'93 KACC (1993. 10. 20~22)

A Study on the Applicability of Augmented Proportional Navigation Guidance

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Abstract

Augmented proportional navigation requires the information of a target acceleration, which must be estimated by a filtering logic. The process necessary accompanies a time lag, which degrades the guidance performance.

A trade-off study between augmented and conventional proportional navigation is conducted with the time lag taken into consideration. The result shows that the conventional proportional navigation has better performance against a target maneuver in the missile-target coplane, while the augmented has better performance against out-of-coplane maneuver.

Nomenclature

a_{cmax}	= missile lateral accelelation comand limit
a_p, a_y, a_{pc}, a_{yc}	= missile-pitch and yaw-axis lateral accelerations and their command signals
a_{pt}, a_{yt}	= aircraft-desired pitch and yaw acceleration components, respectively
a_{tp}, a_{ty}	= aircraft acceleration components measured in missile pitch and yaw axis, respectively
$\hat{a}_{tp}, \hat{a}_{ty}$	= filtered values of a_{tp} and a_{ty}, respectively
C_D	= drag coefficient
C_{D0}	= zero-lift drag coefficient
$C_L, C_{L\alpha}$	= lift coefficient and lift coefficient derivative
D, D_m	= aircraft and missile drag, respectively
g	= acceleration of gravity
g_{bias}	= gravity compensation term in missile guidance system
h	= altitude
k	= aircraft induced-drag coefficient
k_1, k_2	= missile drag coefficients
L	= lift
MD	= miss distance
m	= mass
N_e, N_{e_t}	= missile and aircraft effective navigation constants, respectively
r, r_x, r_y, r_z	= relative range between missile and aircraft and its inertial x, y and z components, respectively
s	= reference area
T	= thrust
t	= time
v	= velocity
v_c	= closing velocity
x, y, z	= inertial coordinates
α, α_0	= angle of attack and zero lift angle
γ, ψ	= flight path and azimuth angles, respectively
η	= introduced angle to rotate line-of-sight rate vector
ρ, ρ_m	= air densities at aircraft and missile altitudes, respectively
$\dot{\bar{\sigma}}, \dot{\sigma}_p, \dot{\sigma}_y$	= line-of-sight rate vector and its pitch and yaw components, respectively
$\dot{\sigma}_{xI}, \dot{\sigma}_{yI}, \dot{\sigma}_{zI}$	= inertial x, y and z components of $\dot{\bar{\sigma}}$, respectively
τ	= missile time constant
τ_1	= target acceleration filter time constant
τ_α, τ_ϕ	= aircraft α and ϕ control time constants, respectively
ϕ	= aircraft roll angle
$(\dot{\ })$	= time derivative
$(\bar{\ })$	= vector
Subscripts	
0	= initial value
c	= control command signal
f	= terminal (final) value
I	= inertial coordinate
max	= maximum value
m	= missile
p	= pitch component
t	= aircraft (target)
y	= yaw or y component

Introduction

The authors have studied the fighter evasive maneuvers against PNG (proportional navigation guidance) missiles, such as the optimal evasive maneuvers, the maximum sustained g turns[1], and the HGBs (high-g barrel roll)[2]. The results show that, the fighter has enough evasive opportunity, if it employs its maximum performance. Many papers suggest the employment of modern control theory in order to improve PNG, but its implementation still seems to be difficult in current hardware point of view. However, APNG (augmented PNG)[3] seems promising in near future. The essential idea of APNG is to introduce a target lateral acceleration compensation term into PNG. Therefore provided with the information on the target lateral acceleration, its application to a current PNG missile is straightforward. Unfortunately the obtained information is deeply contaminated by noise, therefore the true value must be estimated. The use of an extended Kalman filter is assumed here. The superiority of APNG over PNG is well exhibited in our preceding studies[3,4], but in these papers simulations are based on the true target acceleration. As the estimating process is necessary accompanied with a delay time, in this paper the effect is introduced as a first order time lag, and the applicability of APNG in an actual missile is evaluated. Simultaneously, a trade-off study of PNG and APNG are carried out by massive simulations in the parameter space of the missile and target initial geometries and guidance law parameters. In this paper, the mathematical models of the dynamics and the guidance laws of the missile and aircraft are briefly explained first. Next, the results of the simulation study are summarized and discussed.

