# Variable Structure Controller Design for Process with Time Delay

Gwi-Tae park<sup>1)</sup>, Seok-Jin Kim<sup>1)</sup>, Kee-Sang Lee<sup>2)</sup>, Myung-Hyun Song<sup>3)</sup>, Chun-Ping Kuo<sup>4)</sup>, Sung-Ho Kim<sup>5)</sup>

1) Dept. of Electrical Eng., Korea University

<sup>2)</sup>Dept. of Electrical Eng., Dankook University

3) Dept. of Electrical Eng., Sunchon University

<sup>4)</sup>GoldStar Ind. Systems Co.

5) Dept. of Control & Instrumentation, Kunsan University

ABSTRACT: A variable structure control scheme that can be applied to the process with input/output delays are proposed and its control performances are evaluated. The proposed VSCS, which is an output feedback scheme, comprises an integrator for tracking the setpoint and the Smith predictor for compensating the effects of time delay. With The VSCS, the robustness against the parameter variations and external disturbances can be achieved even when the controlled process includes I/O delays. And the desired transient response is obtained by simple adjustment of the coefficients of the switching surface equation.

## 1. Introduction

The variable structure control (VSC) provides the high speed switching input which forces the state trajectory to follow up along the predetermined switching surface. As the VSC has robustness against the parameter variation and disturbance and also has fast response, it has been in broad use for the servo control system.

The VSC was proposed by Russian Emelyanov and his research group in early 1950s, but earnest research and developement for VSC has been activated only after finding the additional characteristics of the VSC and new kinds of control law design in late 1970s. And the initial researches restricted to the 2nd order system which was linearised by modelling of phase parameter have been extended almost to all the control fields including nonlinear system, multi-input multi-output (MIMO) system etc.. But there has been few works on the delayed processes. Some have been reported only by Itkis(1), K.K. Shyu(2), and Young & Rao(3), et al. The work researched by Itkis was able to classify one of VSC methods for the FOPDT process. But it was impossible to accomplish the sliding mode with high speed switching input in the general VSC system because he adopted the on-off type control input. Therefore, it is vey hard to expect the robustness which is the most important characteristic of the VSC, and no progress on this kind of VSC has been reported after Itkis.

Recently, K.K.Shyu, et al have proposed a stabilizatin scheme based on the VSC theory including the stability conditions for uncertain time delayed systems. But he mentioned only about stability problem having the delay of the states without considering the performance of the control system and also without presenting the counterplan of the input/output delay. The works researched by Young & Rao were about the application to the chemical process without modifying the existing VSC method.

The problems caused from the above works could be summarised as follows.

First, a new VSC design method based on the time delayed first order plus dead time (FOPDT) or second order plus dead time (SOPDT) model is required because many real processes are expressed by them.

Second, mismatch problem caused by difference between parameters of the real process and model has not been considered so far, that the VSC method which should guarantee the stability and robustness of the system under any situation consequentially has not been developed yet. Therefore, the problem must be solved to secure the stability and performance of the variable structure control system (VSCS).

This paper for solving the above problems proposes one of the output feedback VSCS for FOPDT and SOPDT processes and analyzes its performance. The proposed VSCS consists of a Smith predictor for compensating the time delay, an integrator for tracking the reference input, and a variable structure controller.

## 2. Problem Representation

The system model is taken by two methods. One is a physical modelling method by which a state space model is derived by the direct application of physical laws. Another one is an identification method of the transfer function expressing the input/output relationship at the frequency region based on the input/output data. The state space model is very useful for system anaysis and design if exact parameters are known, but it is very hard to get the exact parameters from the multivariable, high order, and large scaled systems. Therefore, the identification method of the transfer function is prefered in this case. Also this is more useful as long as designing the controller is concerned even if the state space model is achieved. Especially it is well known that PI(D) controller based on the FOPDT and SOPDT the industrial process is generally in use.

