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Variable Structure Model Folloing Control for Robot Manipulator using Time-varying Sliding Hyperplane

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Abstract

In this paper, the variable structure model following control scheme is proposed for the nonlinear robot manipulator system. The proposed control system guarantees that the system state is in the sliding mode for all time t. Therefore, error transient can be prescribed in advance for all time. Furthermore, overall system is globally exponentially stable. Chattering problem is reduced by the introduction of a boundary layer. Simulation results are given to show the usefulness of the proposed control scheme.

1 Introduction

Model following adaptive control(MFAC) methodologies have recently received great attention in the robot manipulator control design [1]-[4]. These existing designs are based on one of the following adaptive design methodologies for linear plants with unknown parameters. They are the design methods based on hyper-stability theory, the Lyapunov stability methods, and the self-tuning regulator type techniques. But the strict positive realness is invariably required. Furthermore, some of them can only guarantee the error between the states of the model and those of the controlled plant going to zero, the transient behavior of this error is not presented. In addition, model following error goes to zero asymptotically, i.e., we could not say about how fast tracking error decreases.

The advantage of the model following variable structure control lies in its ability to prescribe transient response requirements as well as providing a robust controller [7]-[9]. In the Leung's paper [5], the adaptive variable structure model following control (AVSMFC) design was proposed for accomplishing trajectory tracking in a nonlinear robot system which ensures the stability of the intersection of the surfaces without necessarily stabilizing each individual one. The proposed approach avoided the difficulties linked to the strict positive realness requirment in traditional MFAC by taking advantage of the inherent positive definiteness of manipulators inertia matrix, and is easily extendable to a higher number of links. But the control methodology didn't guarantee that the system states are on the sliding mode from the initial time in the case that the initial conditions for mode and plant are exactly matched. The each sliding functions s_i has gone away from zero as shown in the presented simulation results in [5].

In this paper, variable structure model following control system is considered for accomplishing trajectory tracking control in a nonlinear robot system. The proposed control law guarantees that the system states are on the sliding mode for all time. Therefore, error transient can be prescribed in advance for all time. Furthermore, it is shown that the overall system is globally exponentially stable. Chattering is eliminated by restricting the state of the system to slide within a boundary layer.

2 Modeling for Robot Manipulators

The dynamic equations of motion for a general rigid link manipulator having n degree of freedom can be described as follows:

$$M(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q) = u(t) \tag{1}$$

where $q \in \Re^n$ is joint angles, $u \in \Re^n$ is applied joint torque, $M(q) = M^T(q) > 0$, $M(q) \in \Re^{n\times n}$ is the inertia matrix, $F(q,\dot{q})\dot{q} \in \Re^n$ is the centrifugal and Coriolis torques, and $G(q) \in \Re^n$ is the gravitational torque.

Defining $x \in \Re^{2n}$ to be the vector

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix},$$

equation (1) can be written in a state variable form

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \dot{q} \\ M(q)^{-1}(q)(-F(q,\dot{q})\dot{q} - G(q)) \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix} u \qquad (2) \\ &= \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u \qquad (3) \\ &= Ax + Bu. \qquad (4) \end{aligned}$$

And the reference model is of the form

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} &= \begin{bmatrix} 0 & I \\ A_{m1} & A_{m2} \end{bmatrix}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} + \begin{bmatrix} 0 \\ B_{m1} \end{bmatrix} r \qquad (5) \\ &= A_m x_m + B_m r \qquad (6) \end{aligned}$$

where $x_m = (q_m^T \;\; \dot{q}_m^T)^T \in \Re^{2n}$, $A_m \in \Re^{2n\times 2n}$, $B_m \in \Re^{2n\times n}$ are constant matrices, $r \in \Re^n$ is an external input vector.

3 Controller Design

Define the sliding surface $s \in \Re^n$ as a hyperplane

$$s = G\bar{x} - N(t) = G_1 e + \dot{e} - N(t) = 0 \qquad (7)$$

where $\bar{x} = x_m - x \in \Re^{2n}$ is tracking error vector, $G \in \Re^{n\times 2n}$ is a constant matrix and will be determined as follows.

