system (2.1) may be obtained by deleting those components that have the smallest $\hat{V}_i$.

3. Analytical Expressions of Modal Costs

Usually the dynamics of large structures are modeled by their modal data extracted either by finite element analysis or by experiment. In this case components can be defined by natural frequencies and mode shapes, and hence each component has physical significance. If this is the case, it is possible to get an analytical expression for component costs V_i in (2.5), which we shall call modal costs.

Let a mechanical structure be described as

$$\left.\begin{aligned} &\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = d_i^T w \quad , \quad i = 1, 2, \cdots, N , \\ &y = \sum_{j=1}^{N} p_j\eta_j + \sum_{j=1}^{N} r_j\dot{\eta}_j \end{aligned}\right\} \tag{3.1}$$

where ω_i and ζ_i are, respectively, the natural frequency and damping ratio of mode i. Note that in (3.1) w(t) represents a zero mean white noise with intensity W. For the system (3.1), the explicit solution of the Lyapunov equation (2.4) is known [2,5] to be

$$X_{ij} = \frac{d_i^T W d_j}{\Delta_{ij}} \begin{bmatrix} (2\zeta_i\omega_i + 2\zeta_j\omega_j) & (\omega_i^2 - \omega_j^2) \\ -(\omega_i^2 - \omega_j^2) & \omega_i\omega_j(2\zeta_i\omega_j + 2\zeta_j\omega_i) \end{bmatrix} \tag{3.2}$$

where X_{ij} is the ij-(2×2) block of X in (2.5) with $x = [\eta_1, \dot{\eta}_1, \cdot, \eta_N, \dot{\eta}_N]^T$, and

$$\Delta_{ij} = \omega_i\omega_j(2\zeta_i\omega_i + 2\zeta_j\omega_j)(2\zeta_i\omega_j + 2\zeta_j\omega_i) + (\omega_i^2 - \omega_j^2)^2 . \tag{3.3}$$

Then the modal cost of the i-th mode can be obtained from (2.5a) :

$$V_i = \mathrm{tr}[\sum_{j=1}^{N} X_{ij}\, C_j^T Q C_i] \quad , \quad i = 1, 2, \cdots, N \tag{3.4a}$$

where

$$C_i = [p_i \;\; r_i] \quad \text{and} \quad A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} . \tag{3.4b}$$

Note that for (3.4) the i-th component is defined by $x_i = [\eta_i \;\; \dot{\eta}_i]^T$, i.e., each component consists of only one mode shape. Although it is a formidable task to calculate by (3.4) all V_i's for a large scale system, it is certainly easier than trying to solve the Lyapunov equation (2.5) numerically.

For a lightly damped structure the modal cost V_i of (3.4) can be approximated by setting $\zeta_i \approx 0$ for all i :

$$\begin{aligned} V_i \approx\; & \frac{d_i^T W d_i}{4\zeta_i\omega_i^3}(p_i^T Q p_i + \omega_i^2 r_i^T Q r_i) \\ & + \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{d_i^T W d_j}{\omega_i^2 - \omega_j^2}(p_i^T Q r_j - p_j^T Q r_i) . \end{aligned} \tag{3.5}$$

The approximate formula for MCA suggested by Skelton, et al [2] can be obtained by taking the first term from (3.5), or equivalently by assuming, for all i and j ≠ i, either $d_i^T W d_j = 0$ or $p_i^T Q r_j = 0$:

$$V_i \approx \frac{d_i^T W d_i}{4\zeta_i\omega_i^3}(p_i^T Q p_i + \omega_i^2 r_i^T Q r_i) . \tag{3.6}$$

4. Weighted Modal Cost Analysis

In the previous sections, we assumed that the input noise w is a white noise process. By considering the dynamics of finite-bandwidth actuators which drive the plant to be reduced, we will derive an MCA formula for more realistic cases. We shall call this Weighted Modal Cost Analysis (WMCA).