Mathematical Simulation Model

The model employed is the same as that of Ref. (4), but stated here for the reader's convenience. Figure 1 shows missile and aircraft symbols. Point mass models are used for both vehicles, and a no side-slip condition is assumed for the aircraft. The equations employed are as follows:

Aircraft Dynamics

$$\dot{v}_t = \frac{1}{m_t}(T_t \cos\alpha - D) - g \sin\gamma_t \quad (1)$$

$$\dot{\gamma}_t = \frac{1}{m_t v_t}(L + T_t \sin\alpha)\cos\phi - \frac{g}{v_t}\cos\gamma_t \quad (2)$$

$$\dot{\psi}_t = \frac{(L + T_t \sin\alpha)}{m_t v_t \cos\gamma_t}\sin\phi \quad (3)$$

$$\dot{x}_t = v_t \cos\gamma_t \cos\psi_t \quad (4)$$

$$\dot{y}_t = v_t \cos\gamma_t \sin\psi_t \quad (5)$$

$$\dot{h}_t = v_t \sin\gamma_t \quad (6)$$

where

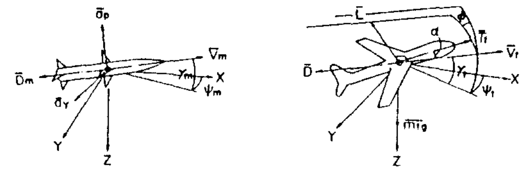

Fig. 1 Missile and aircraft symbols

$$L = 1/2\rho v_t^2\, s_t\, C_L \quad (7)$$

$$C_L = C_{L\alpha}(\alpha - \alpha_0) \quad (8)$$

$$D = 1/2\rho v_t^2\, s_t\, C_D \quad (9)$$

$$C_D = C_{D0} + k\, C_L^2 \quad (10)$$

Missile Dynamics

$$\dot{v}_m = \frac{1}{m_m}(T_m - D_m) - g \sin\gamma_m \quad (11)$$

$$\dot{a}_p = (a_{pc} - a_p)\,/\,\tau \quad (12)$$

$$\dot{a}_y = (a_{yc} - a_y)\,/\,\tau \quad (13)$$

$$\dot{\gamma}_m = (a_p - g\cos\gamma_m)\,/\,v_m \quad (14)$$

$$\dot{\psi}_m = \frac{a_y}{v_m \cos\gamma_m} \quad (15)$$

$$\dot{x}_m = v_m \cos\gamma_m \cos\psi_m \quad (16)$$

$$\dot{y}_m = v_m \cos\gamma_m \sin\psi_m \quad (17)$$

$$\dot{h}_m = v_m \sin\gamma_m \quad (18)$$

where

$$D_m = k_1\, v_m^2 + k_2 \frac{a_p^2 + a_y^2}{v_m^2} \quad (19)$$

where

$$k_1 = 1/2\,\rho_m\, s_m\, C_{D0m} \quad (20)$$

$$k_2 = k_m\, m_m^2/(1/2\,\rho_m\, s_m) \quad (21)$$

Missile Control

PNG

Roll stabilized proportional navigation is assumed with signal saturation taken into account. The pitch and yaw-axis acceleration commands a_{pc} and a_{yc} are given by

$$a_{pc} = \begin{cases} N_e\, v_c\, \dot{\sigma}_p + g_{bias} & for\ |a_{pc}| \le a_{cmax} \\ a_{cmax}\, \mathrm{sign}(a_{pc}) & for\ |a_{pc}| > a_{cmax} \end{cases} \quad (22)$$

$$a_{pc} = \begin{cases} N_e\, v_c \dot{\sigma}_y & for\ |a_{yc}| \le a_{cmax} \\ a_{cmax}\, \mathrm{sign}(a_{yc}) & for\ |a_{yc}| > a_{cmax} \end{cases} \quad (23)$$

where g_{bias} is the compensation term for gravity, and given by

$$g_{bias} = g\cos\gamma_m \quad (24)$$

In Eqs. (22) and (23), N_e is the effective navigation constant, $\dot\sigma_p$ and $\dot\sigma_y$ are the target LOS (line-of-sight) rate vector pitch and yaw components measured in the missile body axes, and v_c is the closing velocity given by

