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Simultaneous Position and Vibration Control of the Flexible Object While Using Dual-Arm Manipulators

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Abstract

In this paper, we consider the handling of a flexible object using dual-arm manipulators. We choose both the side arms as rigid, and the object to be manipulated as flexible. Our purpose is to realize position control for the flexible object while suppressing its vibration. In particular, the problem taken up here is the stability of the control system while manipulating the object. We propose that the traditional approach to investigate the robot system be expanded to include the object's characteristics (thus transferring the stability of the robot system into the full assembly system.) We define a handling characteristic while manipulating the object. Finally, the relationship between the handling characteristic and the positional constraint condition in the hold position of the arms is studied while considering the stability of the control system.

1 Introduction

The problem addressed in this paper concerns the holding of an object by the dual-arm manipulators (Figure 1). The dual-arm manipulators are made up of links and the end effectors. We consider the object to be a flexible beam. So, we consider both the arms and the object together as the entire system, and make a mathematical model for the combined system. This model derives from the positional and velocity considerations in the hold position. For the system's stability, we need to understand both the robot system's stability, and also the mutual relation between the robot system's stability and the full assembly system's stability. For the control system design, we conclude that the control input derived by the control law can be visualized to be as that of the actuators of the dual-manipulators plus virtual actuators. So it is clear that the stabilization problem for the mixed system (dual-arm-manipulators and the object) is not the same as the stabilization of the object using only dual-manipulators. Thus, to perform work by only the dual-manipulators, we propose that the control input from the virtual actuators of the object be distributed to the dual manipulators using the Jacobian matrix and the inverse kinematics of the robots. We next, consider a cooperative control system design for the dual-arm manipulators from the point of view of the handling characteristic. The control system design from this characteristic can take into account any uncertainty of modelling error of the manipulator and the object. This method can also represent the mutual relationship between the mixed system's uncertainty and the positional boundary conditions at the handling point. A brief summary of our results and the organization of the paper is as follows: In Section 2, we present the manipulator's equation of motion and the modelling of the object. Section 3 gives the modelling of the handling system. Section 4 gives the control method for the mixed model presented in Section 3. Section 5 the cooperative control system design for the dual-arm manipulators from the handling characteristic point of view is discussed. Finally, Section 6 the conclusions of this work are given.

2 Kinematics and dynamics

2.1 Manipulator's equation of motion

Each of the manipulators move in a plane. The symbol L and R represent the left arm and the right arm. Using Lagrange's formulation, the dual-manipulator's equation of motion is written as follows:

$$\boldsymbol{J}_L(\boldsymbol{\theta}_L)\ddot{\boldsymbol{\theta}}_L + \boldsymbol{C}_L(\boldsymbol{\theta}_L, \dot{\boldsymbol{\theta}}_L) + \boldsymbol{D}_L\dot{\boldsymbol{\theta}}_L + \boldsymbol{P}_L(\boldsymbol{\theta}_L) = \boldsymbol{\tau}_L \quad (1)$$

$$\boldsymbol{J}_R(\boldsymbol{\theta}_R)\ddot{\boldsymbol{\theta}}_R + \boldsymbol{C}_R(\boldsymbol{\theta}_R, \dot{\boldsymbol{\theta}}_R) + \boldsymbol{D}_R\dot{\boldsymbol{\theta}}_R + \boldsymbol{P}_R(\boldsymbol{\theta}_R) = \boldsymbol{\tau}_R \quad (2)$$

where $\boldsymbol{\theta}_L \in \boldsymbol{R}^{m_1\times 1}$, $\boldsymbol{\theta}_R \in \boldsymbol{R}^{m_2\times 1}$ are the joint angle vectors. $\boldsymbol{J}_L \in \boldsymbol{R}^{m_1\times m_1}$, $\boldsymbol{J}_R \in \boldsymbol{R}^{m_2\times m_2}$ are the inertial force coefficient matrices. $\boldsymbol{C}_L \in \boldsymbol{R}^{m_1\times 1}$, $\boldsymbol{C}_R \in \boldsymbol{R}^{m_2\times 1}$ are centrifugal force terms. $\boldsymbol{D}_L \in \boldsymbol{R}^{m_1\times m_1}$, $\boldsymbol{D}_R \in \boldsymbol{R}^{m_2\times m_2}$ are the damping frictional force coefficients. $\boldsymbol{P}_L \in \boldsymbol{R}^{m_1\times 1}$, $\boldsymbol{P}_R \in \boldsymbol{R}^{m_2\times 1}$ are the gravity terms, $\boldsymbol{\tau}_L \in \boldsymbol{R}^{m_1\times 1}$, $\boldsymbol{\tau}_R \in \boldsymbol{R}^{m_2\times 1}$ are torque input vectors.

2.2 Modelling of the object

We choose the object to be a flexible beam. While handling of the object by using the dual-arm manipulators, we assume that both ends of the beam are free, and control the dual-arms to regulate its vibration.