The FOPDT and SOPDT processes are expressed as follows.

$$G_{pl}(s) = \frac{Y_{pl}(s)}{U_1(s)} = \frac{k_{pl}e^{-r_{pl}s}}{T_{pl}s+1}$$
 (1a)

$$G_{p2}(s) = \frac{Y_{p2}(s)}{U_2(s)} = \frac{k_{p2}e^{-r_{p3}s}}{(T_{p3}s+1)(T_{p3}s+1)}$$
(1b)

where  $T_{p1}$ ,  $T_{p2}$ ,  $T_{p3}$ ,  $k_{p1}$ ,  $k_{p2}$ ,  $t_{p1}$ ,  $t_{p2}$  mean the time constants, gains and delay time of the 1st and 2nd order processes respectively and the subscript p means a process. The model for the above process is taken generally from the

measured data, and its equivalent corresponding to (la)(lb) can be assumed as follows.

$$G_{ml}(s) = \frac{Y_{ml}(s)}{U_{l(s)}} = \frac{k_{ml}e^{-r_{ml}s}}{T_{ml}s+1}$$
 (2a)

$$G_{m2}(s) = \frac{Y_{m2}(s)}{U_2(s)} = \frac{k_{m2}e^{-\tau_{mas}}}{(T_{m2}s+1)(T_{m3}s+1)}$$
 (2b)

where  $T_{m1}$ ,  $T_{m2}$ ,  $T_{m3}$ ,  $k_{m1}$ ,  $k_{m2}$ ,  $r_{m1}$ ,  $r_{m2}$  mean the time constants, gains and delay time of 1st and 2nd order process models respectively and the subscript m means a model.

The servo problem to be dealt in this paper concerns the establishment of the design method of the VSCS tracking to the reference input with the output  $y_p(t)$  of (1a),(1b) based on (2a),(2b). The most important point in this problem is to design a VSCS including the robustness and the stability under conditions of parameter mismatch between the process and model, parameter variation, and disturbance.

## 3. Variable Structure Control System

In this chapter, we propose a VSCS which forces the time delayed system to get the sliding mode, and present the design method for the switching surface and the switching control input for the FOPDT and SOPDT.

#### 3-1 Construction of VSCS

The proposed VSCS adopts an integrator for servo parameter to perform the tracking action and includes a Smith predictor for estimating the state vector which is necessary to construct the switching fuction and control input as well as processing the effect of the time delay effectively. Fig. 1 shows the overall structure of the proposed VSCS.

The utmost outer loop is for compensating the reduction of the control performance caused by the mismatch between parameters of the process and model, e.g. deviation caused by the time constant, DC gain, time delay, and disturbance. The inner loop is for compensating the effect of time dely by making the estimated value to feedback. Therefore, the proposed VSCS equips the sliding mode which is robust to disturbances and parameter variations.

# 3-2 VSCS for FOPDT process

We first propose a design method for the 1st order process with time delay. For the convenience of expression, (2a) is expressed with state space as follows.

$$\dot{x_m}(t) = -a_m x_m(t) + \dot{b}_m u(t)$$

$$y_m(t) = x_m (t - \tau_m)$$
(3)

The design process of the VSCS consists of a step for forming the extended model including the servo variable, a step for designing the switching surface, and a step for determining the switching input.

## (1) Construction of extended system

Let's take the servo variable x, as follows.

$$\dot{x_s} = R - x_m - (y_p - y_m) \tag{4}$$

where  $x_m$ ,  $y_p$ ,  $y_m$  are predicted values by the Smith predictor, process output, and model output respectively. The extended system for designing the VSCS can be taken by defining the  $x_1=x_a$ ,  $x_2=x_m$  from (3)(4) as follows.

$$\dot{x}_1 = -x_2 - (y_p - y_m) + R 
\dot{x}_2 = -a_m x_2 + b_m u$$
(5)

(2) Design for switching surface and equivalent control input

The switching surface  $\sigma$  is derived by using (5) as follows.

$$\sigma = c_1 x_1 + c_2 x_2 \tag{6}$$

To get the design condition, lets's find arrange the  $\dot{\sigma} = 0$  for the U and the equivalent control input Ueq.

$$\sigma = c_1 x_1 + c_2 x_2 
= c_1 (R - x_2 - y_p + y_m) + c_2 (-a_m x_2 + b_m U) = 0 
U_{eq} = -\frac{1}{c_2 b_m} \left[ c_1 (R - x_2 - y_p + y_m) - c_2 a_m x_2 \right]$$
(7)

Let's define the  $k_1 = -\frac{c_1}{c_2 b_m}$ ,  $k_2 = \frac{a_m}{b_m}$ , then Ueq becomes

to 
$$U_{eq} = k_1(R - x_2 - y_p + y_m) + k_2 x_2$$
 (8)

or 
$$U_{eq} = k_1(R - y_p + y_m) + (k_2 - k_1)x_2$$
 (8')

The equivalent control system receiving the equivalent input turns to be like fig. 2. The general characteristics of the equivalent control system can be analyzed by following next procedures.