$$\dot{\bar\sigma} = \frac{\bar r \times (d/dt)\,\bar r}{\bar r \cdot \bar r} = \frac{1}{r^2}\begin{bmatrix} r_y\,\dot r_z & - & r_z\,\dot r_y \\ r_z\,\dot r_x & - & r_x\,\dot r_z \\ r_x\,\dot r_y & - & r_y\,\dot r_x \end{bmatrix} = \begin{bmatrix} \dot\sigma_{xI} \\ \dot\sigma_{yI} \\ \dot\sigma_{zI} \end{bmatrix}_I \quad (25)$$

$$\dot\sigma_p = -\sin\psi_m\,\dot\sigma_{xI} + \cos\psi_m\,\dot\sigma_{yI} \quad (26)$$

$$\dot\sigma_y = \sin\gamma_m(\cos\psi_m\,\dot\sigma_{xI} + \sin\psi_m\,\dot\sigma_{yI}) + \cos\gamma_m\,\dot\sigma_{zI} \quad (27)$$

and

$$v_c = -\dot r = \frac{-(r_x\dot r_x + r_y\dot r_y + r_z\dot r_z)}{r} \quad (28)$$

where r is missile-aircraft relative range and r_x, r_y, and r_z are its inertial three axes components.

APNG

It is will known that PNG is obtained as the optimal control against a nonmaneuvering target, while APNG is against a maneuvering target.[3] In APNG, the target (aircraft) lateral acceleration must be employed. Practically, this value must be estimated under a noisy condition, and an extended Kalman Filter[5] may be used. Figure 2 shows an example of the estimated target acceleration. The output is approximated through a first order lag to true target acceleration, where the time constant τ_1 is a function of noise level and sampling time. The pitch and yaw acceleration commands a_{pc} and a_{yc}, with APNG, are given by

$$a_{pc} = \begin{cases} N_e(v_c\,\dot\sigma_p + \frac{1}{2}\,\hat a_{tp}) + g_{bias} & for\ |\,a_{pc}\,| \le a_{cmax} \\ a_{cmax}\ \mathrm{sigh}(a_{pc}) & for\ |\,a_{pc}\,| > a_{cmax} \end{cases} \quad (29)$$

$$a_{yc} = \begin{cases} N_e(v_c\,\dot\sigma_y + \frac{1}{2}\,\hat a_{ty}) & for\ |\,a_{yc}\,| \le a_{cmax} \\ a_{cmax}\ \mathrm{sigh}(a_{yc}) & for\ |\,a_{yc}\,| > a_{cmax} \end{cases} \quad (30)$$

where

$$\dot{\hat a}_{tp} = (a_{tp} - \hat a_{tp})/\tau_1 \quad (31)$$

$$\dot{\hat a}_{ty} = (a_{ty} - \hat a_{ty})/\tau_1 \quad (32)$$

where a_{tp} and a_{ty} are true target acceleration pitch and yaw components given by

$$a_{tp} = -\sin\gamma_m(\cos\psi_m\,\ddot x_t + \sin\psi_m\,\ddot y_t) - \cos\psi_m\,\ddot z_t \quad (33)$$

$$a_{ty} = -\sin\gamma_m\,\ddot x_t + \cos\gamma_m\,\ddot y_t \quad (34)$$

Aircraft Control

The LOS rate vector from the aircraft to the missile is obtained by changing the sign of $\bar r$ in Eq. (25), which results in the same equation as (25). The pitch and yaw components of $\dot{\bar\sigma}$: $\dot\sigma_{pt}$ and $\dot\sigma_{yt}$ measured in the aircraft body axes are given by

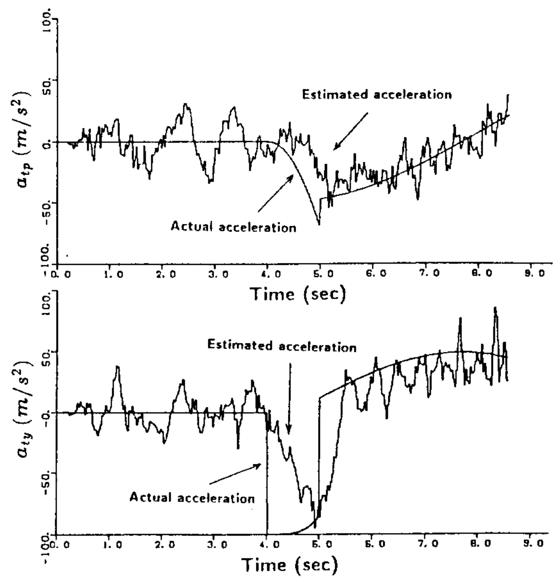

Fig. 2 An estimation example by extended Kalman filter (Sampling time 30ms)

