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A Design of Discrete-Time Integral Controllers Under Skewed Sampling

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Abstract

First, we propose a transparent and efficient design of discrete-time integral controllers accounting sampling skew. Based on the proposed controller, we derive a state-space representation of doubly coprime factorization including integral action. The representation is then used to obtain a convenient state-space parametrization of discrete-time two-degree-of-freedom integral controllers accounting sampling skew.

1. Introduction

Integral controllers have been widely used to achieve zero steady-state errors for step references and/or step disturbances. Mita [1] has proposed a novel state feedback design of discrete-time integral controllers taking account of the full sampling delay. Ishihara *et al.*[2] have reformulated Mita's design and have proposed a transparent design of observer-based integral controllers accounting computation delay which is a multiple of sampling period. Taking this controller as a basic controller, Guo *et al.*[3] have proposed a convenient state-space parametrization of discrete-time two-degree-of-freedom integral controllers.

Recently, digital controller design accounting sampling skew has drawn renewed attention [4][5]. In this paper, by generalizing results given in [1]-[3], we propose a transparent and efficient design of discrete-time integral controllers accounting sampling skew. First, the state feedback design [1] for the full sampling delay case is generalized to skewed sampling case. An observer-based controller is constructed for the output feedback case. Using the observer-based controller as a basic controller, we derive a state space representation of doubly coprime factorization with integral action. Using the representation, we give a convenient parametrization of two-degree-of-freedom integral controllers accounting the sampling skew.

2. Preliminaries

Consider a continuous-time plant described by

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) \qquad (2.1)$$

where $x \in R^n$ is a state vector, $u \in R^m$ is a control vector and $y \in R^m$ in an output vector. Define the skewed output as

$$y_\delta(k) = y(kT-\delta), \quad k=1,2,\ldots,$$

where T is sampling period and δ is a skew factor. Then the discrete-time model of (2.1) under the skewed sampling is given by

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k) \\ y_\delta(k+1) &= H_\delta x(k) + D_\delta u(k) \end{aligned} \qquad (2.2)$$

where

$$\begin{aligned} &F = e^{AT}, \quad G = \int_0^T e^{A\tau}B\,d\tau \\ &H_\delta = Ce^{A(T-\delta)}, \quad D_\delta = C\int_0^{T-\delta} e^{A\tau}B\,d\tau \end{aligned} \qquad (2.3)$$

In the following discussion, we assume that the pair (F,G) is controllable, the pair (H_δ,F) is observable and $\det CB \neq 0$. Define

$$E_\delta = \begin{bmatrix} F-I & G \\ H_\delta & D_\delta \end{bmatrix} \qquad (2.4)$$

and assume $\det E_\delta \neq 0$.

For the output feedback design, we use a full order observer given by

$$\begin{aligned} \hat{x}(k+1) &= F\hat{x}(k) + Gu(k) + K_\delta[y_\delta(k+1) - \hat{y}_\delta(k+1)] \\ \hat{y}_\delta(k+1) &= H_\delta \hat{x}(k) + D_\delta u(k) \end{aligned} \qquad (2.5)$$

where K_δ is an observer gain matrix.

In the next section, we will propose an output feedback design of integral controllers accounting the sampling skew.

3. Integral Controller Design

First, we extend the reformulated version [2] of the state feedback design of integral controllers [1] to the skewed sampling case. Assume that the output is obtained by the skewed sampling while the states are measurable without the sampling skew. This assumption is unrealistic but is convenient for constructing an observer-based controller.

As a structure of an integral controller, we consider

$$\begin{aligned} u(k) &= -O_\delta x(k) + s_\delta(k) + J_\delta[\, r_\delta(k) - y_\delta(k)] \\ s_\delta(k+1) &= s_\delta(k) + J_\delta[\, r_\delta(k) - y_\delta(k)] \end{aligned} \qquad (3.1)$$

where, $s_\delta(k)$ is the state of an integrator, $r_\delta(k)$ is the skewed-sampled value of a continuous time step signal $r(t)$.

For an efficient determination of the matrices O_δ and J_δ in (3.1), we have the following result.

