An Efficient Solution Algorithm of the Optimal Load Distribution for Multiple Cooperating Robots

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Abstract

An efficient solution algorithm of the optimal load distribution problem with joint torque constraints is presented. Multiple robot system where each robot is rigidly grasping a common object is considered. The optimality criteria used is the sum of weighted norm of the joint torque vectors. The maximum and minimum bounds of each joint torque in arbitrary form are considered as constraints, and the solution that reduces the internal force to zero is obtained. The optimal load distribution problem is formulated as a quadratic optimization problem in R^l, where l is the number of robots. The general solution can be obtained using any efficient numerical method for quadratic programming, and for dual robot case, the optimal solution is given in a simple analytical form.

1. Introduction

In recent years, growing research efforts have been focused on the subject of cooperative multiple robot systems. Multiple robot arms in cooperation can perform many tasks that would be impossible to perform for a single robot arm. Examples of these tasks are manipulation of objects without auxiliary equipments such as jigs or fixtures, and handling of heavy or large objects which can not be handled by a single robot arm.

When multiple robots work in cooperation to move an object along a given trajectory, joint torques of the robots that will produce the required motion have to be determined. This problem is referred to as a load distribution problem. When multiple robot arms grasp a common object, they form a closed kinematic chain, and as a result, the number of degrees of freedom of the multi-robot system becomes less than the total number of robot joints. The linear mapping from joint torque vector space to the space of resultant force/torque vector on the object has a null space. As a consequence, infinite number of joint torque solutions exist that can be applied to produce the given motion of the object. In order to obtain the optimal joint torque solution, a suitable objective function need to be introduced.

Optimization algorithms for solving the load distribution problem have been developed in many literatures. The structure of the solution algorithms are closely related to the order of the objective function. When the objective function is linear, the solution is commonly obtained by using linear programming method. Orin and Oh [1] have defined a linear combination of energy and load balancing as the objective function, and used LP method to calculate the joint torques for the OSU hexapod vehicle. Cheng and Orin [2] developed the Compact-Dual LP method that gives an efficient solution for general linear optimization problem with equality and inequality constraints, by eliminating the equality constraints and utilizing the duality theory. When the objective function is quadratic, the solution is commonly obtained by nonlinear programming (NLP) method. Nakamura *et al.* [3] have defined minimum norm forces as the objective function and applied a NLP method based on Lagrange multiplier. Klein and Kittivatcharapong [4] have devised the so-called exterior method, using Rosen's gradient projection algorithm [5]. Zheng and Luh [6] proposed a solution in analytic form for two cooperating robots, and Zheng and Luh [7] presented a numerical method based on direct approximate programming. Choi *et al.* [8] developed a solution method utilizing force ellipsoid that can be used when the norm of joint torque vector is bounded. Lu and Meng [9] have proposed a two loop procedure algorithm which can be applied to general quadratic optimization problem with constant torque bounds. In this method, the constrained optimization problem is transformed into an unconstrained optimization problem by forming a new objective function which linearly combines the quadratic objective function with the maximum normalized torque by an adaptive weighting factor. The solution is then obtained by an iterative procedure using any popular unconstrained optimization techinque.

If the solution of the optimal load distribution is to have practical significance in the control of the multi-robot system, the joint torque bounds must be taken into the problem formulation. The joint torque solution exceeding the maximum torque bound would be meaningless. When the objective function is linear, the joint torque bounds can be included in the formulation without a major increase in computation. For example, the efficient algorithm in Cheng and Orin [2] deals with a general linear programming problem with inequality as well equality constraints. However, when the objective function is quadratic, an addition of inequality constraints can result in a substantial increase in the computational burden. For this reason, an efficient solution algorithm is scarce that can be applied to solve the optimal load distribution problem where the objective function is quadratic and joint torque bounds are included as inequality constraints. Solution procedures in Nakamura *et al.* [3], Klein and Kittivatcharapong [4], and Zheng and Luh [6] ignore the torque bounds. Although Zheng and Luh [7] presented a nonlinear programming method that considers the joint torque bounds, no remark was given on the computational efficiency of the algorithm, and as the authors commented, the convergence of the algorithm is not quaranteed. The solution procedure in Choi *et al.* [8] is efficient, but the joint torque constraint is in the form of the bound on the norm of joint torque vector. The algorithm proposed by Lu and Meng [9] can utilize any efficient unconstrained optimization technique, although the procedure is iterative by its nature.