Let $\bar{x} = (e^T \;\; \dot{e}^T)^T$, $e = q_m - q$, $\dot{e} = \dot{q}_m - \dot{q}$, then the sliding hyperplane becomes

$$\begin{aligned} s &= G\bar{x} - N(t) = G_1 e + \dot{e} - N(t), \\ G_1 &= diag\{g_1, \cdots, g_n\}, \\ N_i(t) &= (g_i e(0) + \dot{e}(0))e^{-\lambda_i t}, \end{aligned}$$

where, $g_i, \lambda_i > 0$ for all $i = 1, 2, \cdots, n$. If the sliding mode exists on $s = 0$, then from the theory of variable structure system, the sliding mode is governed by the following linear differential equationn whose behaviour is dictated by the sliding hyperplane design martix G_1 and $N(t)$

$$\dot{e} + G_1 e = N(t). \qquad (8)$$

In order to derive the control law, the following assumptions are required.

Assumption 1:

$$(I - BB^+)B_m = 0 \qquad (9)$$

$$(I - BB^+)(A_m - A) = 0 \qquad (10)$$

$$(I - BB^+)(A_m + A_n) = 0 \qquad (11)$$

where

$$A_n = \begin{bmatrix} 0 & -I \\ G_1 & (I + G_1) \end{bmatrix}$$

and B^+ is pseudo inverse of B given by $B^+ = [0 \;\; B_1^{-1}]$, where B_1^{-1} always exists because B_1^{-1} is inertia matrix $M(q)$.

Remark: We refer to equation (9)-(11) as matching conditions, which can always be satisfied due to the special structures of B, B_m, A, A_m, A_n, and B^+.

Define the control law of the following form

$$\begin{aligned} u &= K_1 x + K_2 r + K_3\bar{x} + K_4 s + K_5\Lambda N \\ &\quad -(\alpha_1\|x\| + \alpha_2\|r\| + \alpha_3\|\bar{x}\| + \alpha_4\|s\| \\ &\quad +\alpha_5\Lambda|N| + \eta)sgn(s) \qquad (12) \end{aligned}$$

where $K_1, K_3 \in \Re^{n\times 2n}$, $K_2, K_4, K_5 \in \Re^{n\times n}$, $K_3 \in \Re^{n\times 2n}$ are constant matrices, $\eta = [\eta_1, \eta_2, \cdots, \eta_n]^T$, $\eta_i > 0$ for $i = 0, 1, \cdots, n$, $|N| = [|N_1|, |N_2|, \cdots, |N_n|]^T$, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, and

$$sgn(s) = \begin{cases} 1 & \text{if } s > 0 \\ 0 & \text{if } s = 0 \\ -1 & \text{if } s < 0 \end{cases}.$$

Differentiating s in the equation (7) with respect to time gives

$$\begin{aligned} \dot{s} &= G\dot{\bar{x}} + \Lambda N(t) \\ &= G(A_m x_m + B_m r - Ax - Bu) + \Lambda N(t) \\ &= G[A_n(x - x_m) + (A_m + A_n)x_m + B_m r \\ &\quad -(A + A_n)x - Bu] + \Lambda N(t) \\ &= -s + GB\left[B^+(A_m - A)x + B^+B_m r \right. \\ &\quad \left. +B^+(A_m + A_n)\bar{x} - u\right] + \Lambda N(t) \end{aligned}$$

and then, multiply both sides by B_1^{-1} and insert the control law. Then, we obtain the following equation.

$$\begin{aligned} B_1^{-1}\dot{s} &= -B_1^{-1}s + B_1^{-1}GB\left[(B^+(A_m - A) - K_1)x \right. \\ &\quad +(B^+B_m - K_2)r + (B^+(A_m + A_n) - K_3)e \\ &\quad -K_4 s - K_5\Lambda N \end{aligned}$$

$$-(\alpha_1\|x\| + \alpha_2\|r\| + \alpha_3\|\bar{x}\| + \alpha_4\|s\| + \alpha_5\Lambda|N| + \eta)sgn(s)] + B_1^{-1}\Lambda N. \quad (13)$$

Here, we assume that the following condition is satisfied.