$$\dot\sigma_{pt} = -\sin\psi_t\,\dot\sigma_{xI} + \cos\psi_t\,\dot\sigma_{yI} \quad (35)$$

$$\dot\sigma_{yt} = -\sin\gamma_t(\cos\psi_t\,\dot\sigma_{xI} + \sin\psi_t\,\dot\sigma_{yI}) - \cos\gamma_t\,\dot\sigma_{zI} \quad (36)$$

Analogus to the missile PNG, the aircraft has to produce the acceleration components a_{p0} and a_{y0} in the pitch and yaw directions, but for evasion purposes, those sign should be reversed.

$$a_{p0} = -N_{et}\,v_c\,\dot\sigma_{pt} \quad (37)$$

$$a_{y0} = -N_{et}\,v_c\,\dot\sigma_{yt} \quad (38)$$

Our previous studies showed that the high-g barrel roll[2] is quite an efficient evasive maneuver against PNG and APNG missiles ; the prominent feature of the maneuver is producing LOS rate change by rotating the LOS vector in a pitch-yaw plane. Motivated by this fact, the arbitrary angle η, shown in Fig. 3, is introduced to rotate the LOS vector. Then, the desired pitch and yaw acceleration components of the aircraft are

$$a_{pt} = a_{p0}\cos\eta + a_{y0}\sin\eta \quad (39)$$

$$a_{yt} = -a_{p0}\sin\eta + a_{y0}\cos\eta \quad (40)$$

When η=0, the aircraft evades in a coplane with the missile, while with η=±90°, the aircraft evades the missile by rotating the LOS vector normal to the current maneuvering plane. In order to obtain the desired aircraft acceleration, the aircraft α and ϕ are determined as follows:

$$(L + T_t\sin\alpha)\sin\phi = m_t\,a_{yt} \quad (41)$$

$$(L + T_t\sin\alpha)\cos\phi - m_t\,g\cos\gamma_t = m_t\,a_{pt} \quad (42)$$

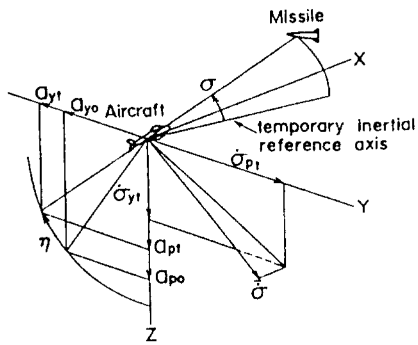

Fig. 3 LOS, its rate vector and η employed in the aircraft control

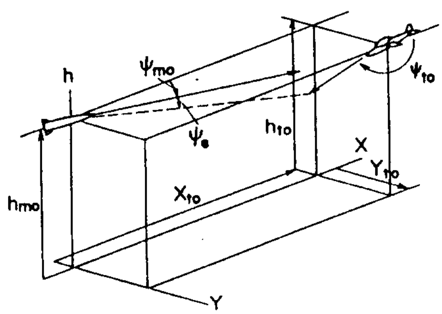

Fig. 4 Initial relative geometry

By approximating $\sin\alpha \cong \alpha$ and from the foregoing equations, we obtain

$$\phi = \tan^{-1}[a_{yt} \,/\, (a_{pt} + g\cos\gamma_t)\,] \qquad (43)$$

and

$$\alpha = m_t\,[\,(a_{pt} + g\cos\gamma_t)^2 + a_{yt}^2]^{\frac{1}{2}} / \,(1/2\rho v_t^2\, s_t\, C_{L\alpha} + T_t) \quad (44)$$

Simulation Conditions and Results

Figure 4 shows the initial geometry of a missile and an aircraft. A set of nominal parameters for both vehicles are given in Table 1. The initial altitude of vehicles is set at 3000m; the modeled aircraft can produce a maximum normal acceleration of 7g's at this altitude. Under a high-g condition, it becomes rather difficult to maneuver an aircraft with a large roll rate; therefore, the maximum roll rate command $\dot{\phi}_{cmax}$, is treated as a function of α.