The objective of this paper is to propose an algorithm that gives an efficient solution of the optimal load distribution problem. Multiple robot arms rigidly grasping a common object is considered. The objective function is the weighted norm of joint torque vectors, and the torque bounds of joint actuators are included in the problem formulation. An additional constraint is imposed so that the internal force in the object is zero. The solution procedure proposed in this paper differs from other algorithms in that the dimension of the feasible solution space is considerably reduced, and the optimal solution is obtained by solving a quadratic optimization problem in R^l, where l is the number of robots. In particular, for dual robot case, the optimal solution is given in a simple analytical form.

This paper is organized as follows. In section 2, the general formulation of the optimal load distribution problem is presented. In section 3, an alternative formulation is derived using the additional constraint of zero internal force, and in section 4, the two robot arm example is studied and the optimal solution is derived in an analytical form.

2. General Problem Formulation for the Optimal Load Distribution

Nomenclatures

l	=	number of robots
m	=	dimension of robots' operational space
n	=	degree of freedom of each robot
${}^iJ$	=	$m \times n$ manipulator Jacobian matrix of robot i
${}^iF^i$	=	$m \times 1$ force/torque applied by robot i at the gripping position
F	=	$m \times 1$ resultant force/torque applied at object reference point
${}^ih$	=	$m \times 1$ force/torque applied by robot i at object reference point
${}^i\tau$	=	$n \times 1$ joint torque/force of robot i
M	=	3 x 3 mass matrix of object
I	=	3 x 3 inertia matrix
p	=	3 x 1 position vector of object
ω	=	3 x 1 angular velocity of the object
g	=	3 x 1 gravitational acceleration vector
${}^ir$	=	3 x 1 position vector of the contact point of robot i from the object reference point

In a multiple robot cooperation, robots can either grasp the object rigidly or hold the object between the robot fingers by frictional forces. In this paper, the former case is considered which is suitable for a large number of industrial robot application. A model of multiple robot arms grasping an rigid body object is shown in Fig. 1. The object is held rigidly so that no relative motion is possible between the object and the robot grippers. It is assumed, for the purpose of convenience, that the robots have the same number of degrees of freedom, and do not pass through singular positions so that the manipulator Jacobians always have full ranks.

The dynamic equation of motion of each robot is given by the following equation.

$$ {}^iD({}^iq)\,{}^i\ddot{q} + {}^iH({}^iq,{}^i\dot{q}) + {}^iG + {}^iJ^{T\,i}F = {}^i\tau \qquad (1)$$

In general, the motion of a rigid body object is completely described by the three dimensional position and orientation vectors, and the maximum value of the dimension of its operational space is six. In some cases, the robots operate in a reduced operational space, for example, in two dimensional space. In this paper, n is assumed to be six, unless specified otherwise. The dynamic equation of motion of the object is described as below.

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \ddot{p} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \times I\omega \end{bmatrix} + \begin{bmatrix} Mg \\ 0 \end{bmatrix} = F \quad (2)$$

where M is a 3 x 3 diagonal matrix whose diagonal elements are equal to the mass of the object, I is a 3 x 3 inertia matrix of the object.

Let the force and torque components of ${}^iF$ be denoted by ${}^if$ and ${}^im$ respectively. Then, the force/torque applied at the object reference point by robot i is given as below.

$$ {}^ih = \begin{bmatrix} {}^if \\ {}^im + {}^ir \times {}^if \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ {}^iS & I_3 \end{bmatrix}\begin{bmatrix} {}^if \\ {}^im \end{bmatrix} = {}^iR\,{}^iF \quad (3)$$

$$ {}^iS = \begin{bmatrix} 0 & -{}^ir_z & {}^ir_y \\ {}^ir_z & 0 & -{}^ir_x \\ -{}^ir_y & {}^ir_x & 0 \end{bmatrix},$$

$$ {}^ir = \begin{bmatrix} {}^ir_x \\ {}^ir_y \\ {}^ir_z \end{bmatrix}, \quad {}^iR = \begin{bmatrix} I_3 & 0 \\ {}^iS & I_3 \end{bmatrix}$$

Then,

$$ {}^iF = {}^iR^{-1}\,{}^ih \qquad (4)$$

Given the trajectory of the robots and the object, $\ddot{p}$, ω, $\dot{\omega}$, ${}^iq$, ${}^i\dot{q}$, ${}^i\ddot{q}$ are known. Let ${}^iu$ denote ${}^iD({}^iq)\,{}^i\ddot{q} + {}^iH({}^iq,{}^i\dot{q}) + {}^iG$. Then, (1) can be written as below.