2.2.1 The state-space description of the fundamental equation

The fundamental equation of the beam when external forces and torques are also present is given as follows:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + E^* I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = f(x_u,t)\delta(x-x_u) + \tau(x_v,t)\delta'(x-x_v) \quad (3)$$

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \quad (4)$$

where $w(x,t)$ is the bending displacement at x, $\eta_i(t)$ is an unknown function, A is the cross-sectional-area of the beam, E it's vertical elastic coefficient, E^* the damping coefficient, I the area moment of inertia; $f(x_u,t)$ is the force input at $x = x_u$, $\tau(x_v,t)$ is the moment input at $x = x_v$, δ is the delta function, x_i is the measured position, and L is the beam length. The mode function and the boundary condition of the free ends beam are given by:

$$\phi(x) = \frac{\cosh\left(\frac{k_i x}{L}\right) + \cos\left(\frac{k_i x}{L}\right)}{\cosh(k_i) - \cos(k_i)} - \frac{\sinh\left(\frac{k_i x}{L}\right) + \sin\left(\frac{k_i x}{L}\right)}{\sinh(k_i) - \sin(k_i)} \quad (5)$$

$$\left(\frac{\partial^2 \phi(x)}{\partial x^2}\right)_{x=0,L} = \left(\frac{\partial^3 \phi(x)}{\partial x^3}\right)_{x=0,L} = 0 \ . \quad (6)$$

k_i can be approximated by:

$$1 - \cos(k_i)\cosh(k_i) = 0 \ . \quad (7)$$

Using the Galerkin's method, the state space description of the beam relative to the unknown function $\eta_i(t)$ is obtained as:

$$\dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{u}_b, \quad \boldsymbol{y}_b = \boldsymbol{C}\boldsymbol{z} \quad (8)$$

where

$$\boldsymbol{z} = \begin{bmatrix} \eta_1 & \dot{\eta}_1 & \cdots & \eta_i & \dot{\eta}_i \end{bmatrix}^T \in \boldsymbol{R}^{2i\times 1}$$

$$\boldsymbol{y}_b = \begin{bmatrix} W(x_{w_1},t) & W(x_{w_2},t) & \cdots \end{bmatrix}^T \in \boldsymbol{R}^{\infty\times 1}$$

$$\boldsymbol{u}_b = \begin{bmatrix} f(x_u,t) & \tau(x_v,t) \end{bmatrix}^T \in \boldsymbol{R}^{2\times 1}$$

$$\boldsymbol{A} = \text{block diag}\,[\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3 \cdots] \in \boldsymbol{R}^{\infty\times\infty}$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1^T & \boldsymbol{B}_2^T & \boldsymbol{B}_3^T & \cdots \end{bmatrix}^T \in \boldsymbol{R}^{\infty\times 2}$$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} & \cdots \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \in \boldsymbol{R}^{\infty\times\infty}$$

$$\boldsymbol{A}_i = \begin{bmatrix} 0 & 1 \\ -\frac{EI}{\rho A}\left(\frac{k_i}{L}\right)^4 & -\frac{E^* I}{\rho A}\left(\frac{k_i}{L}\right)^4 \end{bmatrix} \in \boldsymbol{R}^{2\times 2}$$

$$\boldsymbol{B}_i = \begin{bmatrix} 0 & 1 \\ \frac{\phi_i(x_u)}{\rho A} & -\frac{\phi_i'(x_v)}{\rho A} \end{bmatrix} \in \boldsymbol{R}^{2\times 2}$$

$$\boldsymbol{C}_{ji} = \begin{bmatrix} \phi_i\left(x_{w_j}\right) & 0 \end{bmatrix} \in \boldsymbol{R}^{1\times 2} \ .$$

2.2.2 Reduction of the object into a finite dimensional system

In this section, we describe the reduction of the object beam. The control theory for the linear finite dimensional system cannot be applied to the distributed-parameter system. If the original system is approximated by some lower-order-modes neglecting the higher-order-modes, the control system may generate a spill over and unstabilize. So we reduce the original system using the stabilization method [1] for the elastic vibrating system, which stabilizes the considered system in spite of modelling errors. The method is as follows:

Let the state variable vector $\boldsymbol{z}^i$ be the state response of the system to the input $\boldsymbol{u}^i$. In the beam's state-space equation (8), the r dimensional vector $\boldsymbol{R}\boldsymbol{z}^i$ is given as the linear combination of the state response generated by the impulse input. Let $\tilde{\boldsymbol{z}}$ be the new state variable for the reduced system. We apply the vector $\boldsymbol{R}\boldsymbol{z}^i$ to the following reduced model:

$$\frac{d\tilde{\boldsymbol{z}}}{dt} = \boldsymbol{A}_r\tilde{\boldsymbol{z}} + \boldsymbol{B}_r\boldsymbol{u}_b \ . \quad (9)$$