Proposition 3.1: Let L is a arbitrary matrix that makes the matrix $F-GL$ stable. Determine the matrices O_δ and J_δ in (3.1) by the matrix equation

$$[\, O_\delta \;\; J_\delta]E_\delta = [\, LF \;\; I+LG] \qquad (3.2)$$

Then, the closed-loop system consisting (2.2) and (3.1) is asymptotically stable and the zero steady-state error for a step reference signal $r_\delta(k)$ is achieved. In addition, the behavior of the error vector

$$\xi(k) = [\, x'(k) - x'(\infty) \;\;\; u'(k) - u'(\infty)\,]' \qquad (3.3)$$

where $x(\infty)$ and $u(\infty)$ are the steady values of the state and the control input, respectively, is described by

$$\xi(k+1) = \Phi\xi(k) \qquad (3.4)$$

where

$$\Phi = \begin{bmatrix} F & G \\ -LF & -LG \end{bmatrix} \qquad (3.5)$$

The eigenvalues of the error transition matrix (3.5) consist of the eigenvalues of $F-GL$ and m zeros.

Proof: The closed-loop system consisting of (2.2) and (3.1) is described by

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} &= \Psi \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} 0 \\ J_\delta \end{bmatrix} r_\delta(k) \\ y_\delta(k+1) &= [\, H_\delta \;\; D_\delta\,] \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \end{aligned} \qquad (3.6)$$

where

$$\Psi = \begin{bmatrix} F & G \\ -O_\delta(F-I) - J_\delta H_\delta & I - (O_\delta G + J_\delta D_\delta) \end{bmatrix}$$

Note that (3.7) can be expressed as

$$\Psi = I + \begin{bmatrix} I & 0 \\ -O_\delta & -J_\delta \end{bmatrix} \begin{bmatrix} F-I & G \\ H_\delta & D_\delta \end{bmatrix} \qquad (3.8)$$

Assume that the closed loop system is asymptotically stable. Then, it follows from (3.6) and (3.8) that the steady state is expressed as

$$\begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} = E_\delta^{-1} \begin{bmatrix} I & 0 \\ J_\delta^{-1} O_\delta & J_\delta^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ J_\delta r \end{bmatrix} \qquad (3.9)$$

Consequently, the steady state output is given by

$$y_\delta(\infty) = [\, H_\delta \;\; D_\delta\,] E_\delta^{-1} \begin{bmatrix} 0 \\ r \end{bmatrix} = r \qquad (3.10)$$

Next, we show the closed loop stability. It follows from (3.6) and (3.8) that the behavior of the error can be described as

$$\xi(k+1) = \begin{bmatrix} F & G \\ 0 & 0 \end{bmatrix} \xi(k) + \begin{bmatrix} 0 \\ I \end{bmatrix} v(k) \qquad (3.11)$$

where $v(k) = u_e(k+1)$ is a new control input defined by

$$\begin{aligned} v(k) &= -[\, O_\delta(F-I) + J_\delta H_\delta \;\;\; O_\delta G + J_\delta D_\delta - I\,]\xi(k) \\ &= -\{[\, O_\delta \; J_\delta]E_\delta + [\, 0 \;\; -I\,]\}\xi(k) \end{aligned} \qquad (3.12)$$

For the matrices O_δ and J_δ satisfying (3.2), (3.12) can be rewritten as

$$v(k) = -[LF \quad LG]\xi(k) \tag{3.13}$$

and the error system can be described by (3.4). Using the technique used by Mita [1], it can easily be shown that the eigenvalues of the error transition matrix (3.5) are those of $F-GL$ and m zeros. ■

The above result show that the state feedback matrices O_δ and J_δ can easily determined by solving the linear matrix equation (3.2) for arbitrary state feedback gain matrix L for the standard regulator problem for the plant.