Assumption 2:

$$\|B^+(A_m - A) - K_1\| < \alpha_1$$
$$\|B^+B_m - K_2\| < \alpha_2$$
$$\|B^+(A_m + A_n) - K_3\| < \alpha_3$$
$$\|B_1^{-1}(I + A_2) + K_4\| < \alpha_4$$
$$\|B_1^{-1} - K_5\| < \alpha_5.$$

Remark: Due to the mechanical characteristics of robotic manipulators and the boundness of the reference model, the assumption is valid [5].

Then, we can derive the following theorem.

Lemma 1 *For the robot manipulator plant (4), and model (6), and the control law (12), the sliding mode exists from a given initial state.*

Proof

Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}s^T B_1^{-1} s. \quad (14)$$

Differentiating V with respect to time t and using equation (13), we can obtain the following equality.

$$\begin{aligned}\dot{V} &= s^T\left[B_1^{-1}\dot{s} - B_1^{-1}A_2 s\right] \\ &= s^T\left[(B^+(A_m - A) - K_1)x + (B^+B_m - K_2)r\right. \\ &\quad + (B^+(A_m + A_n) - K_3)e - (B_1^{-1}(I + A_2) + K_4)s \\ &\quad + (B_1^{-1} - K_5)\Lambda N - (\alpha_1\|x\| + \alpha_2\|r\| + \alpha_3\|w\| \\ &\quad \left. + \alpha_4\|s\| + \alpha_5\Lambda|N| + \eta)sgn(s)\right] + B_1^{-1}\Lambda N.\end{aligned}$$

Because of Assumption 2, the following inequality can be derived.

$$\dot{V} \le -\sum_{i=1}^{n}\eta_i|s_i| < 0. \quad (15)$$

Therefore, V is really a Lyapunov function. And the Lyapunov function $V(t)$ is equal to zero for all time because $V > 0$, $V = 0$ is true only for $s = 0$ and $\dot{V} \le 0$, $\dot{V} = 0$ is true only for $s = 0$, and $V(0) = 0$. This also implies that

$$s = 0 \qquad \forall t \ge 0 \quad (16)$$

Thus, the system state is forced to stay in the sliding mode from a given initial state.

□

Remark: The evolution of the i-th joint's model tracking error, $e_i(t)$, can be predicted as

$$e_i(t) = \begin{cases} \frac{1}{k_i - \lambda_i}[(\dot{e}_i(0) + k_i e_i(0))e^{-\lambda_i t} \\ \quad -(\dot{e}_i(0) + \lambda_i e_i(0))e^{-k_i t}] & \text{if } k_i \ne \lambda_i, \\ e_i(0)e^{-\lambda_i t} + (\dot{e}_i(0) + \lambda_i e_i(0))te^{-\lambda_i t} & \text{if } k_i = \lambda_i, \end{cases} \quad (17)$$

for all time $t \ge 0$.

Proof

Using equation (16), we can obtain the following equation.

$$s_i(t) = \dot{e}_i(t) + \lambda_i e_i(t) - (\dot{e}_i(0) + \lambda_i e_i(0))e^{-k_i t} = 0 \qquad \forall t \ge 0$$

where $i = 1, 2, \cdots, n$. By solving the above differential equation, we can easily conclude that the tracking error is given by equation (17). Since the detail derivation is somewhat tedious, we omit the details.

□

From the above lemma, we can conclude that the time history of the tracking error for each joint can be predicted completely for all time and they are decoupled each other. Therefore, we can derive the following theorem.