The error between the original model and the reduced model is given as follows:

$$\boldsymbol{d}^i = \boldsymbol{R}\frac{d\boldsymbol{z}^i}{dt} - \left(\boldsymbol{A}_r\boldsymbol{R}\tilde{\boldsymbol{z}}^i + \boldsymbol{B}_r\boldsymbol{u}_b^i\right) \ . \quad (10)$$

In the above equation (10), the system matrices of the reduced model are given as follows:

$$\boldsymbol{A}_r = \boldsymbol{R}\boldsymbol{A}\boldsymbol{W}_c\boldsymbol{R}^T\left(\boldsymbol{R}\boldsymbol{W}_c\boldsymbol{R}^T\right)^{-1}, \quad \boldsymbol{B}_r = \boldsymbol{R}\boldsymbol{B} \quad (11)$$

which minimizes the mean value of the state response by the r linearly independent impulse inputs. $\boldsymbol{W}_c$ is the controllability Gramian matrix of (8). Choose $\boldsymbol{R}$ as follows:

$$\boldsymbol{R} = \begin{bmatrix} \phi_1(x_{w_1}) & 0 & \phi_2(x_{w_1}) & 0 & \cdots \\ 0 & \phi_1(x_{w_1}) & 0 & \phi_2(x_{w_1}) & \cdots \\ \phi_1(x_{w_2}) & 0 & \phi_2(x_{w_2}) & 0 & \cdots \\ 0 & \phi_1(x_{w_2}) & 0 & \phi_2(x_{w_2}) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \ . \quad (12)$$

The state variables (unknown function $\eta_i(t)$) of the system (9) changes to the state variables (sensor displacement $w(x_{w_i}, t)$) of the reduced model. This is desirable in the control system design, because the displacement detected by the sensor can be applied to the state feedback control. We assume that the feedback gain in the closed loop is denoted by $\boldsymbol{K}$. Then the error $\boldsymbol{\Delta}(\varphi)$ between the original model and the reduced one is given by:

$$\begin{aligned} \boldsymbol{\Delta}(\varphi) &= \boldsymbol{K}\left\{\boldsymbol{R}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}-(s\boldsymbol{I}-\boldsymbol{A}_r)^{-1}\boldsymbol{B}_r\right\} \\ &= \boldsymbol{K}\boldsymbol{\Delta}(s) \,. \end{aligned} \tag{13}$$

Therefore, to get the precise reduced model, it is important to select appropriate values for the elements of $\boldsymbol{R}$ (sensor positions). Of course, it is possible to install the sensor at the handling point.

3 Modelling of the handling system

In this section, we consider the constitution of the mixed model combining the dual-manipulators with the object while satisfying the positional boundary conditions at the handling point. Transforming equations (1) & (2), into the first order differential equations, and combining these with equation (9), we get

$$\begin{aligned}
\begin{bmatrix} \dot{\theta}_L \\ \ddot{\theta}_L \\ \hline \dot{\tilde{z}} \\ \hline \dot{\theta}_R \\ \ddot{\theta}_R \end{bmatrix} &= \begin{bmatrix} 0^{m_1\times 1} \\ -\boldsymbol{J}_L^{-1}(\theta_L)\left\{\boldsymbol{C}_L(\dot{\theta}_L,\theta_L)+\boldsymbol{P}_L(\theta_L)\right\} \\ \hline 0^{r\times 1} \\ \hline 0^{m_2\times 1} \\ -\boldsymbol{J}_R^{-1}(\theta_R)\left\{\boldsymbol{C}_R(\dot{\theta}_R,\theta_R)+\boldsymbol{P}_R(\theta_R)\right\} \end{bmatrix} \\
&+ \left[\begin{array}{c:c:c} 0^{m_1\times n_1} & 0^{2m_1\times 2} & 0^{2m_1\times 2n_2} \\ \boldsymbol{J}_L^{-1}(\theta_L) & & \\ \hline 0^{r\times n_1} & \boldsymbol{B}_r & 0^{r\times 2n_2} \\ \hline 0^{2m_2\times n_1} & 0^{2m_2\times 2} & 0^{m_2\times n_2} \\ & & \boldsymbol{J}_R^{-1}(\theta_R) \end{array}\right] \begin{bmatrix} \tau_L \\ u_h \\ \tau_R \end{bmatrix} \\
&+ \left[\begin{array}{c:c:c} \boldsymbol{A}_{L0} & 0^{2m_1\times r} & 0^{2m_1\times 2m_2} \\ \hline 0^{r\times 2m_1} & \boldsymbol{A}_r & 0^{r\times 2m_1} \\ \hline 0^{2m_2\times 2m_1} & 0^{2m_2\times r} & \boldsymbol{A}_{R0} \end{array}\right] \begin{bmatrix} \theta_L \\ \dot{\theta}_L \\ \hline \tilde{z} \\ \hline \theta_R \\ \dot{\theta}_R \end{bmatrix} \\
&\equiv \boldsymbol{W}(\boldsymbol{x})+\boldsymbol{B}(\boldsymbol{x})\boldsymbol{\tau}+\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x}
\end{aligned} \tag{14}$$