For the output feedback case, we can construct an integral controller by replacing the state $x(t)$ in (3.1) with the estate estimate generated by an observer. Then, the algorithm of the observer-based controller is described by

$$\begin{aligned} u(k) &= -O_\delta \hat{x}(k) + s_\delta(k) + J_\delta[r(k) - y_\delta(k)] \\ s_\delta(k+1) &= s_\delta(k) + J_\delta[r(k) - y_\delta(k)] \end{aligned} \tag{3.14}$$

4. Doubly Coprime Factorization with Integral Action

Williamson [4] has obtained a state space representation of the doubly coprime factorization accounting the sampling skew by a direct use of the well-known continuous-time result [6] which is based on an LQG controller. The result is summarized as follows.

Lemma 4.1: Let $M_\delta(z)$ and $C_{LQG}(z)$ denote the transfer function matrix of the plant and that of the LQG controller with the state feedback gain matrix L and the observer gain matrix K_δ, respectively. Then $M_\delta(z)$ and $C_{LQG}(z)$ can be expressed in coprime forms as

$$\begin{aligned} M_\delta(z) &= z^{-1}[D_\delta + H_\delta(zI-F)^{-1}G] \\ &= \tilde{Q}_\delta^{-1}(z)\tilde{P}_\delta(z) = P_\delta(z)Q_\delta^{-1}(z) \end{aligned} \tag{4.1}$$

$$C_{LQG}(z) = \tilde{S}_\delta^{-1}(z)\tilde{R}_\delta(z) = R_\delta(z)S_\delta^{-1}(z) \tag{4.2}$$

where

$$P_\delta(z) = z^{-1}[D_\delta + (H_\delta - D_\delta L)(zI-F+GL)^{-1}G] \tag{4.3}$$

$$Q_\delta(z) = I - L(zI-F+GL)^{-1}G \tag{4.4}$$

$$\tilde{P}_\delta(z) = z^{-1}[D_\delta + H_\delta(zI-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)] \tag{4.5}$$

$$\tilde{Q}_\delta(z) = I - H_\delta(zI-F+K_\delta H_\delta)^{-1}K_\delta \tag{4.6}$$

$$R_\delta(z) = zL(zI-F+GL)^{-1}K_\delta \tag{4.7}$$

$$S_\delta(z) = I + (H_\delta - D_\delta L)(zI-F+GL)^{-1}K_\delta \tag{4.8}$$

$$\tilde{R}_\delta(z) = zL(zI-F+K_\delta H_\delta)^{-1}K_\delta \tag{4.9}$$

$$\tilde{S}_\delta(z) = I + L(zI-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta) \tag{4.10}$$

The eight matrices in (4.3)-(4.10) are proper and stable and constitute the doubly coprime factorization

$$\begin{bmatrix} \tilde{R}_\delta(z) & \tilde{S}_\delta(z) \\ -\tilde{P}_\delta(z) & \tilde{Q}_\delta(z) \end{bmatrix} \begin{bmatrix} Q_\delta(z) & -S_\delta(z) \\ P_\delta(z) & R_\delta(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{4.11}$$

□

Since the above representation is based on an LQG controller, it is inconvenient to parametrize all stabilizing integral controllers. Taking the observer-based integral controller (3.14) as a basic controller, we give a convenient state-space parametrization.

First, we need the following results for the solutions of the matrix equation (3.2) and its dual equation.

Lemma 4.2: (a) The solution of matrices O_δ, J_δ satisfying the matrix equation $[O_\delta \ J_\delta]E_\delta=[LF \ I+LG]$ can be expressed as follows.

$$\begin{aligned} J_\delta &= [D_\delta + (H_\delta - D_\delta L)(I-F+GL)^{-1}G]^{-1} \\ O_\delta &= L + J_\delta(H_\delta - D_\delta L)(I-F+GL)^{-1} \end{aligned} \tag{4.12}$$

(b) For an observer gain matrix K_δ, consider the matrix equation

$$E_\delta \begin{bmatrix} \bar{O}_\delta \\ \bar{J}_\delta \end{bmatrix} = \begin{bmatrix} FK_\delta \\ I+H_\delta K_\delta \end{bmatrix} \tag{4.13}$$

with respect to the matrices $\bar{O}_\delta$ and $\bar{J}_\delta$. Then the solution can be expressed as

$$\begin{aligned} \bar{J}_\delta &= [D_\delta + H_\delta(I-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)]^{-1} \\ \bar{O}_\delta &= K_\delta + (I-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)\bar{J}_\delta \end{aligned} \tag{4.14}$$

Proof: The results are obtained by straightforward matrix calculations. Details are omitted. ■

Using Lemma 4.1 and Lemma 4.2, we can obtain the following result.