The transfer functions of the  $\frac{Y_p(s)}{R(s)}$ ,  $\frac{Y_p(s)}{T_L(s)}$  can be

taken as follows.

$$\frac{Y_p(s)}{R(s)} = \frac{k_1 \frac{b_p}{s + a_p} e^{-r_s s}}{1 + \frac{b_m}{s + a_m} (k_1 - k_2) + \frac{k_1 b_p}{s + a_p} e^{-r_s s} - \frac{k_1 b_m}{s + a_m} e^{-r_s s}}$$
(9)

$$\frac{Y_{p}(s)}{T_{Ls}(s)} = 1 - \frac{k_{1} \frac{b_{p}}{s + a_{p}} e^{-r_{p}s}}{1 + (k_{1} - k_{2}) \frac{b_{m}}{s + a_{m}} + \frac{k_{1}b_{p}}{s + a_{p}} e^{-r_{p}s} - \frac{k_{1}b_{m}}{s + a_{m}} e^{-r_{p}s}} \times \frac{b_{p}}{s + a_{p}} e^{-r_{p}s}$$
(10)

Since we do not know about  $G_p(s)$  when designing the control system, let's assume  $r_m = r_p$ ,  $b_m = b_p$ ,  $a_m = a_p$ . Then (9) becomes to

$$\frac{Y_{p}(s)}{R(s)} = \frac{\frac{k_{1}b_{m}}{s+a_{m}}e^{-r-s}}{1+(k_{1}-k_{2})\frac{b_{m}}{s+a_{m}}} = \frac{-\frac{c_{1}}{c_{2}b_{m}}\frac{b_{m}}{s+a_{m}}e^{-r-s}}{1+(-\frac{c_{1}}{c_{2}b_{m}}-\frac{a_{m}}{b_{m}})\frac{b_{m}}{s+a_{m}}}$$

$$= \frac{-\frac{c_{1}}{c_{2}}}{s-\frac{c_{1}}{c_{2}}}e^{-r-s}$$
(11)

Equation (11) gives the slelecting reference for the coefficient of the switching function. It means that  $c_1,c_2$  must be selected to have a relationship of  $c_1/c_2 < 0$ , and  $-\alpha = c_1/c_2$  must be a desired pole of the equivalent control system. Therefore, when selecting the  $[c_1,c_2]$  satisfying these conditions, next equation is established.

$$\lim_{t \to \infty} y_p(t) = R \tag{12}$$

The disturbance effect in the equivalent control system can be analyzed by (10). When assuming the perfect model as above, we can get relationship like the next from (10).

$$\frac{Y_{p}(s)}{T_{Is}(s)} = \left(1 - \frac{\frac{k_{1}b_{m}}{s + a_{m}} e^{-r_{m}s}}{1 + (k_{1} - k_{2}) - \frac{b_{m}}{s + a_{m}}}\right) - \frac{b_{m}}{s + a_{m}} e^{-r_{m}s}$$

$$= \left(1 - \frac{a}{(s + a)} e^{-r_{m}s}\right) - \frac{b_{m}}{s + a_{m}} e^{-r_{m}s}$$
(13)

where  $y_m=x_2(t-r_m)$  and  $y_n$  is a measured output.

In case of  $T_L(s) = \frac{k}{s}$ , next relationship is established.  $\lim_{t \to \infty} y_p(t) = 0$  (14)

As a result, we can conclude that the proposed control system tracks the set point regardless of disturbance if we take the perfect model and the process is stable.