where

$$\begin{aligned} \boldsymbol{A}_{L0} &= \begin{bmatrix} 0^{m_1\times m_1} & \boldsymbol{I}^{m_1\times m_1} \\ 0^{m_1\times m_1} & -\boldsymbol{J}_L^{-1}(\theta_L)\boldsymbol{D}_L \end{bmatrix} \\ \boldsymbol{A}_{R0} &= \begin{bmatrix} 0^{m_2\times m_2} & \boldsymbol{I}^{m_2\times m_2} \\ 0^{m_2\times m_2} & -\boldsymbol{J}_R^{-1}(\theta_L)\boldsymbol{D}_R \end{bmatrix} \end{aligned}$$

and $\boldsymbol{A}(\boldsymbol{x}) \in \boldsymbol{R}^{m\times m}$, $\boldsymbol{B}(\boldsymbol{x}) \in \boldsymbol{R}^{m\times n}$, $\boldsymbol{W}(\boldsymbol{x}) \in \boldsymbol{R}^{m\times 1}$, $m = 2m_1 + 2m_2 + r$, $n = n_1 + n_2 + 2$. Define the positional constraints to combine the equations of the dual-manipulators with that of the object:

$$\boldsymbol{f}_c(\boldsymbol{x}) \equiv 0 \,. \tag{15}$$

If the number of constraints is k, dimension of $\boldsymbol{f}_c(x)$ is $\boldsymbol{R}^{k\times 1}$. It is possible to control if and only if the following equation holds:
first,

$$k < n \,. \tag{16}$$

(16) is a necessary condition for the existence of the unobservable space capable of setting all the poles of the whole system, and

$$k < n_1 + n_2 \,. \tag{17}$$

(17) is a necessary condition for the existence of the unobservable space capable of setting all the poles of the dual-arm manipulators. The state space equations of the system with the boundary conditions is given by:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{\tau} + \boldsymbol{W}(\boldsymbol{x}) + \frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\boldsymbol{\lambda} \tag{18}$$

where $\boldsymbol{\lambda}$ is the unknown multiplier vector. The standard assumption for the boundary condition $\boldsymbol{f}(\boldsymbol{x})$ is $\mathrm{rank}(\partial \boldsymbol{f}(\boldsymbol{x})/\partial \boldsymbol{x}) = k$.
When the boundary condition is met, note by (15) we deduce that the following equation holds:

$$\frac{d\boldsymbol{f}(\boldsymbol{x}(t))}{d\boldsymbol{x}} = \left[\frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right]^T \dot{\boldsymbol{x}} = \boldsymbol{0} \,. \tag{19}$$

From (18) and (19), the unknown multiplier vector $\boldsymbol{\lambda}$ is given by:

$$\boldsymbol{\lambda} = -\left\{\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)^T \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right\}^{-1} \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)^T [\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x}+\boldsymbol{B}(\boldsymbol{x})\boldsymbol{\tau}+\boldsymbol{W}(\boldsymbol{x})] \,. \tag{20}$$

Consequently, the state equation of the mixed model is given by:

$$\begin{aligned} \dot{\boldsymbol{x}} &\equiv \left[\boldsymbol{I}^{m\times m} - \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)^T \left\{\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)^T \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right\}^{-1} \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)^T\right] \\ &\quad \times \; [\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{\tau} + \boldsymbol{W}(\boldsymbol{x})] \qquad (21) \\ &\equiv \boldsymbol{F}(\boldsymbol{x})\,[\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{\tau} + \boldsymbol{W}(\boldsymbol{x})] \,. \qquad (22) \end{aligned}$$

4 The control method for the handling system

For the mixed model (22), let the control input be given by:

$$\begin{aligned} \boldsymbol{\tau} \;=\; & \boldsymbol{Q}(\boldsymbol{x})\,[\dot{\boldsymbol{x}}_d + \boldsymbol{K}\boldsymbol{x}_d \\ & -\{\boldsymbol{K} + \boldsymbol{F}(\boldsymbol{x})\boldsymbol{A}(\boldsymbol{x})\}\boldsymbol{x} - \boldsymbol{F}(\boldsymbol{x})\boldsymbol{W}(\boldsymbol{x})] \,, \end{aligned} \tag{23}$$

then the closed system come to the PD servo system, where $\boldsymbol{x}_d$ is the reference signal vector. To stabilize the closed system, let the proportional gain $\boldsymbol{K}$ be given by:

$$\boldsymbol{K} = \mathrm{diag}\{\kappa_1, \kappa_2, \cdots, \kappa_m\}, \;\; \kappa_i > 0 \tag{24}$$