Proposition 4.1: Consider the observer-based integral controller (3.14). Let $C_{INT}(z)$ denote the transfer function matrix from $y_\delta(k)$ to $-u(k)$. Then $C_{INT}(z)$ can be expressed in coprime factorizatin forms as

$$C_{INT}(z) = \tilde{U}_\delta^{-1}(z)\tilde{V}_\delta(z) = V_\delta(z)U_\delta^{-1}(z) \tag{4.15}$$

where

$$\tilde{U}_\delta(z) = z^{-1}(z-1)[I+O_\delta(zI-F+K_\delta H_\delta)(G-K_\delta D_\delta)] \quad (4.16)$$

$$U_\delta(z) = z^{-1}(z-1)[I+(H_\delta-D_\delta L)(zI-F+GL)^{-1}\tilde{O}_\delta] \quad (4.17)$$

$$\tilde{V}_\delta(z) = J_\delta + (z-1)O_\delta(zI-F+K_\delta H_\delta)^{-1}K_\delta \quad (4.18)$$

$$V_\delta(z) = \tilde{J}_\delta + (z-1)L(zI-F+GL)^{-1}\tilde{O}_\delta \quad (4.19)$$

The matrices (4.16)-(4.19) are proper and stable and constitute the doubly coprime factorization with the matrices (4.3)-(4.6), *i.e.*,

$$\begin{bmatrix} \tilde{U}_\delta(z) & \tilde{V}_\delta(z) \\ -\tilde{P}_\delta(z) & \tilde{Q}_\delta(z) \end{bmatrix} \begin{bmatrix} Q_\delta(z) & -V_\delta(z) \\ P_\delta(z) & U_\delta(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (4.20)$$

Proof: The left factorization in (4.15) can easily be obtained by considering z-transformations of (3.14) and the observer algorithm (2.5). Although the right coprime factorization can directly be proved, it requires extensive matrix calculations. Instead, we give a constructive proof which requires less calculation.

Consider the doubly coprime factorization (4.11) in Lemma 4.1. Then, for an arbitrary proper and stable matrix $\Theta(z)$, the following identity holds

$$\begin{bmatrix} \tilde{S}_\delta(z)-\Theta(z)\tilde{P}_\delta(z) & \tilde{R}_\delta(z)+\Theta(z)\tilde{Q}_\delta(z) \\ -\tilde{P}_\delta(z) & \tilde{Q}_\delta(z) \end{bmatrix} \times \begin{bmatrix} Q_\delta(z) & -[R_\delta(z)+Q_\delta(z)\Theta(z)] \\ P_\delta(z) & S_\delta(z)-P_\delta(z)\Theta(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (4.21)$$

It follows from the above identity that

$$\begin{aligned} &[\tilde{S}_\delta(z)-\Theta(z)\tilde{P}_\delta(z)]^{-1}[\tilde{R}_\delta(z)+\Theta(z)\tilde{Q}_\delta(z)] \\ &= [R_\delta(z)+\Theta(z)Q_\delta(z)][S_\delta(z)-\Theta(z)P_\delta(z)]^{-1} \end{aligned} \quad (4.22)$$

Consequently, if there exists a stable matrix $\Theta(z)$ such that

$$\begin{aligned} \tilde{U}_\delta(z) &= \tilde{S}_\delta(z) - \Theta(z)\tilde{P}_\delta(z) \\ \tilde{V}_\delta(z) &= \tilde{R}_\delta(z) + \Theta(z)\tilde{Q}_\delta(z) \end{aligned} \quad (4.23)$$

are the left factorization factors of (4.15), then the right hand of (4.22) must be a right factorization of $C_{INT}(z)$. Using the left factorization of (4.16) and (4.18), we can express the matrix $\Theta(z)$ satisfying (4.23) as

$$\Theta(z) = [\tilde{V}_\delta(z) - \tilde{R}_\delta(z)]\tilde{Q}_\delta^{-1}(z) \quad (4.24)$$