Let the plant be given by

$$\left.\begin{aligned} &\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = b_i^T u + d_i^T w_p \quad , \\ &y = \sum_{j=1}^{N} p_j\eta_j + \sum_{j=1}^{N} r_j\dot{\eta}_j \quad , \quad i = 1, 2, \cdots, N \end{aligned}\right\} \tag{4.1}$$

where w_p is an additional plant noise with intensity W_p and u is the actuator output signal which is now colored by the actuator dynamics given by

$$\left.\begin{aligned} &\dot{x}_a = A_a x_a + D_a w_a \quad , \quad x_a \in R^m , \\ &u = C_a x_a + H_a w_a \end{aligned}\right\} \tag{4.2}$$

where w_a is a zero mean white noise with intensity W_a. A state space description of the combined system (4.1) and (4.2) is obtained as

$$\left.\begin{aligned} &\dot{\lambda} = \Theta\,\lambda + \delta\,\omega , \\ &y = \Gamma\,\lambda \end{aligned}\right\} \tag{4.3a}$$

where

$$\lambda^T = [\ \eta_1,\ \dot{\eta}_1,\ \cdots\ \eta_N,\ \dot{\eta}_N,\ x_a^T\ ]\ ,\quad \omega^T = [\ w_p^T\ ,\ w_a^T\ ]$$ and

$$\theta = \begin{bmatrix} A_p & A_2 \\ 0 & A_a \end{bmatrix},\ \delta = \begin{bmatrix} D_p & D_2 \\ 0 & D_a \end{bmatrix},\ \Gamma = [\ C_p\ ,\ 0\ ]\ , \quad (4.3b)$$

$$A_p = \text{block diag}\left[\cdots\begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}\cdots\right]\ , \quad (4.3c)$$

$$A_2 = \begin{bmatrix} \vdots \\ \begin{bmatrix} 0 \\ b_i^T C_a \end{bmatrix} \\ \vdots \end{bmatrix},\ D_p = \begin{bmatrix} \vdots \\ \begin{bmatrix} 0 \\ d_i^T \end{bmatrix} \\ \vdots \end{bmatrix},\ D_2 = \begin{bmatrix} \vdots \\ \begin{bmatrix} 0 \\ b_i^T H_a \end{bmatrix} \\ \vdots \end{bmatrix}, \quad (4.3d)$$

$$C_p = [\cdots[\ p_i\ ,\ q_i\ ]\cdots]\ . \quad (4.3e)$$

Note that by setting $w_p = 0$ we have only the colored noise input u. Now that the system (4.3) is in the standard form of a linear time-invariant system driven by white noise, its steady-state covariance matrix satisfies the following equation :

$$0 = \theta\,\Xi + \Xi\,\theta^T + \delta\,\Theta\,\delta^T \quad (4.4a)$$

where

$$\Theta = \begin{bmatrix} W_p & 0 \\ 0 & W_a \end{bmatrix}. \quad (4.4b)$$

Let

$$\Xi = \begin{bmatrix} X_p & X_2 \\ X_2^T & X_a \end{bmatrix}\ . \quad (4.5)$$

Then (4.4) can be partitioned into 3 equations :

$$0 = A_a X_a + X_a A_a^T + D_a W_a D_a^T \quad (4.6a)$$

$$0 = A_p X_2 + X_2 A_a^T + A_2 X_a + D_2 W_a D_a^T \quad (4.6b)$$

$$0 = A_p X_p + X_p A_p^T + D_p W_p D_p^T + A_2 X_2^T + X_2 A_2^T + D_2 W_a D_2^T. \quad (4.6c)$$

Since the number of actuators is usually relatively small, the solution of (4.6a) can be easily obtained by any numerical method. However, for completeness, we assume here that actuator dynamics is described in 2nd order modal coordinates (instead of in state space form), and derive an analytical expression for X_a as follows : let actuators be represented by

$$\left.\begin{aligned} &\ddot{\eta}_{a_i} + 2\zeta_{a_i}\omega_{a_i}\dot{\eta}_{a_i} + \omega_{a_i}^2\eta_{a_i} = d_{a_i}^T w_a\ , \\ &u = \sum_{j=1}^{M} p_{a_j}\eta_{a_j} + \sum_{j=1}^{M} r_{a_j}\dot{\eta}_{a_j} + H_a w_a\ , \\ &i = 1,\ 2,\ \cdots\ ,\ M\ . \end{aligned}\right\} \quad (4.7)$$