Simulations are conducted for a combination of missile guidance law and that of the aircraft. As for the latter, the determined strategy is to rotate the relative LOS vector, and the arbitrary rotation angle η is changed over the $-90°$ to $90°$ deg range. The effective navigation constant

Table 1 Nominal parameters

Aircraft		
m_t	= 7,500 kg	
s_t	= 26.0 m^2	
v_{t0}	= 290 m/s	
h_{t0}	= 3,000 m	
x_{t0}	= 4,000 m	
$C_{L\alpha}$	= 4.01 / rad	
C_{D0}	= 0.0169	
k	= 0.179	
α_{max}	= 0.13 rad	
T_t	= 65,000 N	
τ_α	= 0.3 s	
τ_ϕ	= 0.2 s	
$\dot{\phi}_{cmax}$	= 16 rad/s	($\alpha = 0$)
	8 rad/s	($\alpha = 0.065$ rad)
	3.2 rad/s	($\alpha = 0.13$ rad)
Missile (with sustainer)		
m_m	= 165kg(t=0)	150kg(t=6s)
s_m	= 0.032 m^2	
v_{m0}	= 600 m/s	
h_{m0}	= 3000 m	
x_{m0}	= 0 m	
$C_{L\alpha}$	= 35.0 / red	(at Mach 2)
C_{D0}	= 0.74	(at Mach 2)
k	= 0.03	(at Mach 2)
T_m	= 5880N ($0 \le t \le 6$s)	
a_{cmax}	= 30 g	
τ	= 0.3 s	

of the missile N_e is set to 3 or 4 ; that of the aircraft N_{e_t} is set to 4 or 6. In the simulations, the aircraft is given a command in the first second to take the maximum angle of attack and a preset roll angle. After that, the aircraft evasive control algorithm stated in the preceding section is activated. This initial ϕ command is changed from -180 to 180 deg at intervals of 45 deg. Figures 5 through 8 show MD (miss distance) distributions in relation to η. The four kinds of differently patterned areas indicate the number of cases where MDs are 0~4m, 4~8m, 8~12m and 12~20m, respectively. Figure 5 shows a PNG missile case. Small MD regions are concentrated in the neighborhood of η=0 deg. The MD is within 8m in the range over η=-60 deg to 60 deg, and within 4m from η=-35 deg to 25 deg. Figure 6 shows a APNG missile case with τ_1=0; which means the case where a target acceleration is ideally estimated and no lag exists. On the contrary to the result of PNG, small MD regions are concentrated in the neighborhood of η=$\pm$90 deg. However, even in the worst case where η=10~15 deg, more than 50 % cases MDs are still less than 4m. Figures 7

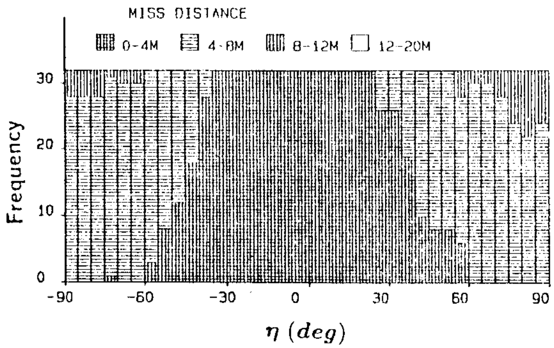

Fig. 5 Miss distance distribution (PNG missile)

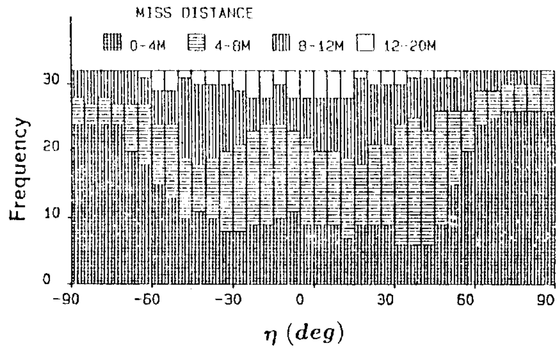

Fig. 7 Miss distance distribution (APNG missile, τ_1=0.3s)