From (4.9) and (4.18), we can write the numerator matrix in (4.24) as

$$\begin{aligned} \tilde{V}_\delta(z) - \tilde{R}_\delta(z) = J_\delta[&I+(H_\delta-D_\delta L)(I-F+GL)^{-1}K_\delta] \\ &[I-H_\delta(zI-F+K_\delta H_\delta)^{-1}K_\delta] \end{aligned} \quad (4.25)$$

Using (4.25) and (4.6) in (4.24), we have

$$\Theta(z) = J_\delta[I+(H_\delta-D_\delta L)(I-F+GL)^{-1}K_\delta] \quad (4.26)$$

By simple matrix calculation, we have an expression dual to (2.26) as

$$\Theta(z) = [I+L(I-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)]\tilde{J}_\delta \quad (4.27)$$

The matrix $\Theta(z)$ is apparently proper and stable. We can easily check that the matrix (4.27) also satisfies the first equation of (4.23). Hence, we have shown that there exists $\Theta(z)$ satisfying (4.23).

Substituting (4.26) and (4.27) in the right hand side of (4.22), we can obtain the right factorization factors of $C_{INT}(z)$ as follows

$$\begin{aligned} U_\delta(z) &= S_\delta(z) - \Theta(z)P_\delta(z) \\ &= z^{-1}(z-1)\{I+(H_\delta-D_\delta L)(zI-F+GL)^{-1} \\ &\quad [K_\delta+(I-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)\tilde{J}_\delta]\} \end{aligned} \quad (4.28)$$

$$\begin{aligned} V_\delta(z) &= R_\delta(z) + \Theta(z)Q_\delta(z) \\ &= \tilde{J}_\delta+(z-1)L(zI-F+GL)^{-1}[K_\delta+(I-F \\ &\quad +K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)\tilde{J}_\delta] \end{aligned} \quad (4.29)$$

Using the expressions given in Lemma 4.2, we can easily check that (4.28) and (4.29) reduced to (4.17) and (4.19), respectively. ■

5. Parametrization of Two-Degree-of-Freedom Integral Controllers

The doubly coprime factorization given in the previous section can conveniently be used to obtain parametrization of two-degree-of-freedom integral controllers accounting the sampling skew.

In the following discussion, we denote the set of proper and stable rational matrices by R_S.

For the plant given by (2.2), we consider a two-degree-of-freedom controller described by the transfer function matrices

$$\begin{aligned} C_T(z) &= [\,C_{T1}(z) \quad C_{T2}(z)\,] \\ &= \tilde{D}_{T2}^{-1}(z)[\,\tilde{N}_{T1}(z) \quad \tilde{N}_{T2}(z)] \end{aligned} \quad (5.1)$$

from the input vector $[r'(t) \; -y'(t)]'$ to the control input $u(t)$. Noting that the z-transform of the unit step reference input $r_\delta(t)$ is factored as

$$r_\delta(z) = \tilde{D}_r^{-1}(z)\tilde{N}_r(z) \quad (5.2)$$

where

$$\tilde{D}_r(z) = z^{-1}(z-1)I, \quad \tilde{N}_r(z) = I \tag{5.3}$$

we have the following result as a special case of the parametrization of general two-degree-of-freedom controllers.

Lemma 5.1: Let $T(M_\delta)$ denote the class of stabilizing two-degree-of-freedom controller for the plant M_δ given by (4.1) and a step reference input. Then this class can be parametrized by two free parameters $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$ as

$$\begin{aligned} \tilde{D}_{T2}(z) &= \tilde{U}_\delta(z) - \Theta_2(z)\tilde{R}_\delta(z) \\ \tilde{N}_{T1}(z) &= \tilde{V}_\delta(z) + \Theta_1(z)\tilde{D}_r(z) \\ \tilde{N}_{T2}(z) &= \tilde{V}_\delta(z) + \Theta_2(z)\tilde{Q}_\delta(z) \end{aligned} \tag{5.4}$$

where $\tilde{U}_\delta(z)$, $\tilde{V}_\delta(z)$, $\tilde{Q}_\delta(z)$ and $\tilde{R}_\delta(z)$ are defined in Lemma 4.1 and $\tilde{D}_r(z)$ is defined in (4.3). □

Note that a controller given by the above parametrization does not necessarily guarantee integral action. As a special case of the well known result for general robust tracking problem [7], the necessary and sufficient conditions guaranteeing the integral action can be stated as follows.