## (3) Existence condition and Switching control input

In this chapter, we select a switching control input and verify the existence of the sliding mode.

$$U = U_{eq} - k_d \operatorname{sign}(\sigma) = U_{eq} - k_d(\sigma / \| \sigma \|^2)$$
 (15)

Let's take the Lyapunov function as follows.

$$H(\sigma) = \frac{1}{2} \sigma^2 \tag{16}$$

In this case, the existence condition is  $\sigma \dot{\sigma} < 0$ .

$$\dot{\sigma} = c_1 \dot{x}_1 + c_2 \dot{x}_2 
= x_1 (R - x_2 - y_p + y_m) + c_2 (-a_m x_2 + b_m \mu) 
= c_2 b_m (-k_d \frac{\sigma}{\|\sigma\|^2})$$
(17)

$$\sigma \dot{\sigma} = -c_2 b_m k_d \frac{\sigma^2}{\parallel \sigma \parallel^2} \tag{18}$$

Therefore, the existence conditions for the sliding mode are always satisfied if  $c_2b_mk_d>0$ .

#### 3-3. VSCS for SOPDT process

The model of (2b) turns to be like fig. 3 when expressing with the 1st order cascade representation, and to be as follows when turning to the state space model.

where 
$$b_m = \frac{1}{T_{m3}}, a_m = \frac{1}{T_{m2}}, c_m = \frac{k_m}{T_{m2}T_{m3}}$$

The processes to get the equation for the extended system, design for the switching surface, and determination of the control input are same to the above.

## (1) Construction of Extended system

Let's take the servo variable x, as follows.

$$\dot{x_s} = R - x_{ml} - (y_p - y_m)$$
 (4')

where  $x_{m1}$ ,  $y_p$ ,  $y_m$  are the predicted value, process output, and model output respectively. The design for the extended system can be expressed as follows if  $x_1=x_a$ ,  $x_2=x_{m1}$ ,  $x_3=x_{m2}$  in (4') and (19).

$$x_1 = -x_2 - (y_p - y_m) + R$$

$$x_2 = -b_m x_2 + x_3$$

$$x_3 = -a_m x_3 + c_m U$$
(20)

where  $y_m=x_2(t-r_m)$  and  $y_p$  is a process output.

(2) Design for switching surface and equivalent control input

The switching input  $\sigma$  is selected as follows by using (20).

$$\sigma = c_1 x_1 + c_2 x_2 + c_3 x_3 \tag{21}$$

In the design conditions for  $c_1$ ,  $c_2$ , and  $c_3$ ,  $\sigma = 0$  should be solved from the condition of the sliding mode, and arranged against to the U for taking the equivalent control input Ueq.

$$\dot{\sigma} = c_1 \dot{x}_1 + c_2 \dot{x}_2 + c_3 \dot{x}_3 
= c_1 (R - x_2 - y_p + y_m) - c_2 b_m x_2 + (c_2 - c_3 a_m) x_3 + c_3 c_m U = 0$$
(22)

$$U_{eq} = -\frac{1}{c_3 c_m} \left[ c_1 (R - x_2 - y_p + y_m) - c_2 b_m x_2 + (c_2 - c_3 a_m) x_3 \right]$$

or 
$$U_{eq} = -\frac{1}{c_3 c_m} \left[ c_1 (R - y_p + y_m) - (c_1 + c_2 b_m) x_2 + (c_2 - c_3 a_m) x_3 \right]$$
 (23)

From above equation, if we take

$$k_1 = -\frac{c_1}{c_3 c_m}, k_2 = \frac{c_2 b_m}{c_3 c_m}, k_3 = -\frac{(c_2 - c_3 a_m)}{c_3 c_m}$$
(24)

then Ueq can be expressed as follows.

$$Ueq = k_1(R - x_2 - y_p + y_m) + k_2 x_2 + k_3 x_3$$
  
=  $k_1(R - y_p + y_m) + (k_2 - k_1)x_2 + k_3 x_3$  (25)

The fig. 4 shows the structure of equivalent control system expressed by (25). The transfer function of  $\frac{Y_{P(s)}}{R(s)}$  in fig. 4 can be derived as follows.

$$\frac{Y_{p}(s)}{R(s)} = \frac{k_{1} \frac{c_{p}}{(s+a_{p})(s+b_{p})} e^{-t_{p}t}}{1 - (\frac{c_{m}k_{3}}{s+a_{m}} + \frac{c_{m}(k_{2}-k_{1})}{(s+a_{m})(s+b_{m})} - \frac{c_{m}k_{1}e^{-t_{p}t}}{(s+a_{m})(s+b_{m})} - \frac{c_{p}k_{1}e^{-t_{p}t}}{(s+a_{p})(s+b_{p})}}$$