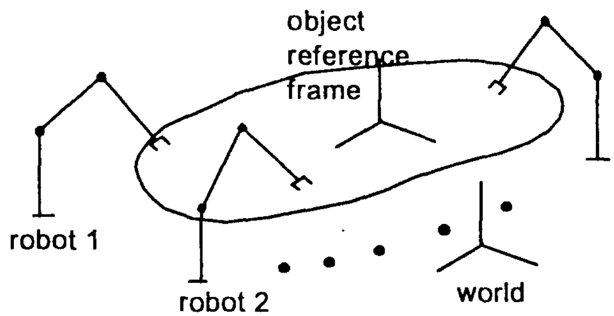


Fig. I Coordination of multiple robots

$$ {}^i\tau = {}^iJ^{T}\,{}^iR^{-1}\,{}^ih + {}^iu $$

$$ = {}^iV\,{}^ih + {}^iu \ , \ \text{where } {}^iV = {}^iJ^{T}\,{}^iR^{-1} \qquad (5) $$

The resultant force/torque F is the sum of the force/torques applied by the robots. Therefore, the force balance equation is

$$ F = \Sigma_{i=1}^{l}\ {}^ih \qquad (6) $$

where F is given by (2). The force/torques applied by joint torques of the robots must satisfy the constraint (6). The joint torque constraints are expressed by the maximum and minimum bounds of the joint actuator torques.

$$ {}^i\tau_{min} \le {}^i\tau \le {}^i\tau_{max} \ , \ i = 1,2,\ldots,l \qquad (7) $$

The general quadratic objective function that minimizes the sum of weighted norms of joint torques can be expressed as

$$ \Phi = \Sigma_{i=1}^{l}\ {}^i\tau^{T}\,{}^iQ\,{}^i\tau \qquad (8) $$

where

$$ {}^iQ = diag\{w_{i1},\ldots, w_{in}\} \ , \ w_{ij} \in R^1 $$

The element w_{ij} represents the weighting factor for j-th component of the torque vector of robot i. The objective function in (8) has frequently been used when the minimum energy criteria was employed [6-9]. Combining (6)-(8), the load distribution problem can be formulated as a quadratic optimization problem as below

$$ \begin{aligned} \text{Minimize} \quad & \Phi = \Sigma_{i=1}^{l}\ {}^i\tau^{T}\,{}^iQ\,{}^i\tau \\ \text{subject to} \quad & \Sigma_{i=1}^{l}\ {}^ih = F \\ & {}^i\tau \le {}^i\tau_{max} \\ & -{}^i\tau \le -{}^i\tau_{min} \ , \quad i = 1,2,\ldots,l \end{aligned} \qquad (9) $$

The force/torque term ${}^ih$ is related to the joint torques by (5). The formulation in (9) is a nonlinear optimization problem with quadratic objective function, and equality as well as inequality constraints. The optimal solution, if it exists, belongs to a nl - dimensional space.

3. Alternative Formulation of the Optimal Load Distribution

When the contact between the object and robot fingers is a frictional contact, each robot must maintain a suitable normal contact force at the contact point so that the contact between the object and the finger is not lost. However, when multipie robots rigidly grasp a common rigid body object, as the case is in this paper, the normal contact force need not be applied. Moreover, in order to prevent the damage to the object, it is generally desirable to keep the internal force to the minimum. Internal force is defined [10] as the component of the force applied by the robots on the object that are cancelled by each other, producing no motion of the object, but resulting in the stress in the object. From the above discussions, a reasonable control strategy in multiple robot coordination which can be employed in the industrial applications, is to maintain the internal force in the object to zero. This strategy has been used without considering the joint torque constraints in [6]. If the zero internal force strategy is adopted, then, the force/torque applied by the robot i , ${}^ih$, at the object reference point must be parallel to the resultant force F. Hence, ${}^ih$ can be expressed as

$$ {}^ih = \alpha_i F \qquad (10) $$

$$ \Sigma_{i=1}^{l}\ \alpha_i = 1 \ \text{ and } 0 \le \alpha_i \le 1, \quad i = 1,\ldots,l \qquad (11) $$