Lemma 5.2: The two-degree-of-freedom controller (5.1) solves the discrete-time robust tracking problem for a step reference input if and only if the following three conditions are satisfied.

$$\begin{aligned} &1)\ \ \tilde{D}_{T2}(z)Q_\delta(z) + \tilde{N}_{T2}(z)P_\delta(z) = I \\ &2)\ \ (z-1)^{-1}\tilde{D}_{T2}(z) \in R_S \\ &3)\ \ [\tilde{N}_{T1}(z) - \tilde{N}_{T2}]\tilde{D}_r^{-1}(z) \in R_S \end{aligned} \tag{5.5}$$

□

Let us call a controller satisfying the three conditions by a two-degree-of-freedom integral controller. For our choice of the basic controller, we can obtain the following simple characterization.

Lemma 5.3: Let $T_{INT}(M_\delta)$ denote the class of all two-degree-of-freedom integral controllers for the plant M_δ given by (4.1). Then the controller belonging to this class can be represented by the parametrization of $T(M_\delta)$ given in Lemma 5.1 with the free parameters $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$ satisfying $\Theta_2(1)=0$.

Proof: For a controller described by (5.4) in Lemma 5.1 with the free parameters $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$, the three conditions in Lemma 4.2 can be restated as follows.

1) Note that the first condition in Lemma 5.2 is independent of $\Theta_1(z)$. In addition, since the matrices $P_\delta(z)$, $Q_\delta(z)$, $\tilde{U}_\delta(z)$ and $\tilde{V}_\delta(z)$ satisfy the relation of the doubly coprime factorization, the first condition in Lemma 5.1 is satisfied for any $\Theta_1(z) \in R_S$. Consequently, the first condition is satisfied for arbitrary free parameters in R_S.

2) Substituting (5.4) into the second condition in Lemma 5.2, we have

$$\begin{aligned} &z^{-1}[\,I + O_\delta(zI-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)] \\ &\quad + z^{-1}(z-1)^{-1}\Theta_2(z)[\,D_\delta + H_\delta \\ &\quad (zI-F+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)] \in R_S \end{aligned} \tag{5.6}$$

Since the first term belongs to R_S and the zeros of $H_\delta(zI-A+K_\delta H_\delta)^{-1}(G-K_\delta D_\delta)$ do not locate at $z=1$ by the assumption $\det E_\delta \neq 0$, the second condition in Lemma 5.2 is equivalent to

$$z^{-1}(z-1)^{-1}\Theta_2(z) \in R_S \tag{5.7}$$

3) Using (5.3) and (5.4) in the third condition in Lemma 5.2, we obtain

$$\Theta_1(z) - z(z-1)^{-1}\Theta_2(z)\tilde{Q}_\delta(z) \in R_S \tag{5.8}$$

which is equivalent to

$$z(z-1)^{-1}\Theta_2(z)\tilde{Q}_\delta(z) \in R_S \tag{5.9}$$

under the assumption $\Theta_1(z) \in R_S$.

Assume that the controller described by $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$ achieves robust tracking. Then, it follows from (5.7) and (5.9) that $\Theta_2(1)=0$. Conversely, if we consider a controller described by the parametrization given in Lemma 5.1 with $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$ satisfying $\Theta_2(1)=0$, then the condition (5.7) and (5.9) are satisfied. Consequently, the three condition in Lemma 5.2 are equivalent to $\Theta_1(z) \in R_S$ and $\Theta_2(z) \in R_S$ satisfying $\Theta_2(1)=0$. ■

Although the above result gives a simple parametrization of the integral controllers, the free parameter $\Theta_2(z)$ must satisfy the constraint. The following result gives a simple trick to remove the constraint.