For the state space form (4.2) we have $x_a^T = [\ \eta_{a_1},\ \dot{\eta}_{a_1},\ \ldots,\ \eta_{a_M},\ \dot{\eta}_{a_M}]$ and

$$A_a = \text{block diag}\left[\cdots\begin{bmatrix} 0 & 1 \\ -\omega_{a_i}^2 & -2\zeta_{a_i}\omega_{a_i} \end{bmatrix}\cdots\right]\ , \quad (4.8a)$$

$$D_a = \begin{bmatrix} \vdots \\ \begin{bmatrix} 0 \\ d_{a_i}^T \end{bmatrix} \\ \vdots \end{bmatrix}\ ,\quad C_a = [\cdots[p_{a_i}\quad r_{a_i}]\cdots]\ . \quad (4.8b)$$

In the same manner as in section 3, we get the Lyapunov solution for (4.6a) :

$$X_{a_{ij}} = \frac{d_{a_i}^T W_a d_{a_j}}{\Delta_{a_{ij}}}\begin{bmatrix} (2\zeta_{a_i}\omega_{a_i} + 2\zeta_{a_j}\omega_{a_j}) & (\omega_{a_i}^2 - \omega_{a_j}^2) \\ -(\omega_{a_i}^2 - \omega_{a_j}^2) & \omega_{a_i}\omega_{a_j}(2\zeta_{a_i}\omega_{a_j} + 2\zeta_{a_j}\omega_{a_i}) \end{bmatrix} \quad (4.9a)$$

where

$$\Delta_{a_{ij}} = \omega_{a_i}\omega_{a_j}(2\zeta_{a_i}\omega_{a_i} + 2\zeta_{a_j}\omega_{a_j})(2\zeta_{a_i}\omega_{a_j} + 2\zeta_{a_j}\omega_{a_i}) + (\omega_{a_i}^2 - \omega_{a_j}^2)^2\ . \quad (4.9b)$$

Once X_a for (4.6a) is known, (4.6b) can be analytically solved due to the special structure of A_p, A_2 and D_2 of (4.4). Let

$$X_2^T \equiv [\cdots[\alpha_i\ ,\ \beta_i]\cdots]\ . \quad (4.10)$$

Then the solution of (4.6b) is given by

$$\alpha_i = [A_a^2 - 2\zeta_i\omega_i A_a + \omega_i^2 I_a]^{-1}(X_a C_a^T + D_a W_a H_a^T)b_i\ , \quad (4.11a)$$

$$\beta_i = -A_a\alpha_i\ ,\quad i = 1,\ 2,\ \ldots,\ N \quad (4.11b)$$

where I_a is an identity matrix of size of A_a. Finally for (4.6c), consider the ij-(2×2) block and let

$$[X_p]_{ij} \equiv \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ X_{ij}^{21} & X_{ij}^{22} \end{bmatrix}\ . \quad (4.12a)$$

After some algebraic manipulation, we have the solution of (4.6c) :

$$\begin{aligned} X_{ij}^{11} = \frac{1}{\Delta_{ij}}[\ &(2\zeta_i\omega_i + 2\zeta_j\omega_j)(2\zeta_i\omega_i b_j^T C_a\alpha_i + 2\zeta_j\omega_j b_i^T C_a\alpha_j \\ &+ d_i^T W_p d_j + b_i^T H_a W_a H_a^T b_j - b_j^T C_a A_a\alpha_j - b_j^T C_a A_a\alpha_i) \\ &+ (\omega_i^2 - \omega_j^2)(b_i^T C_a\alpha_j - b_j^T C_a\alpha_i)] \end{aligned} \quad (4.12b)$$

$$\begin{aligned} X_{ij}^{12} = \frac{1}{\Delta_{ij}}[\ &(2\zeta_i\omega_i + 2\zeta_j\omega_j)(\omega_i^2 b_j^T C_a\alpha_i - \omega_j^2 b_i^T C_a\alpha_j) \\ &+ (\omega_i^2 - \omega_j^2)(d_i^T W_p d_j + b_i^T H_a W_a H_a^T b_j \\ &- b_i^T C_a A_a\alpha_j - b_j^T C_a A_a\alpha_i)] \end{aligned} \quad (4.12c)$$