Theorem 1 *For the robot manipulator (1) and the control law (12), the overall system is globally exponentially stable.*

Proof

Let's choose the constants A_i, B_i as follows:

$$\begin{aligned} A_i &= \frac{1}{|k_i - \lambda_i|}\left[\ |\dot{e}_i(0) + k_i e_i(0)| + |\dot{e}_i(0) + \lambda_i e_i(0)|\ \right] \\ B_i &= \min\{\lambda_i, k_i\} \end{aligned}$$

where $i = 1, 2, \cdots, n$. Then from equation (17), it is obvious that the following inequality is guaranteed for all i,

$$|e_i(t)| \le A_i e^{-B_i t}, \qquad \forall t \ge 0.$$

Therefore, the overall system is globally exponentially stable.

□

4 Simulation Results

A 2-link robotic manipulator model used by Young [9] was used for the simulation. Figure 1 shows the manipulator. The dynamic equation is given by

$$M(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q) = u$$

where $q = [q_1\ q_2]^T$,

$$\begin{aligned}
M_{11} &= (m_1+m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2\cos q_2 + J_1 \\
M_{12} &= M_{21} = m_2r_2^2 + m_2r_1r_2\cos q_2 \\
M_{22} &= m_2r_2^2 + J_2 \\
F_{11} &= -2m_2r_1r_2\dot{q}_2\sin q_2 \\
F_{12} &= -m_2r_1r_2\dot{q}_2\sin q_2 \\
F_{21} &= m_2r_1r_2\dot{q}_1\sin q_2 \\
F_{22} &= 0 \\
G_1 &= \{(m_1+m_2)r_1\cos q_1 + m_2r_2\cos(q_1+q_2)\}\,g \\
G_2 &= m_2r_2g\cos(q_1+q_2).
\end{aligned}$$

Parameter values used are same as those of [9].

$$\begin{gathered}
r_1 = 1m, \quad r_2 = 0.8m, \\
J_1 = 5kg\cdot m, \quad J_2 = 5kg\cdot m, \\
m_1 = 0.5kg, \\
0.5kg \ < \ m_2 \ < \ 6.25kg,
\end{gathered}$$

In order to avoid the chattering phenomenon, the function $sgn(s)$ in the controller (14) has been replaced by $sat(s)$. The function $sat(s)$ is defined as follows

$$sat(s) = \begin{cases} 1 & s > \delta \\ s/\delta & |s| \le \delta \\ -1 & s < -\delta \end{cases},$$

where $\delta = 0.05$.

Figure 2 shows that each joint angle error decreases exponentially to zero. And Figure 3 shows the control input torque for that case. The control input is smooth for the introduction of saturation function with $\delta = 0.05$ instead of sign function. Figure 4 shows that the sliding function s_1 and s_2 were confined to the predetermined boundary $\delta = 0.05$, and the fact means that the system state is in the sliding mode for all time t. Figure 5 shows the phase portrait for joint 1 and 2 of the robot manipulator.

In the Leung's controller simulation results, Figure 12 and 13 have shown that the sliding function started from zero ($s_1 = 0$, $s_2 = 0$) is initially away from zero. In addition, the sliding function s_1 gone away from the predetermined boundary layer $\delta_1 = 0.05$ as shown in Figure 12 of Leung's paper [5]. However, for the proposed control law designed by the concept that the overall system ensures the stability of the intersection of the sliding surfaces without necessarily stabilizing each individual one, all s_i is confined to predetermined boundary layer for all time t. Therefore, error transient can be predetermined in advance.

5 Conclusions

In this paper, the variable structure model following control method is proposed. The proposed control scheme ensures the system state is on the intersection of the surfaces from a given initial state without necessarily stabilizing each individual one. Hence, the control system also guarantees that the controller designer can prescribe the transient response of the joint error. Furthermore, overall system is globally exponentially stable. So, we can assign the tracking error decreasing rate, though MFAC method can guarantee only the asymptotic stability. Chattering during the transient phase can be reduced by using the boundary layer technique. Simulation results show the good performance of the robot manipulator.