The inequality constraint (11) on the range of α_i is needed to comply with the control strategy that the internal force be zero. The objective function Φ can be expressed as a function of α_i using (5) and (10).

$$ \begin{aligned} \Phi &= \Sigma_{i=1}^{l}\ {}^i\tau^{T}\,{}^iQ\,{}^i\tau \\ &= \Sigma_{i=1}^{l}\ [\,\alpha_i\,{}^iV\,F + {}^iu\,]^{T}\,{}^iQ\,[\,\alpha_i\,{}^iV\,F + {}^iu\,] \\ &= \Sigma_{i=1}^{l}\ (\,F^{T}\,{}^iV^{T}\,{}^iQ\,{}^iV\,F\,)\,\alpha_i^2 \\ &\quad + 2\,(\,{}^iu^{T}\,{}^iQ\,{}^iV\,F\,)\,\alpha_i + {}^iu^{T}\,{}^iQ\,{}^iu \end{aligned} $$

$$
\begin{aligned}
&= \Sigma_{i=1}^{l}\ a_i\,\alpha_i^2 + b_i\,\alpha_i + c_i \\
&= \alpha^{\mathrm{T}} A\,\alpha + B\,\alpha + C \qquad (12)
\end{aligned}
$$

where

$$
\begin{aligned}
a_i &= F^{\mathrm{T}}\,{}^iV^{\mathrm{T}}\,{}^iQ\,{}^iV F,\quad b_i = 2\,(\,{}^iu^{\mathrm{T}}\,{}^iQ\,{}^iV F\,), \\
c_i &= {}^iu^{\mathrm{T}}\,{}^iQ\,{}^iu \\
\alpha &= [\,\alpha_1, \ldots, \alpha_l\,]^{\mathrm{T}},\quad A = diag\{\,a_1, \ldots, a_l\,\} \\
B &= [\,b_1, \ldots, b_l\,],\quad C = c_1 + c_2 + \ldots + c_l
\end{aligned}
$$

Since ${}^iQ$ is positive definite, a_i is strictly positive , and it follows that A is also positive definite. The joint torque constraint (7) can be rewritten using (5) and (10).

$$
{}^i\tau_{\min} - {}^iu \;\le\; \alpha_i\,{}^iV F \;\le\; {}^i\tau_{\max} - {}^iu \qquad (13)
$$

The expression in (13) represents n inequality constraints for the range of the scalar variable α_i , and thus, its feasible region can be obtained by the intersection of n constraints in (13). Let $(\bullet)_j$ denote the j-th element of $\bullet$ vector. Then, assuming $({}^iVF)_j \ne 0$ for all j , the constraints on α_i is obtained from (13) as below.

$$
{}^iLB_j \le \alpha_i \le {}^iUB_j\ ,\ j = 1, 2, \ldots n
$$

$$
\text{where}\quad {}^iUB_j = \frac{({}^i\tau_{\max})_j\,[({}^iVF)_j > 0] + ({}^i\tau_{\min})_j\,[({}^iVF)_j < 0] - ({}^iu)_j}{({}^iVF)_j}
$$

$$
{}^iLB_j = \frac{({}^i\tau_{\min})_j\,[({}^iVF)_j > 0] + ({}^i\tau_{\max})_j\,[({}^iVF)_j < 0] - ({}^iu)_j}{({}^iVF)_j} \qquad (14)
$$

Taking the intersection of n constraints,

$$
\begin{aligned}
&{}^iLB \le \alpha_i \le {}^iUB\,, \\
&{}^iLB = \max_j\ {}^iLB_j\,,\ {}^iUB = \min_j\ {}^iUB_j \qquad (15)
\end{aligned}
$$

where $[({}^iVF)_j > 0\,]$ evaluates to 1 if $({}^iVF)_j > 0$, and zero otherwise. Similarly, $[({}^iVF)_j < 0]$ evaluates to 1 if $({}^iVF)_j < 0$, and zero otherwise.