$$\begin{aligned} X_{ij}^{22} = \frac{1}{\Delta_{ij}}[\ &\omega_i\omega_j(2\zeta_i\omega_j + 2\zeta_j\omega_i)(d_i^T W_p d_j + b_i^T H_a W_a H_a^T b_j \\ &- b_i^T C_a A_a\alpha_j - b_j^T C_a A_a\alpha_i) \\ &- (\omega_i^2 - \omega_j^2)(\omega_i^2 b_j^T C_a\alpha_i - \omega_j^2 b_i^T C_a\alpha_j)] \end{aligned} \quad (4.12d)$$

$$X_{ij}^{21} = -X_{ij}^{12} \quad (4.12e)$$

where Δ_{ij} and α_i are given by (3.3) and (4.11a), respectively. Now having the explicit solution given by (4.9), (4.11) and (4.12) for (4.5), we define the cost function as given in (2.3) and get the analytical expression of modal costs for the plant ;

$$\begin{aligned} V_i &= \mathrm{tr}[\sum_{j=1}^{N}[X_p]_{ij} C_j^T Q C_i] \\ &= \sum_{j=1}^{N}[X_{ij}^{11} p_i^T Q p_j + X_{ij}^{22} r_i^T Q r_j + X_{ij}^{12}(p_i^T Q r_j - p_j^T Q r_i)]\ . \end{aligned} \quad (4.13)$$

As we can see in (4.12), the $[X_p]_{ij}$ are weighted by actuator parameters and so V_i in (4.13) are called Weighted Modal Costs. Notice that by setting $b_i = 0$ for all i and $W = W_p$, (4.12) leads to (3.2), which is for the standard white noise input case. As an approximation we take only the j = i term from (4.13) as we did for MCA (this is justified when all ζ_k are small and

$d_i^T W d_j = 0$, or $p_i^T Q r_j = 0$):

$$V_i \approx \frac{p_i^T Q p_i + \omega_i^2 r_i^T Q r_i}{4\zeta_i \omega_i^3} \{ d_i^T W_p d_i + b_i^T H_a W_a H_a^T b_i - 2 b_i^T C_a A_a [A_a^2 - 2\zeta_i \omega_i A_a + \omega_i^2 I_a]^{-1} (X_a C_a^T + D_a W_a H_a^T) b_i \} + \frac{p_i^T Q p_i}{\omega_i^2} b_i^T C_a [A_a^2 - 2\zeta_i \omega_i A_a + \omega_i^2 I_a]^{-1} (X_a C_a^T + D_a W_a H_a^T) b_i . \quad (4.14)$$

Notice again that by setting $b_i = 0$ for all i and $W = W_p$, (4.14) leads to the standard formula for approximate MCA, V_i in (3.6).

5. Application : MINIMAST

The MINIMAST considered here is schematically represented by Figure 1. From a finite element model we have the following data :

$$\ddot{\eta}_i + 2\zeta_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = b_i^T u + d_i^T w_p ,$$
$$y = \sum_{j=1}^{149} p_i \eta_i , \quad i = 1, 2, \cdots, 149 , \quad (5.1)$$

where w_p is the noise input with intensity $W_p = 1976.5\ I_3\ (\text{Newton})^2$ from the shakers located at 3 corners of Bay 9 . The natural frequencies and damping ratios of the MINIMAST structure are shown in Figure 2. The description of some global modes is given in Table 1. Damping ratios are obtained by the Rayleigh model: $2\zeta_i \omega_i = \alpha + \beta \omega_i^2$, $i = 5, 6, \cdots, 149$, $\zeta_5 = 0.01194$ and $\zeta_{149} = 0.05$. There are three noisy Torque Wheel Actuators (TWA) on the Tip Plate at Bay 18. Each TWA is modeled as

$$\dot{x}_a = A_a x_a + B_a u_a + D_a w_a ,$$
$$u = C_a x_a + H_a w_a \quad (5.2)$$