Because we can not know about  $G_p(s)$  when designing the control system, let's assume  $\tau_m = \tau_p$ ,  $b_m = b_p$ ,  $a_m = a_p$  and  $c_m = c_p$ . Then (26) becomes to

$$\frac{Y_{p}(s)}{R(s)} = \frac{c_{mk1e}^{-t_{m1}}}{s^{2} + (a_{m} + b_{m} - c_{mk3})s + (a_{m}b_{m} - c_{m}b_{m}k_{3} - c_{m}(K_{2} - K_{1}))}$$
(27)

As we know from (27), the effect of time delay does not influence the characteristic equation at all. When applying the (24) to (27),

$$\frac{Y_p(s)}{R(s)} = \frac{c_m k_1 e^{-\tau_m s}}{s^2 + (a_m + b_m - c_m k_3)s + c_m K_1}$$
(28)

The equation (28) gives the selecting reference for coefficient of the switching function. It means that the root of the characteristic equation should be located at the left half region of the s plane. Therefore, equation (29) is derived.

$$c_m K_1 = -\frac{C_1}{c_3} > 0$$
 (29)  
 $a_m + b_m - c_m K_3 = b_m + \frac{c_2}{c_3} > 0, \quad \frac{c_2}{c_2} > -b_m$ 

If the condition of (29) is satisfied, the final value theorem can be applied to (28). And the next equation for the step input whose magnitude is R is established.

$$\lim_{t \to \infty} y_{\rho}(t) = \lim_{s \to 0} s Y_{\rho}(s)$$

$$= \lim_{s \to 0} s \frac{R}{s} \frac{c_{m} k_{1} e^{-\tau_{m} s}}{s^{2} + (a_{m} + b_{m} - c_{m} k_{3})s + c_{m} K_{1}} = R$$
(30)

We can identify the tracking characteristic against to the step input from (30). The transfer function  $\frac{Y_{\rho}(s)}{T_{\ell}(s)}$  can be derived from fig. 4.

$$\frac{Y_{p}(s)}{T_{L}(s)}$$

$$= \frac{\frac{C_{\ell}}{(s+\sigma_{p})[s+b_{p}]} e^{-t_{p}t}(1 - \frac{C_{p}k_{2}}{s+\sigma_{m}} - \frac{C_{m}(k_{2}-k_{1})}{(s+\sigma_{m})[s+b_{m}]} - \frac{C_{p}k_{1}e^{-t_{p}t}}{(s+\sigma_{m})[s+b_{m}]})}{1 - (\frac{C_{p}k_{2}}{s+\sigma_{m}} + \frac{C_{m}(k_{2}-k_{1})}{(s+\sigma_{m})[s+b_{m}]}) + \frac{C_{p}k_{1}e^{-t_{p}t}}{(s+\sigma_{m})[s+b_{m}]} - \frac{C_{p}k_{1}e^{-t_{p}t}}{(s+\sigma_{p})[s+b_{p}]}}$$
(31)

When assuming  $c_m=c_p$ ,  $a_m=a_p$ , and  $b_m=b_p$  in (31), it becomes to

$$\frac{Y_{P}(s)}{T_{L}(s)} = \frac{(s^{2} + (a_{m} + b_{m} - c_{m}K_{3})s + c_{m}K_{1} - c_{m}K_{1}e^{-(s-t)})}{s^{2} + (a_{m} + b_{m} - c_{m}K_{3})s + (a_{m}b_{m} - c_{m}b_{m}K_{3} - c_{m}(K_{2} - K_{1})) \cdot (s + a_{m})(s + b_{m})} (32)$$

$$= \frac{(s^{2} + (a_{m} + b_{m} - c_{m}K_{3})s + c_{m}K_{1}(1 - e^{-(s-t)})}{s^{2} + (a_{m} + b_{m} - c_{m}K_{3})s + c_{m}K_{1}} \cdot \frac{c_{m}e^{-(s-t)}}{(s + a_{m})(s + b_{m})}$$

$$\lim_{t \to \infty} y_p(t) = \lim_{s \to 0} s Y_p(s)$$

$$= \lim_{s \to 0} s \frac{c_m k_1 (1 - e^{-\tau_m s})}{c_m K_1} \frac{c_m e^{-\tau_m s}}{a_m b_m} \frac{T_t}{s} = 0$$
(33)

If the  $T_L$  is a general step input( disturbance ) and the model is perfect, (32) means that the final value is not influenced by the injected disturbance. Therefore, the proposed VSCS can track the objective value well regardless of disturbance injection if the VSCS is designed to satisfy the condition of (29).