Note that it has not been assumed that the joint torque bounds , $({}^i\tau_{\max})_j$ and $({}^i\tau_{\min})_j$ are constants. They may be indeed arbitrary functions as long as their values are known before the calculation of (14). If $({}^iVF)_j = 0$ for some j, then (13) reduces to

$$
({}^i\tau_{\min})_j \le ({}^iu)_j \le ({}^i\tau_{\max})_j\,,\quad j = 1, \ldots \mathrm{n} \qquad (16)
$$

In this case, there is no extra load for the j-th actuator of robot i to move the object, apart from that to move its own link. If $({}^iu)_j$ does not satisfy the constraint (16) , then it implies that the joint torque required to move the links of the robot i exceeds the torque bound of j-th actuator. Consequently, the given trajectory of the robots and the object can not be generated and the trajectory of the robots and the object must be replanned. Hence the assumption that $({}^iVF)_j \ne 0$ can be used without loss of generality.

By definition, α_i must also satisfy (11). Combining (11) and (15), the feasible region of α_i is given by

$$
\begin{aligned}
&\alpha_{i,\min} \le \alpha_i \le \alpha_{i,\max}\ ,\quad i = 1, \ldots\, l \qquad (17) \\
&\quad where\quad \alpha_{i,\min} = \max\{\,0,\ {}^iLB\,\}\,, \\
&\quad \alpha_{i,\max} = \min\{\,1,\ {}^iUB\,\}
\end{aligned}
$$

It is noted that if $\alpha_{i,\min} \ge \alpha_{i,\max}$, then no feasible α_i exists and the load distribution problem has no solution. In this case, the trajectory of the object has to be replanned.

Summarizing the above results, the optimal load distribution problem can be formulated as a quadratic optimization problem using the vector variable α.

$$
\begin{aligned}
&\text{Minimize}\ \ \Phi = \alpha^{\mathrm{T}} A\,\alpha + B\,\alpha + C \\
&\text{subject to}\quad \eta^{\mathrm{T}}\alpha = 1 \\
&\qquad\qquad\quad \alpha_{\min} \le \alpha \le \alpha_{\max} \qquad (18)
\end{aligned}
$$

where $\eta = [\,1\ 1\ 1 \ldots 1\,]^{\mathrm{T}}$, $\alpha_{\min} = [\,\alpha_{1,\min}, \ldots, \alpha_{l,\min}\,]^{\mathrm{T}}$, $\alpha_{\max} = [\,\alpha_{1,\max}, \ldots, \alpha_{l,\max}\,]^{\mathrm{T}}$

The optimization problem in (18) searches for an optimal solution in a l - dimensional space. Note that compared to the formulation in (9), the search space was reduced from nl -dimensional space to l - dimensional space.

5. Two robot example

When two robots rigidly grasp a common object, the optimal load distribution solution can be obtained in a simple analytical form. For two robot case, the optimization problem in (18) can be written as

$$
\begin{aligned}
\text{Minimize}\ \ \Phi &= \alpha^{\mathrm{T}} A\,\alpha + B\,\alpha + C \\
&= a_1\,\alpha_1^2 + a_2\,\alpha_2^2 + b_1\,\alpha_1 + b_2\,\alpha_2 + c_1 + c_2
\end{aligned}
$$

$$
\begin{aligned}
&\text{subject to} \\
&\qquad \alpha_1 + \alpha_2 = 1 \\
&\qquad \alpha_{1,\min} \le \alpha_1 \le \alpha_{1,\max} \\
&\qquad \alpha_{2,\min} \le \alpha_2 \le \alpha_{1,\max} \qquad (19)
\end{aligned}
$$

Using $\alpha_2 = 1 - \alpha_1$, the objective function is reformed as

$$
\begin{aligned}
\Phi = &\ (\,a_1 + a_2\,)\,\alpha_1^2 + (\,b_1 - 2a_2 - b_2\,)\,\alpha_1 \\
&+ (\,a_2 + b_2 + c_1 + c_2\,) \qquad (20)
\end{aligned}
$$