where u_a is the command signal to TWA which we shall set to zero, and w_a is a white noise with intensity $W_a = 1.8382\ (\text{Newton-Meter})^2$. (For complete system data for MINIMAST, see [6,7].) Selected outputs are the translational displacements of 3 corners and the centroidal rotations at Bay 10, 14 and 18. For these selected outputs, $r_i = 0$ for all i (no velocity or acceleration outputs). The following modal costs are given:

$$V_i = \sum_{j=1}^{149} X_{ij}^{11} p_i^T Q p_j , \quad i = 1, 2, \cdots, 149 \quad (5.3)$$

where X_{ij}^{11} is the (1,1) element of (3.2) for MCA and is given in (4.12a) for the weighted MCA. Since $r_i = 0$ for all i, the approximate modal costs (3.5) and (3.6) are exactly the same. The approximate (unweighted) modal costs are

$$V_i \approx \frac{(d_i^T W d_i)(p_i^T Q p_i)}{4\zeta_i \omega_i^3} , \quad i = 1, 2, \cdots, 149, \quad (5.4)$$

where we use $W = W_p$ or $W = W_u$ (= intensity of u as a white noise) if either w_p or u is considered as a white noise input. For the approximate weighted modal costs, (4.14) with $r_i = 0$ for all i will be used.

The normalized modal costs of the 50 highest-ranked modes are given in Figure 3. Similar plots are shown in Figure 4 when the actuator dynamics are included (hence WMCA). Table 2 shows the corresponding rankings of modes. From Figures 3, 4 and Table 2, one should notice that the cost rankings obtained by the exact expressions (4.4) and (4.13) are quite different from those by the approximate expressions (4.5) and (4.14), even with fairly small damping (1 to 5 %). One of the main reasons for this is that MINIMAST has a dense frequency spectrum(see Figure 2 and Equations (3.4), (3.5) and (4.13)). Notice also that the 5 highest-ranked modes give the same normalized modal costs with exact and approximate expressions. These 5 modes are 4 bending and 1 torsion modes (see Table 1). Based on these costs and rankings given in Figures 3, 4 and Table 2, six reduced-order models are obtained by retaining the highest-ranked modes in each case. Four cases are generated by only w_a inputs : the exact and the approximate of both the weighted and unweighted MCA. The remaining two cases are the exact and approximate MCA with only the w_p inputs. Observe, from Tables 1 and 2, that more global modes (e.g., modes 121, 122, 128 and 129) will be retained in a low-order reduced model when w_a is used as an input noise than when w_p is used.

The output covariance errors are calculated for each of the six cases to evaluate each reduced-order model. Figures 5 and 6 show the relative covariance errors of rotations at Bay 10. First of all, as expected in the cost analysis, the errors of those reduced-order models by exact analysis (either MCA or WMCA) are quite smaller than those by approximation, except for low-order models (less than 8 modes). Figure 5 indicates that when the MINIMAST is subjected to the shaker noise, w_p, we need more modes to get the "Relative Error" down to a small number. On the other hand, when the system is subjected to the TWA noise(w_a), we can get slightly improved reduced-order models if we use the weighted MCA instead of the MCA (see the first plots of Figures 5 and 6). In general, when different input sources are used for MCA, we shall have different reduced-order models. If there are different sets of input sources (e.g. actuator noises and shaker noises in MINIMAST), we recommend performing as many cost analyses as input sets and to take union of sets of the highest-ranked modes in order to get a reduced-order model which is "good" with respect to an overall performance.