(3) Existance condition of Sliding mode and Switching control input

In this chapter, we select a switching control input of the following form to show that the sliding mode exists under the control input.

$$U = U_{eq} - K_d sign(0)$$

$$= U_{eq} - K_d - \frac{\sigma}{\parallel \sigma \parallel}$$
(3.4)

Let's take the following Lyapunov function.

$$H(\sigma) = \frac{1}{2}\sigma^2 \tag{35}$$

In this case, the existence condition of the sliding mode comes to be  $\sigma$   $\dot{\sigma} < 0$ .

$$\sigma \dot{\sigma} = (C_1 \dot{x}_1 + C_2 \dot{x}_2 + C_3 \dot{x}_3) \sigma$$

$$= -C_3 C_m K_{cl} \frac{\sigma^2}{\parallel \sigma \parallel^2}$$
(36)

To satisfy the existence condition for (36), the equation (37) must be established.

$$C_3 C_m K_d > 0 \tag{37}$$

The particular points on the above existence condition and switching control input are that they comprise parameters of the nominal model or Smith predictor, and all the uncertainties are included in the (  $y_0{}^{\dots}\ y_m$  ) term. Therefore, the existence condition of the sliding model is always established as long as the term (  $y_0{}^{\dots}\ y_m$  ) is fed back regardless of the deviation and uncertainty of all the model parameter.

## 4. Simulation

In this chapter, we design a VSCS for the proposed FOPDT and SOPDT process and evaluate its performance with the computer simulation.

## (1) FOPDT process

We design the proposed VSCS for the FOPDT including the parameters of  $a_m{=}2.0,\ b_m{=}1.0,\ r_m{=}1.0$ , and examine the mismatch characteristics when  $a_p,\ b_p,\ r_p$  of the process are mismatched to  $a_m,\ b_m,\ r_m$  of the model.

The switching surface is designed to follow  $c_1/c_2 < 0$ , e.g.  $c_1 = -1.0$  and  $c_2 = 1.0$ , and  $k_1 = -\frac{c_1}{c_2 b_m} = 1, k_2 = \frac{a_m}{b_m} = 2$  are determined.

mined by the values of  $c_1$ ,  $c_2$ ,  $a_m$ ,  $b_m$ .  $K_a$  in the control input of (15) is taken to be 1.0.

Fig. 5 shows the responses when the parameters of the process and model are matched perfectly. The curve of a) at which  $a_p=2.0$ ,  $b_p=1.0$  and  $r_p=1.0$  become a reference, and the curves of b), c), and d) show each response only when each parameter is doubled than the corresponding parameter of a) respectively. That is only when  $a_p=4.0$  in b),  $b_p=2.0$  in c), and  $r_p=2.0$  in d) while the other parameters are not varied. The curve e) shows the response when all the parameters are doubled than a). These responses indicate that the proposed system has a desired stability and robustness even when each parameter is increased or all parameters are mismatched. The numbers under figures mean the values of  $a_p$ ,  $b_p$ ,  $r_p$ ,  $c_1$ , and  $c_2$  in sequence.

Fig. 6 shows the responses occurring when each parameter is reduced as much as 50 % than the reference in b), c), and d) but all parameters are reduced to half in e). As in the fig. 5, the system is stable and robust.

Fig. 7 shows the robustness under the disturbance injection. The proposed system expressed by (13) keeps tracking the objective even when the step—disturbance is entered under mismatched conditions of  $a_p=1$ ,  $b_p=0.5$ ,  $c_p=0.5$ ,  $c_1=-1.0$ , and  $c_2=1.0$  in c). The disturbance of the magnitude of 0.2 is injected at t=30 sec..

#### (2) SOPDT process

We consider SOPDT process whose parameters of the nominal model are  $a_m = 2.0$ ,  $b_m = 1.0$ , and  $r_m = 1.0$ . The switching surface is designed to satisfy the condition of (29), e.g.  $c_1 = -1$ ,  $c_2 = 1.0$ , and  $c_3 = 1.0$ .  $K_d$  in the control input of (34) is taken to be 1.0. In this case, the control input is as follows.

$$U = U_{eq} - k_d sign(\sigma)$$

$$= (R - x_2 - y_p + y_m) + x_2 + x_3 - sign(\sigma)$$
(38)

Fig. 8 shows the responses under  $c_1$ =1.0,  $c_2$ =1.0, and  $c_3$ =1.0. The response a) is for perfect model, and b), c), d) for models having doubled each parameter, but e) for model having all the doubled parameters.