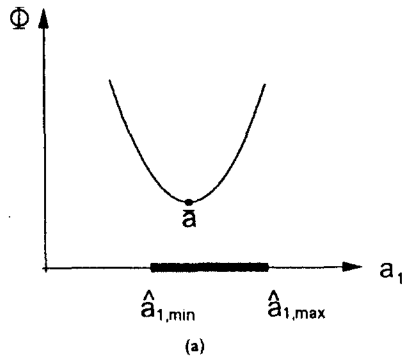
, and the constraints can be written as

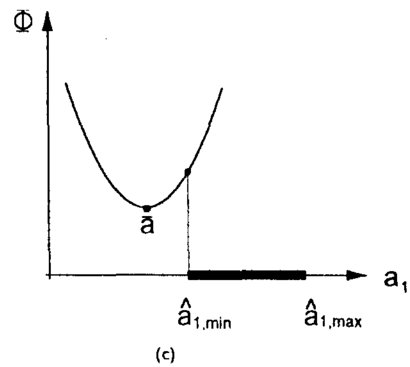
$$
\begin{aligned}
&\alpha_{1,\min} \le \alpha_1 \le \alpha_{1,\max} \\
\text{and}\quad &1 - \alpha_{2,\max} \le \alpha_1 \le 1 - \alpha_{2,\min} \qquad (21)
\end{aligned}
$$

Then, the range of α_1 is finally formed as

$$
\begin{aligned}
&\hat{\alpha}_{1,\min} \le \alpha_1 \le \hat{\alpha}_{1,\max}\ , \\
&\text{where}\quad \hat{\alpha}_{1,\min} = \max\{\alpha_{1,\min},\, 1 - \alpha_{2,\max}\} \\
&\qquad\qquad \hat{\alpha}_{1,\max} = \min\{\alpha_{1,\max},\, 1 - \alpha_{2,\min}\} \qquad (22)
\end{aligned}
$$

Summarizing the above results, the optimization problem is simplified to





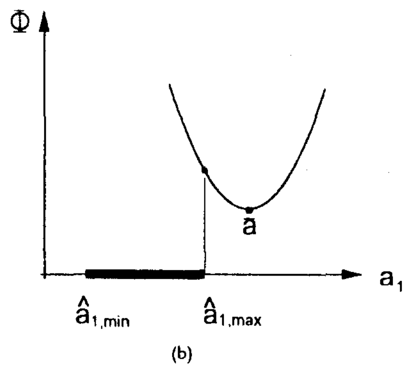


Fig.2 Solution for two robot example.
(a) $\hat{\alpha}_{1,min} \le \bar{\alpha} \le \hat{\alpha}_{1,max}$ and $\alpha_1^* = \bar{\alpha}$
(b) $\bar{\alpha} \ge \hat{\alpha}_{1,max}$, and $\alpha_1^* = \hat{\alpha}_{1,max}$
(c) $\bar{\alpha} \le \hat{\alpha}_{1,min}$, and $\alpha_1^* = \hat{\alpha}_{1,min}$

Minimize

$$\Phi = (a_1 + a_2)\alpha_1^2 + (b_1 - 2a_2 - b_2)\alpha_1 + (a_2 + b_2 + c_1 + c_2)$$

$$\text{subject to } \hat{\alpha}_{1,min} \le \alpha_1 \le \hat{\alpha}_{1,max} \tag{23}$$

The solution of the above problem is now given below. The objective function Φ has a unique global optimal , and the solution is divided into three cases depending on the position of this global optimal point. Three cases are illustrated in Fig. 2.

Let α_1^* denote the optimal solution , and
$\bar{\alpha} = \frac{-(b_1 - 2a_2 - b_2)}{2(a_1 + a_2)}$ denote the global optimal of Φ .

i) If $\hat{\alpha}_{1,min} \le \bar{\alpha} \le \hat{\alpha}_{1,max}$, then, $\alpha_1^* = \bar{\alpha}$

ii) If $\bar{\alpha} \ge \hat{\alpha}_{1,max}$, then, $\alpha_1^* = \hat{\alpha}_{1,max}$

iii) If $\bar{\alpha} \le \hat{\alpha}_{1,min}$, then, $\alpha_1^* = \hat{\alpha}_{1,min}$

Once α_1^* and $\alpha_2^* = 1 - \alpha_1^*$ is found, the joint torques can be obtained using (5) and (10).

6. Conclusions

The optimal load distribution with joint torque constraints is studied and an efficient algorithm is presented. Objective function to be minimized is the sum of weighted norm of joint torque vectors, and joint torque bounds in arbitrary form is considered. By employing an additional constraint of zero internal force in the object, the dimension of the feasible solution space is reduced from nl to l , resulting in a major reduction in computation. The problem is in the form of quadratic optimization problem and any popular optimization technique can be used to obtain the solution. For two robot example, the solution is given in a simple analytical form.

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