7 Conclusion

This paper presents several new results. First, the expressions for modal costs are in explicit closed form. Secondly, frequency weighting has been added to include the case when the inputs are colored noises instead of white noises. These expressions are also in explicit closed form. The final contribution is to apply the theory to a large physical system, NASA's MINIMAST, with real (and therefore finite bandwidth) actuators. Our analysis was based upon a finite element model supplied by NASA. It is shown in [6,7] that these models are useful for control design. The advantage of the exact closed-form expressions is that previous approximate closed-form expressions were small damping approximations. But the system damping might not be small and this section shows that large errors in the reduced-order models may arise from the use of the standard (small damping) modal cost analysis. Also previous theory could not treat the weighted case (e.g. with actuator dynamics), without having to resort to numerical approaches of component cost analysis, requiring the solution of Lyapunov equations. For large scale systems (such as the MINIMAST example in this paper) this would have been impossible with present day computers. Our closed-form results open up the application of model reduction practice to large scale systems beyond "Lyapunov solvable" dimension. In fact these modal cost formulas can be applied to a structural system as large as the finite element code can compute modal data. In the future the inclusion of modal cost analysis into the finite element codes (e.g. NASTRAN) seems desirable.

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Table 1. Description of Some Global Modes

Mode	Description
1	First Bending
2	First Bending
3	First Torsion
4	Second Bending
5	Second Bending
117	Tip Plate
118	Second Torsion
119	Tip Plate with 3rd Bending
120	Tip Plate with 3rd Bending
121	Third Bending
122	Third Bending
123	Mid Plate
124	Tip Plate with 3rd Torsion
127	Third Torsion
128	Fourth Bending
129	Fourth Bending
130	Tip Plate with 4th Torsion
131	Fourth Torsion
136	Tip Plate with 4th Bending
140	Tip Plate with some Torsion
others	Local Modes

Table 2. Rankings of Modes

Ranking	Input Noise w_s				w_p	
	MCA		WMCA		MCA	
	Exact	Appro.	Exact	Appro.	Exact	Appro.
1	3	3	3	3	2	2
2	1	1	1	1	1	1
3	2	2	2	2	5	5
4	4	4	4	4	4	4
5	5	5	5	5	3	3
6	118	130	118	130	118	118
7	130	118	130	118	34	34
8	34	131	34	131	31	31
9	117	117	117	117	32	130
10	128	128	128	128	30	32
11	131	34	31	34	35	131
12	31	140	131	140	39	30
13	32	119	32	119	130	119
14	119	120	119	120	131	39
15	144	144	144	144	47	144
16	120	121	120	121	56	35
17	30	129	30	129	17	117
18	121	32	121	32	119	116
19	129	31	129	31	33	17
20	35	141	35	141	40	115
21	140	127	140	127	117	121
22	141	39	141	39	29	129
23	39	148	39	148	49	122
24	148	137	127	137	52	128
25	127	136	148	136	21	56
26	56	122	56	122	58	127
27	122	6	122	6	65	47
28	47	30	47	30	43	9
29	124	124	124	124	37	13
30	137	146	137	146	76	120
31	17	115	17	115	144	15
32	40	35	40	35	73	40
33	33	47	33	47	28	76
34	29	8	29	8	54	16
35	6	145	6	145	63	33
36	136	134	136	134	70	21
37	52	56	52	56	59	10
38	76	17	76	17	50	137
39	58	139	58	139	68	145
40	65	116	65	116	77	14
41	49	135	49	123	27	65
42	133	123	73	135	36	73
43	73	38	133	38	116	58
44	21	73	21	73	24	49
45	63	7	63	7	115	77
46	28	76	28	76	15	6
47	115	133	68	133	41	52
48	68	58	70	58	48	148
49	70	106	115	106	38	70
50	43	52	43	52	13	68