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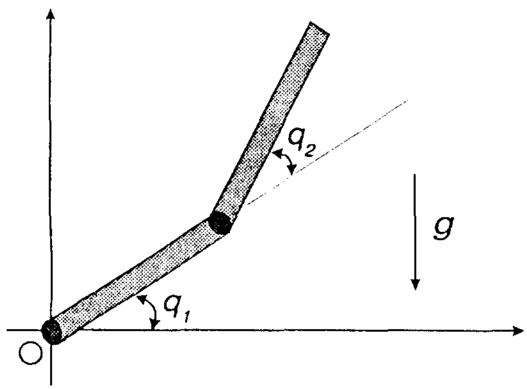

Figure 1. Two Degree of Freedom Robot Manipulator

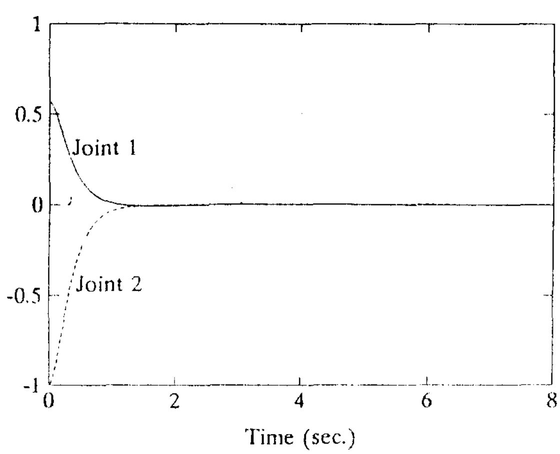

Figure 2. Tracking Error for Joint 1 & 2

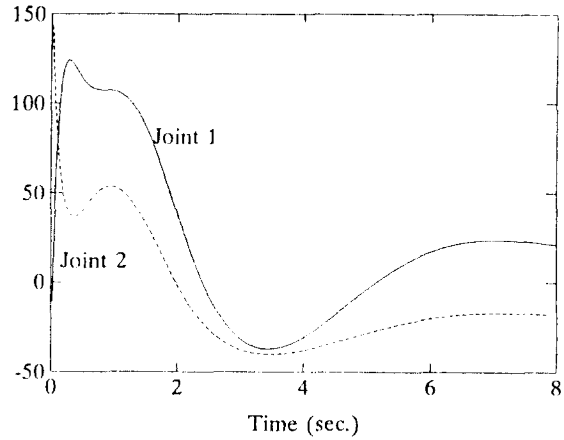

Figure 3. Control Input Torque for Joint 1 & 2

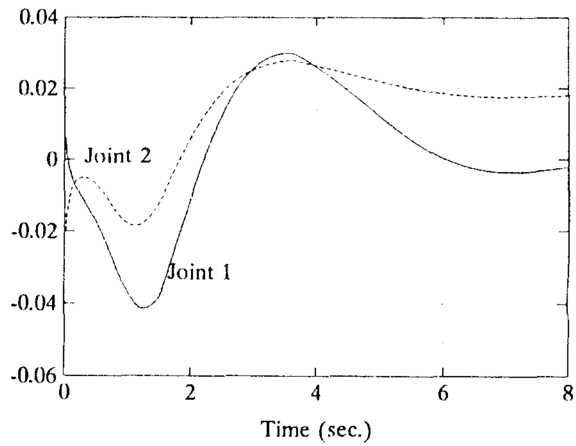

Figure 4. Sliding Function for Joint 1 & 2

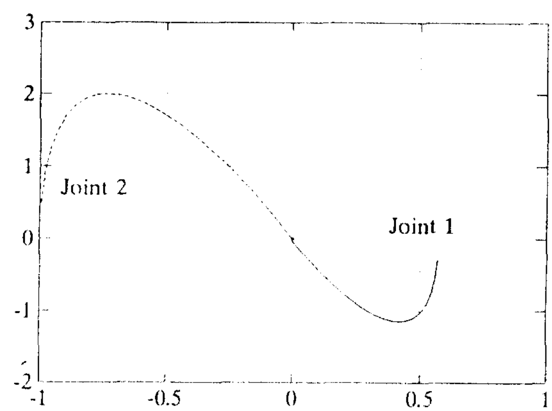

Figure 5. Phase Portrait