Lemma 5.4: In the parametrization $T(M_\delta)$ given in Lemma 5.1, replace the free parameter $\Theta_2(z)$ by

$$\Theta_2(z) = z^{-1}(z-1)\Theta_{FB}(z) \tag{5.10}$$

where $\Theta_{FB}(z) \in R_S$. Then the class $T_{INT}(M_\delta)$ is parametrized by $\Theta_1(z) \in R_S$ and $\Theta_{FB}(z) \in R_S$ both of which are arbitrary in R_s. □

From the above lemmas, we obtain the following result clarifying the relation between the above parametrization and the basic integral controller.

Proposition 5.1: Consider a controller described by the parametrization given in lemma 5.4. Let $u(z)$ denote the z-transform of the control input $u(t)$.

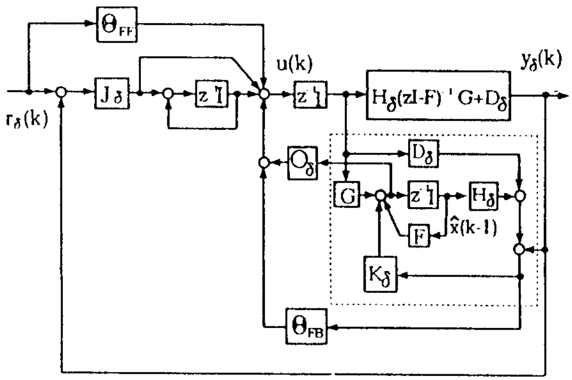

Fig.1 Basic integral controller and free parameters

Then the control input can be expressed as

$$\begin{aligned} u(z) &= z(z-1)^{-1}J_\delta[r_\delta(z)-y_\delta(z)] - O_\delta\hat{x}(z) \\ &\quad -\Theta_{FB}(z)[y_\delta(z)-\hat{y}_\delta(z)] \\ &\quad +\Theta_{FF}(z)r_\delta(z) \end{aligned} \tag{5.11}$$

where

$$\Theta_{FF}(z) = \Theta_1(z) + zO_\delta(zI-F+K_\delta H_\delta)^{-1}K_\delta \tag{5.12}$$

$$\hat{y}_\delta(z) = H_\delta\hat{x}(z) + D_\delta u(z) \tag{5.13}$$

Proof: Using (5.4), we can write the control input of a two-degree-of-freedom integral controller with the free parameters as

$$\begin{aligned} u(z) &= [\tilde{U}_\delta(z) - \Theta_2(z)\tilde{P}_\delta(z)]^{-1} \\ &\quad \Big\{[\tilde{V}_\delta(z)+\Theta_1(z)\tilde{D}_R(z)]r_\delta(z) \\ &\quad + [\tilde{V}_\delta(z)+\Theta_2(z)\tilde{Q}_\delta(z)]y_\delta(z)\Big\} \end{aligned} \tag{5.14}$$

Note that the z-transform of the state estimate generated by the observer (2.5) can be written as

$$\begin{aligned} \hat{x}(z) &= (zI-F+K_\delta H_\delta)^{-1} \\ &\quad [zK_\delta y_\delta(z) + (G-K_\delta D_\delta)u(z)] \end{aligned} \tag{5.15}$$

Using the expressions for the factorization given in Proposition 4.1 and (5.15) in (5.14), we can easily obtain the expression (5.11). ■

Since the term $z(zI-F+K_\delta H_\delta)^{-1}K_\delta$ in (5.13) belongs to R_S, $\Theta_{FF}(z) \in R_S$ if and only if $\Theta_1(z) \in R_S$. Therefore, instead of $\Theta_1(z)$, we can use $\Theta_{FF}(z)$ as a new free parameter in the parametrization. From (5.11), we can readily illustrate the structure of the controller given by the proposed parametrization as in Fig. 1.

6. Conclusions

We have proposed a transparent and efficient design of discrete-time observer-based integral controller accounting sampling skew. Unlike the conventional approach, this design does not require a solution of a higher order regulator problem for an extended system including the integrators.

The free parameters in the parametrization given in Section 5 can be used in various ways to enhance performance. This issue will be discussed elsewhere.

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