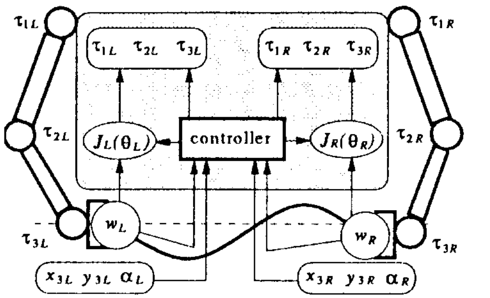

Figure 1: The control system to handle the flexible object.

where $Q(x)$ is the Moore-Penrose inverse matrix of $F(x)B(x)$. If we directly input τ to the mixed model, this method is equivalent to the control by the use of the actuators of the dual-manipulators plus virtual actuators. As it is impossible to give directly the control input to the object, the control is performed by only the dual-manipulators, and it becomes feasible if and only if (17) holds. The relationship between the generalized forces at the end effectors and that at each of the joints is transferred as follows:

$$\tau_{L,R} = J^T_{L,R}(\theta_{L,R}) \begin{bmatrix} 0 & u_b^T \end{bmatrix}^T \tag{25}$$

In the above equation, the control input to the object is distributed to the dual arm's joints. The description is given in Figure 1. Thus, the new control input $\overline{u}$ to the mixed model is given by:

$$\overline{u} = \begin{bmatrix} I & J_L^T(\theta_L) & 0 \\ 0 & 0 & 0 \\ 0 & J_R^T(\theta_R) & I \end{bmatrix} \tau \, . \tag{26}$$

5 The cooperative control system design from the standpoint of handling

The above section mentioned a method for the cooperative control system design for the dual-arm manipulators but it does not compensate for the model uncertainty of the manipulators and the object. It is not known how this uncertainty affects the equations of the boundary conditions. A good method from the standpoint of the system maybe bad for the object. In this paper, to realize accurate handling, we study the dynamic characteristic at the equilibrium point while satisfying the boundary conditions.

5.1 The design of the controller satisfying the boundary conditions

Define the equilibrium point that satisfies the boundary conditions $x = \overline{x}$. Assuming that the following condition hold at the equilibrium point:

$$\dot{\theta}_L = \ddot{\theta}_L = 0^{m_1\times 1},\ \dot{\theta}_R = \ddot{\theta}_R = 0^{m_2\times 1},\ \dot{\tilde{z}} = 0^{r\times 1}, \tag{27}$$

we get the following conditions:

$$\left.\frac{\partial f(x)}{\partial x}\right|_{x=\overline{x}} \overline{\lambda} + \varrho = 0 \tag{28}$$

where

$$\varrho = \begin{bmatrix} 0 \\ J_L^{-1}(\overline{\theta}_L)\overline{\tau}_L - J_L^{-1}(\overline{\theta}_L)\overline{P}_L \\ \hline A_r\tilde{z} + B_r u_b \\ \hline 0 \\ J_R^{-1}(\overline{\theta}_R)\overline{\tau}_R - J_R^{-1}(\overline{\theta}_R)\overline{P}_R \end{bmatrix} \in R^{m\times 1} \, . \tag{29}$$

Let $X(\in R^{l\times m})$ denote the Moore-Penrose inverse of $\partial f(x)/\partial x$. $\overline{\lambda}$ is given by:

$$\overline{\lambda} = -X(x)|_{x=\overline{x}}\varrho \, , \tag{30}$$

and at the equilibrium point, it is necessary that both (30) and the following equation holds:

$$f(x)|_{x=\overline{x}} = 0 \, . \tag{31}$$

We next, consider the vicinity of the equilibrium point as follows:

$$x(t) = \overline{x} + \Delta x(t),\ \lambda(t) = \overline{\lambda} + \Delta\lambda(t),\ \tau(t) = \overline{\tau} + \Delta\tau(t) \, . \tag{32}$$

First, we consider the left arm. The state equation is given by:

$$\begin{bmatrix} \dot{\overline{\theta}}_L + \Delta\dot{\theta}_L \\ \ddot{\overline{\theta}}_L + \Delta\ddot{\theta}_L \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -J_L^{-1}(\overline{\theta}_L + \Delta\theta_L)D_L \end{bmatrix} \begin{bmatrix} \overline{\theta}_L + \Delta\theta_L \\ \dot{\overline{\theta}}_L + \Delta\dot{\theta}_L \end{bmatrix} + \begin{bmatrix} 0 \\ -J_L^{-1}(\overline{\theta}_L + \Delta\theta_L) \end{bmatrix} (\overline{\tau}_L + \Delta\tau_L) + \begin{bmatrix} 0 \\ -J_L^{-1}(\overline{\theta}_L + \Delta\theta_L)P_L(\overline{\theta}_L + \Delta\theta_L) \end{bmatrix} \, . \tag{33}$$