Fig. 9 shows the same case as above but the parameter value is reduced to a half. Fig.8 and fig. 9 indicates the robustness under the parameter mismatch.

Fig. 10 shows the response of the disturbance whose magnitude is 0.2, entered at t=30 sec. The response indicates that the system is stable and robust even when the parameter inismatch and the disturbance injection occur simultaneously.

#### 5. Conclusion

A VSCS structured design method for the 1st and 2nd order process with input/output delay is proposed and evaluated in this paper.

The proposed VSCS, which is an output feedback scheme, includes an integrator for tracking the given set-point and the Smith predictor for compensating the effects of time delay.

With the VSCS, the robust preperties of the conventional VSCS against parameter variations and disturbances can be achieved even when the controlled process includes input/output delays. And it is simple matter to adjust coefficients of the switching surface equation so as to improve the transient response of the VSCS.

Although the proposed VSCS has the robustness against the parameter variations, mismatches, and external disturbances, there are problems of trade off between relative stability and the responsiveness.

Therefore, the development of a robust design method to compromise the trade off problem is required our research.

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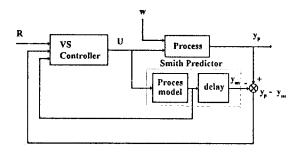


Fig. 1 Proposed control system structure

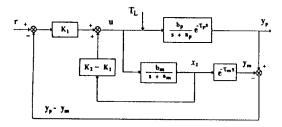


Fig. 2 Equivalent Control System for FOPDT Process

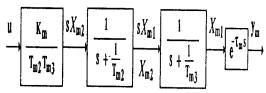


Fig. 3 Cascade first-order representation

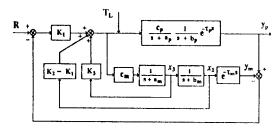


Fig. 4 Equivalent control system for SOPDT Process

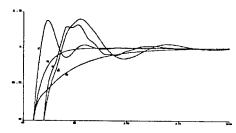


Fig.5 FOPDT Resonse at a) 2,1,1,-1,1 b) 4,1,1,-1,1 c) 2,2,1,-1,1 d) 2,1,2,-1,1 e) 4,2,2,-1,1

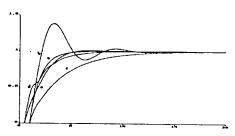


Fig.6 FOPDT response at a) 2,1,1,-1,1, b) 1,1,1,-1,1 c) 2,0.5,1,-1,1 d) 2,1,0.5,-1,1 e) 1,0.5,0.5 -1,1

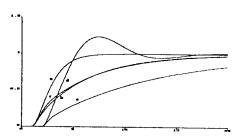


Fig.8 Parameter Mismatch Response of SOPDT at a) 2,1,1,-1,1,1 b) 4,1,1,-1,1,1 c) 2,2,1,-1,1,1 d) 2,1,2,-1,1,1 e) 4,2,2,-1,1,1

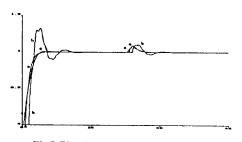


Fig.7 Disturbance response of FOPDT at a) 2,1,1,-1,1 b) 4,2,2,-1,1 c) 1,0.5,0.5,-1,1

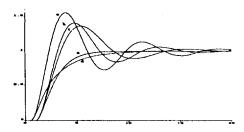


Fig.9 Parameter Mismatch Response of SOPDT at a) 2,1,1,-1,1,1 b) 1,1,1,-1,1,1 c) 2,0.5,1,-1,1,1 d) 2,1,0.5,-1,1,1 e) 1,0.5,0.5,-1,1,1

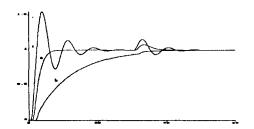


Fig.10 Disturbance Response of SOPDT at a) 2,1,1,-1,1,1,1 b) 4,2,2,-1,1,1 c) 1,0.5,0.5,-1,1,1