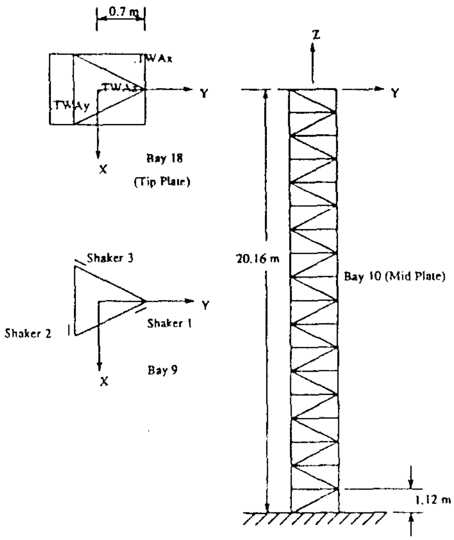

Figure 1. MINIMAST Configuration

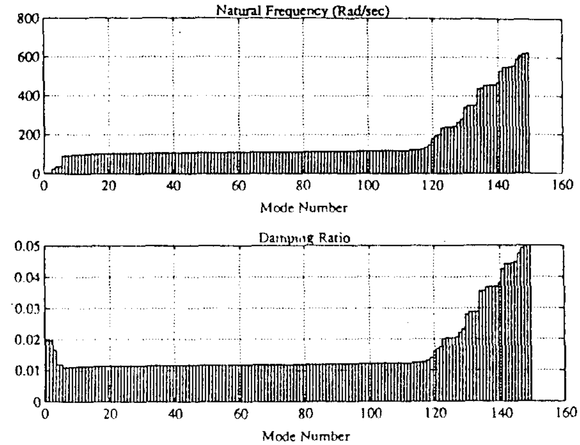

Figure 2. Natural Frequencies and Damping Ratio of MINIMAST

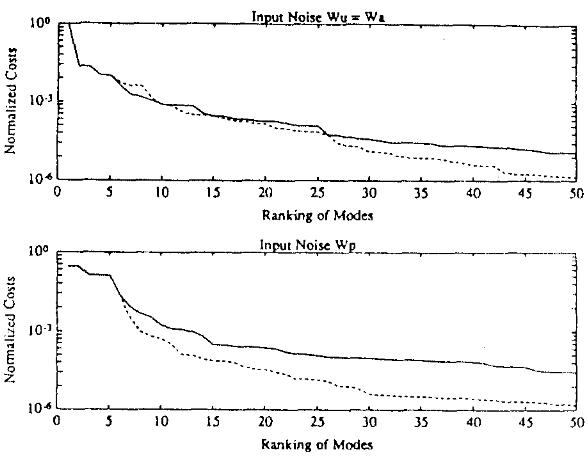

Figure 3. Ranked Modal Costs by MCA

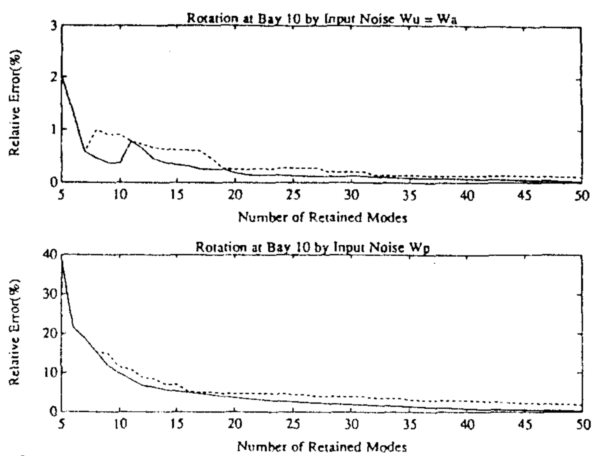

Figure 5. Output Covariance Error by MCA

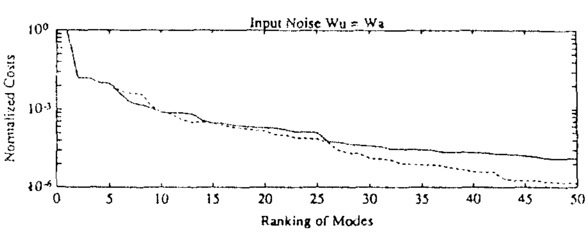

Figure 4. Ranked Modal Costs by Weighted MCA (WMCA)

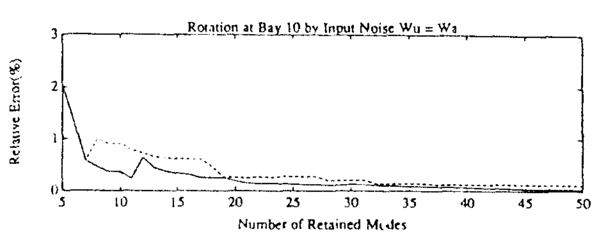

Figure 6. Output Covariance Error by Weighted MCA

solid ; Exact
dotted ; Approximate