$P_L(\overline{\theta}_L + \Delta\theta_L)$ is expanded by using Taylor series. To linearize the series, approximation until the second term is done. Assume that the following equations hold:

$$J_L(\overline{\theta}_L + \Delta\theta_L) = J_L(\overline{\theta}_L) \tag{34}$$

$$\dot{\overline{\theta}}_L = \ddot{\overline{\theta}}_L = 0 \, . \tag{35}$$

$\overline{\theta}_L$ has no affect on the expression. Thus, the following equation holds:

$$\begin{bmatrix} \Delta\dot{\theta}_L \\ \Delta\ddot{\theta}_L \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -J_L^{-1}(\overline{\theta}_L)D_L \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\dot{\theta}_L \end{bmatrix} + \begin{bmatrix} 0 \\ -J_L^{-1}(\overline{\theta}_L) \end{bmatrix} (\overline{\tau}_L + \Delta\tau_L) - \begin{bmatrix} 0 \\ J_L^{-1}(\overline{\theta}_L)\left\{ P_L(\overline{\theta}_L) + \left.\frac{\partial P_L^T(\theta_L)}{\partial\theta_L}\right|_{\theta_L=\overline{\theta}_L} \right\} \Delta\theta_L \end{bmatrix} \, . \tag{36}$$

The right arm's and the object's equations are got in a similar way. The equation for the whole system are given as follows:

$$\Delta\dot{x} = \begin{bmatrix} A_L & 0^{2m_1\times r} & 0^{2m_1\times m_2} \\ 0^{r\times 2m_1} & A_r & 0^{r\times m_2} \\ 0^{2m_2\times 2m_1} & 0^{2m_2\times r} & A_R \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\dot{\theta}_L \\ \tilde{z} \\ \Delta\theta_R \\ \Delta\dot{\theta}_R \end{bmatrix}$$

$$+ \begin{bmatrix} B_L & 0^{2m_1\times 2} & 0^{2m_1\times 2n_2} \\ 0^{r\times n_1} & B_r & 0^{r\times 2n_2} \\ 0^{2m_2\times n_1} & 0^{2m_2\times 2} & B_R \end{bmatrix} \begin{bmatrix} \Delta\tau_L \\ \Delta u_b \\ \Delta\tau_R \end{bmatrix}$$

$$+ \frac{\partial f(\overline{x})}{\partial \overline{x}}\Delta\lambda$$

$$\equiv \overline{A}\Delta x + \begin{bmatrix} \overline{B} & \frac{\partial f(\overline{x})}{\partial \overline{x}} \end{bmatrix} \begin{bmatrix} \Delta\tau \\ \Delta\lambda \end{bmatrix}$$

$$\equiv \overline{A}\Delta x + \begin{bmatrix} \overline{B} & \overline{C}^T \end{bmatrix}\Delta u \qquad (37)$$

where,

$\overline{A} \in R^{m\times m}$, $\overline{B} \in R^{m\times n}$, $\overline{C} \in R^{l\times m}$, $\Delta u \in R^{(n+l)\times 1}$

$$A_{L,R} = \begin{bmatrix} 0^{m_{1,2}\times m_{1,2}} & I^{m_{1,2}\times m_{1,2}} \\ -J_{L,R}^{-1}(\overline{\theta}_{L,R})\left[\frac{\partial P_{L,R}(\overline{\theta}_{L,R})}{\partial \theta_{L,R}}\right]^T & -J_{L,R}^{-1}(\overline{\theta}_{L,R})D_{L,R} \end{bmatrix}$$

$$B_{L,R} = \begin{bmatrix} 0^{m_{1,2}\times n_{1,2}} \\ J_{L,R}^{-1}(\overline{\theta}_{L,R}) \end{bmatrix} .$$

If we design a feedback system that achieves $\Delta y = \overline{C}\Delta x \equiv 0$, then it is possible to realize the boundary conditions at the equilibrium point. From $\overline{C}\Delta\dot{y} = 0$, the following equation holds:

$$\overline{C}\,\overline{A}\Delta x + \overline{C}\begin{bmatrix} \overline{B} & \overline{C}^T \end{bmatrix}\Delta u = 0 . \qquad (38)$$

As a result the following equation holds:

$$\Delta y = 0 . \qquad (39)$$

However, it is difficult to achieve (39) strictly. So we use sliding mode control [2].
Define the deviation $\Delta\overline{x}$ at the equilibrium point as follows:

$$\Delta e = \Delta x - \Delta\overline{x} . \qquad (40)$$

The error equation is given as follows:

$$\Delta\dot{e} = \overline{A}\Delta e + \begin{bmatrix} \overline{B} & \overline{C}^T \end{bmatrix}\Delta u + \overline{A}\Delta\overline{x} . \qquad (41)$$