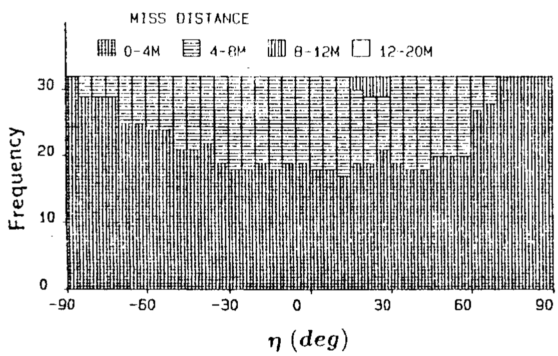

Fig. 6 Miss distance distribution (APNG missile, τ_1=0)

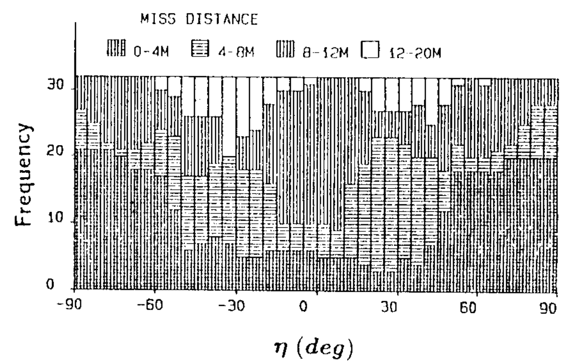

Fig. 8 Miss distance distribution (APNG missile, τ_1=0.5s)

and 8 show the cases where time lags τ_1=0.3s and 0.5s exist, respectively. As τ_1 increases MD naturally increases, however, in the neighborhood of $\eta=\pm 90$ deg, MD is far smaller than that of PNG even in the case where a large time lag τ_1=0.5s exists. These results show that PNG is favorable for a small η and APNG is sperior to PNG for a η in the neighborhood of ±90 deg even in a case where a fairly large time lag exists for the estimation of the target acceleration. As these η values in the neighborhood of 0 deg and ±90 deg mean that the target maneuver lies in or out of missile-target coplane respectively, and the missile can discriminate which maneuver is being taken in the process of the target acceleration estimation. Therefore for a practical use, it is recommended to switch the guidance algorithm between PNG and APNG depending on the kind of the target maneuver.

Conclusions

A trade-off study between conventional and augmented proportional navigation is conducted with the time lag for obtaining the information of the target acceleration, which is employed in the augmented proportional navigation, taken into consideration. Simulations are conducted in relation to the guidance laws and the angle η which expresses the direction of the target line-of-sight rate vector. A η in the neighborhood of 0 deg means the target maneuver lies in the missile-target coplane, while a η in the neighborhood of ±90 deg means out-of coplane. The study result shows that conventional proportional navigation is favorable against the aircraft in-coplane maneuver, while augmented proportional navigation is superior to the former against the aircraft out-of-coplane maneuver even in a case where a fairly large time lag exists in the target acceleration estimation. Therefore it is recommended to switch the guidance algorithm between conventional and augmented proportional navigation depending on the kind of the target maneuver.

Acknowledgments

The authors are indebted to Dr. S. Uehara, Director General for Research and Development of the Japan Defence Agency for giving us useful advice and discussing the contents of this paper.

References

[1] Imado, F. and Miwa, S. , "Fighter Evasive Maneuvers Against Proportional Navigation Missile", Journal of Aircraft, vol. 23, No. 11, Nov. 1986, pp. 825-830.

[2] Imado, F. and Miwa, S. , "Three Dimentional Study of Evasive Maneuvers of a Fighter Against a Missile", AIAA Paper 86-2038, Aug. 1986.

[3] Imado, F. and Miwa, S. , "Fighter Evasive Boundaries Against Missiles", Computers Mathematics with Applications, vol. 18, No. 1-3, 1989, pp. 1-14.

[4] Imado, F. , "Some Aspects of a Realistic Three-Dimensional Pursuit-Evasion Game", Journal of Guidance, Control, and Dynamics, vol. 16, No. 2, 1993, pp. 289-293.

[5] Zhou, H. , and Kamar, K. S. P. , "A Current Statistical Model and Adaptive Algorithm for Estimating Maneuvering Targets", Journal of Guidance, Control, and Dynamics, vol. 7, No. 5, 1984, pp. 596-602.