Define the switching plane as follows:

$$S(t) = \overline{C}\Delta e(t) . \qquad (42)$$

$S(t)$ plane is shown in Figure 2. Let $W(\in R^{(l+n)\times m})$ denote the Moore-Penrose inverse of $\begin{bmatrix} \overline{B} & \overline{C}^T \end{bmatrix}$. Let the control input $\Delta u(t)$ be given by:

$$\Delta u(t) = -WF_rQ_e, \qquad (43)$$

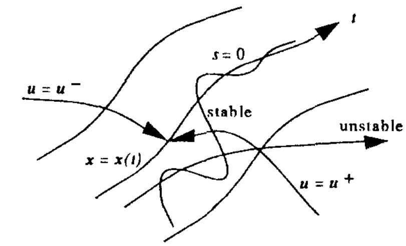

Figure 2: $S(t)$ plain.

then Q_e is given by:

$$Q_e = \begin{bmatrix} |\Delta e_1(t)|\,\mathrm{sgn}(S_1(t)) \\ |\Delta e_2(t)|\,\mathrm{sgn}(S_2(t)) \\ \vdots \\ |\Delta e_m(t)|\,\mathrm{sgn}(S_m(t)) \end{bmatrix} \in R^{m\times 1} . \qquad (44)$$

Define the Lyapunov equation for $S(t)$ as:

$$V = \frac{1}{2}SS^T . \qquad (45)$$

Differentiating V with time respect to $\dot{V}$ is given as follows:

$$\begin{aligned} \dot{V} &= S\dot{S}^T \\ &= S\Delta\dot{e}^T\overline{C}^T \\ &= S[\overline{A}\Delta e + [\overline{B}\ \overline{C}^T]\Delta u + \overline{A}\Delta\overline{x}]^T\overline{C}^T \\ &= |S^{\cdot}|[(\overline{A} - F_p)|\Delta e^{\cdot}| \pm \overline{A}\Delta\overline{x}]^T\overline{C}^T \end{aligned} \qquad (46)$$

where,

$$S = [\ S_1\ S_2\ \cdots\ S_m\ ]^T,\ e = [\ e_1\ e_2\ \cdots\ e_m\ ]^T$$

$$|S^{\cdot}| = [\ |S_1|\ |S_2|\ \cdots\ |S_m|\ ]^T,\ |e^{\cdot}| = [\ |e_1|\ |e_2|\ \cdots\ |e_m|\ ]^T .$$

To maintain Δx at 0, define F_p as follows:

$$F_p \equiv \overline{C}^+\widetilde{F} \qquad (47)$$

$$\dot{V} = |S^{\cdot}|[(\overline{C}\,\overline{A} - \widetilde{F})|\Delta e^{\cdot}| \pm \overline{C}\,\overline{A}\Delta\overline{x}]^T \qquad (48)$$

where $\widetilde{F}$ is positive definite and $\overline{C}^+$ is the Moore-Penrose inverse of $\overline{C}$. Choose $\widetilde{F}$ satisfying the following equation:

$$\overline{C}\,\overline{A} < \widetilde{F} . \qquad (49)$$

Accordingly, $\dot{V}$ reduces to:

$$\dot{V} < 0 , \qquad (50)$$

and the stability near $S = 0$ is compensated for thus. Let $\overline{L}$ be the Moore Penrose inverse of $\overline{C}\begin{bmatrix} \overline{B} & \overline{C}^T \end{bmatrix}$, Then the control input is given by:

$$\Delta u = -\overline{L}\,\overline{C}\,\overline{A}(\Delta e + \Delta\overline{x}) . \qquad (51)$$

Given the control input for $S = 0$ as (51), then Δx nears $\Delta\overline{x}$. If we let $\Delta\overline{y}$ be the deviation from

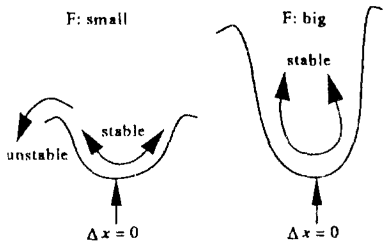

Figure 3: Stability margin in $\boldsymbol{S}$ plain.

the equilibrium output, then from (42) it follows that $\boldsymbol{\Delta y} \to \boldsymbol{\Delta \overline{y}}$.
If $\boldsymbol{\Delta u}$ is given by the following equation:

$$\boldsymbol{\Delta u} = -\boldsymbol{W}\overline{\boldsymbol{C}}^{+}\widetilde{\boldsymbol{F}}\overline{\boldsymbol{C}}^{+}\boldsymbol{Q}_y \qquad (52)$$

where

$$\boldsymbol{Q}_y = \begin{bmatrix} |\Delta y_1 - \Delta \overline{y}_1|\,\mathrm{sgn}(S_1(t)) \\ |\Delta y_2 - \Delta \overline{y}_2|\,\mathrm{sgn}(S_2(t)) \\ \vdots \\ |\Delta y_l - \Delta \overline{y}_l|\,\mathrm{sgn}(S_l(t)) \end{bmatrix},$$

then it is possible to realize the control which satisfies the boundary conditions at the equilibrium point, which realizes $\boldsymbol{\Delta y} \to \mathbf{0}$.

5.2 The cooperative control considered the handling characteristics

The stability margin in the equilibrium point depend on the norm of $\widetilde{\boldsymbol{F}}$ (Figure 3), and the control input measure increase with the norm larger. Our purpose is to realize proper handling by changing the handling characteristic [3], [4] to be within the range of stability at the equilibrium point and by increasing the controllability of the system.
In addition to the control input (52), we add another input $\boldsymbol{\Delta\sigma}$, and we use the following equations.

$$\begin{aligned} \boldsymbol{\Delta\dot{x}} &= \overline{\boldsymbol{A}}\boldsymbol{\Delta x} + [\ \overline{\boldsymbol{B}}\ \ \overline{\boldsymbol{C}}^T\ ]\boldsymbol{\Delta u} \\ \boldsymbol{\Delta x}' &= (\boldsymbol{I} + \boldsymbol{\Delta\sigma})\boldsymbol{\Delta x} \end{aligned} \qquad (53)$$

where $\boldsymbol{\Delta x}'$ is the state error by the new input $\boldsymbol{\Delta x}$. Define f_s as the sum of the forces applied to the object by the end effectors. Let $\xi(-f_s)$ and $\zeta(-f_s)$ be the functions dependent upon the material of the end effectors, represented by $(2n+1)$ order polynomials. Also let α and β be the distribution coefficient of the handling force for the left arm and the right arm, respectively. For example, α and β can be chosen:

$$\begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} : \begin{bmatrix} 1-\alpha(t) \\ 1-\beta(t) \end{bmatrix} = \begin{bmatrix} \{\boldsymbol{J}_L(\boldsymbol{\theta}_L)\boldsymbol{\tau}_L\}_{1,1} \\ \{\boldsymbol{J}_L(\boldsymbol{\theta}_L)\boldsymbol{\tau}_L\}_{2,1} \end{bmatrix} : \begin{bmatrix} \{\boldsymbol{J}_R(\boldsymbol{\theta}_R)\boldsymbol{\tau}_R\}_{1,1} \\ \{\boldsymbol{J}_R(\boldsymbol{\theta}_R)\boldsymbol{\tau}_R\}_{2,1} \end{bmatrix}. \qquad (54)$$

Set up $\boldsymbol{\Delta\sigma}$ as follows:

$$\begin{aligned} \boldsymbol{\Delta\sigma} = \text{block diag} & \\ & \left\{\alpha\xi(-f_s)\begin{bmatrix} \boldsymbol{I}^3 & \mathbf{0}^3 \\ \mathbf{0}^3 & \boldsymbol{I}^3 \end{bmatrix},\ \beta\zeta(-f_s)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\right. \\ & \left.(1-\beta)\zeta(-f_s)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ (1-\alpha)\xi(-f_s)\begin{bmatrix} \boldsymbol{I}^3 & \mathbf{0}^3 \\ \mathbf{0}^3 & \boldsymbol{I}^3 \end{bmatrix}\right\}. \end{aligned} \qquad (55)$$

For the whole system's stability by using this input, we let $\widetilde{\boldsymbol{F}}$ satisfy the following equation around $\boldsymbol{S} = \mathbf{0}$:

$$|\boldsymbol{S}^{\sim}|[(\overline{\boldsymbol{C}}\ \overline{\boldsymbol{A}} - \widetilde{\boldsymbol{F}})|\boldsymbol{\Delta e}^*| \pm \overline{\boldsymbol{C}}\ \overline{\boldsymbol{A}}(\boldsymbol{\Delta\overline{x}} + \boldsymbol{\Delta\sigma})]^T < \mathbf{0} \qquad (56)$$

where

$$|\boldsymbol{\Delta e}^*| = [\ |e_1 - \Delta\sigma_1|\ \ |e_2 - \Delta\sigma_2|\ \ \cdots\ \ |e_m - \Delta\sigma_m|\ ]^T.$$

Finally, we choose $\widetilde{\boldsymbol{F}}$ to satisfy (56) and compute $\boldsymbol{\Delta u}$ in (52), and determine the new input, as

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + [\ \boldsymbol{I}\ \ \overline{\boldsymbol{B}}^{+}\overline{\boldsymbol{C}}^{T}\ ]\boldsymbol{\Delta u}. \qquad (57)$$

6 Conclusions

We have proposed a handling method of a flexible object using dual-arm manipulators. We regard both the arms and the object together as the entire system and use the positional constraints as the boundary conditions. We also design the servo system to make the entire system stable, and thus realize the handling of the object using dual-arm manipulators. In our future work, we purpose to decrease the control input, for instance to decrease the value of $\|\widetilde{\boldsymbol{F}}\|_m + \|\boldsymbol{\Delta\sigma}\|$.

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